Fringe visibility estimators for the Palomar Testbed Interferometer

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ABSTRACT

The **Palomar Testbed Interferometer** (PTI) is a long-baseline infrared **interferometer** located at Palomar Observatory, California. One operational mode of PTI is single-baseline visibility measurement using pathlength modulation with synchronous readout by a NICMOS-3 infrared array. The visibility estimators are similar to those used with photon-counting detectors, except that greater attention is given to correcting biases from the detection process. These include biases attributable to detector offsets and read noise; their effects differ for incoherent and coherent **estimators**. Quality measures such as measured fringe-tracker performance can be used to improve the visibility estimates or their error bars.

fringe visibility

1. Instrument configuration

The Palomar Testbed Interferometer (PTI) (Colavita et al. 1988; Wallace 1998; Colavita et al. 1994) uses coherent fringe demodulation and active fringe tracking, similar to that employed with the Mark III Interferometer (Shao et al. 1988). Differences arise attributable to the use of an infrared array detector with its attendant read noise and required bias corrections.

The beam combiner on PTI accepts the tilt-corrected, delayed beams from the two interferometer apertures. These are combined at a beamsplitter, and the two combined outputs directed to an infrared dewar. One output is imaged onto a single pixel of a NICMOS-3 infrared array. This white-light pixel is band-limited by an astronomical K (2.00–2.40 μm FWHM) filter, yielding an effective wavelength of $\sim$2.2 μm. The other output is dispersed with a prism spectrometer and imaged adjacent to the white-light pixel on the same line of the infrared array. Resolution is variable; one typical configuration uses 7 spectrometer pixels with center wavelengths of 1.993–2.385 μm, yielding average channel widths of 65 nm. The combined light for the spectrometer channels is spatially filtered prior to dispersion with a single-mode infrared fiber. The white-light channel is not explicitly spatially filtered, although some filtering occurs because of the finite pixel size (40 μm pixel and an f/10 relay).

2. Array readout

The infrared array is read out coherently using a 4-bin algorithm with pathlength modulation implemented on the optical delay line. The 100-Hz modulation uses a sawtooth waveform, and the array readout timing varies according to the wavelength of each pixel to achieve a one-wavelength scan for all channels. Clocking constraints and overhead lead to a typical sample integration time of 6.75 ms (out of a sample spacing of 10 ms) for the white-light pixel, scaling proportionally for other wavelengths.

For each sample period, the active and adjacent lines of the array are first cleared, the reset pedestal for each data pixel is read, and each pixel is then read out after each quarter-wave of modulation. Each of these (nondestructive) "reads" is actually an average of 16–64 consecutive 2-μs subreads, used to reduce the effective read noise, typically to a correlated-double-sample (cds) value of 12 e^- for the white-light pixel and 16 e^- for the spectrometer pixels. These 5 reads per sample for the white-light and spectrometer pixels are the fundamental interferometer data.

Denote these 5 reads as z_i, a_i, b_i, c_i, and d_i, where $i = 0$ denotes the white-light pixel and $i = 1 \ldots R$ denote the R spectrometer pixels. The integrated flux in each quarter-wave time bin is calculated as $A_i = a_i - z_i, B_i = b_i - a_i, C_i = c_i - b_i$, and $D_i = d_i - c_i$. From these values, the raw fringe quadratures and total flux (in units of dn) are calculated as

$$X_i = A_i - C_i \tag{1}$$

$$Y_i = B_i - D_i \tag{2}$$

$$N_i = A_i + B_i + C_i + D_i. \tag{3}$$

We can also calculate an energy measure which we denote as

$$\mathrm{NUM}_i = X_i^2 + Y_i^2. \tag{4}$$

From these quantities we can estimate the fringe phase, visibility, and signal-to-noise ratio, but first it is necessary to correct for biases associated with the detection and readout process.

3. Biases

It's convenient to speak of "dc" and "ac" biases. The dc biases are essentially the zero points of A, B, C, and D, and are those values observed with the instrument pointing at dark sky.[1](At high light levels, there are also nonlinearities as the detector saturates, but these effects are small for typical observations.) Expressed in terms of the quadratures and flux, we denote the biases as B^X, B^Y, and B^N, so that the corrected values of these quantities are given as (omitting subscripts for clarity)

$$\widehat{X} = X - B^X \tag{5}$$

$$\widehat{Y} = Y - B^Y \tag{6}$$

$$\widehat{N} = N - B^N. \tag{7}$$

We can also correct NUM for the dc biases as

$$\widehat{\mathrm{NUM}} = \mathrm{NUM} - B^X(2\widehat{X} + B^X) - B^Y(2\widehat{Y} + B^Y). \tag{8}$$

This is equivalent to simply computing $\widehat{\mathrm{NUM}}$ as $\widehat{X}^2 + \widehat{Y}^2$.

There are additional ac biases which apply to $\widehat{\mathrm{NUM}}$ and arise from the squaring of the photon and read noise. Fundamentally, the two terms are given as

$$B^{\mathrm{pn}} = k\widehat{N} \tag{9}$$

and

$$B^{\mathrm{rn}} = 4k^2\sigma^2_{\mathrm{cds}}. \tag{10}$$

In the first equation, k is the gain per pixel in units of dn per electron, typically 0.11 for the PTI array electronics, and $\widehat{N}$ is the corrected flux in dn. This term is the

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[1]With an ideal detector, these biases would be identical, proportional to the dark current and background. In practice, the biases on A, B, C, and D are slightly different.

standard photon-counting bias. In the second equation, $\sigma_{\rm cds}$ is the detector read noise (correlated-double-sample) in dn, with the factor of 4 arising from the 4 bins used to compute NUM. We are usually read-noise limited on the channels of interest, in which case $B^{\rm rn}$ dominates. Correcting $\widehat{\rm NUM}$ for the ac biases yields

$$\widehat{\widehat{\rm NUM}} = \widehat{\rm NUM} - B^{\rm pn} - B^{\rm rn}. \tag{11}$$

4. Bias measurements

The biases for each pixel are measured at the beginning of each night of observation. While these initial values are adequate for proper operation of the real-time system, the biases are also measured repeatedly throughout the night for use in the science data processing.

Initial calibrations A low-level calibration measurement *cal-low* is made at the beginning of each night with the instrument pointed at dark sky. The bias terms B^X, B^Y, and B^N are computed simply as the measured values of X, Y, and N. The bias term $B^{\rm rn}$ is computed as the mean value of $\widehat{\rm NUM}$. This term also incorporates that fraction of the photon-noise bias attributable to finite dark count and background.

A high-level calibration measurement *cal-high* is done at the beginning of each night using an internal white-light source, which illuminates the white-light and spectrometer pixels. The increased value of $\widehat{\rm NUM}$ with light level is used to estimate the pixel responsivity as

$$k = \left(\widehat{\rm NUM} - B^{\rm rn}\right)/\widehat{N}, \tag{12}$$

so that $B^{\rm pn}$ can be computed for other light levels using Eq. 9. These values of B^X, B^Y, B^N, $B^{\rm rn}$, and k are used by the real-time system.

On-going calibrations Repeated measurements of the bias terms throughout the night accommodate drifts and improve the quality of the final data processing. Each typically 125-s scan on a science object is bracketed by several other calibration measurements: total-flux *foreground* and single-aperture *ratio* calibrations precede the scan; a *background* calibration, typically 25-s long, follows it.

A foreground measurement observes the target with the instrumental pathlengths intentionally mismatched to yield zero fringe contrast. In this case, the observed value of $\widehat{\mathrm{NUM}}$ can be used as a direct estimate of the sum $B^{\mathrm{pn}} + B^{\mathrm{rn}}$. The foreground calibration can also be used to estimate B^X and B^Y.

A ratio calibration measurement observes the target with one aperture blocked. Combined with the total flux measured above, the intensity ratio between the interferometer arms can be estimated.

A background measurement is essentially a low-level calibration measurement taken close in time to the stellar observation, and as such provides an estimate of B^X, B^Y, B^N, and B^{rn}.

These five calibration types can be used in different ways in the final data analysis. Typically, the three dc biases and B^{rn} for each scan are estimated from the associated background measurement, while B^{pn} is calculated from the actual flux during the scan using Eq. 9. Averaging of several nearby background measurements using a median filter generally improves the calibration quality. The current data processing pipeline normally uses the foreground and ratio values only as diagnostics.

5. Incoherent estimators

Given the bias-corrected values $\widehat{X}, \widehat{Y}, \widehat{N}$, and $\widehat{\widehat{\mathrm{NUM}}}$ for the white-light and spectrometer channels, we can estimate fringe visibility. Below we adopt a nomenclature for time intervals: a *scan* is a single measurement of an astronomical target, typically 120–150 s of recorded data, accompanied by local calibration measurements as described above. A scan is divided into *blocks*, typically 25 s in length; the fluctuations of estimators among the blocks of a scan provides an estimate of their internal errors. Each block comprises a number of *frames*, which are typically 0.5 s long, and synchronized to the half-second tick. The significance of a frame is that intentional fringe hops to correct unwrapping errors in the real-time system are introduced only at frame boundaries. Each frame consists typically of up to 50 *samples*, which are data at the fastest rate in the system, typically 10 ms, which is of order of the atmospheric coherence time. The actual number of samples per frame will be less than 50 if fringe acquisition or loss occurs mid-frame; partial frames with less than typically 10 samples are discarded in the data processing. Squared visibility V^2 is estimated for each channel as (see Mozurkewich et al. 1991)

$$V^2 = \frac{\pi^2}{2} \frac{\langle\langle \widehat{\widehat{\mathrm{NUM}}} \rangle\rangle}{\langle\langle \widehat{N} \rangle\rangle^2}, \tag{13}$$

where $\langle\langle\rangle\rangle$ represents an incoherent average over a block.[2] While we're usually not photon-noise limited, the photon-noise-limited SNR is estimated similarly as

$$\mathrm{SNR}^2 = 2\frac{\langle\langle \widehat{\widehat{\mathrm{NUM}}} \rangle\rangle}{\langle\langle \widehat{N} \rangle\rangle}. \tag{14}$$

The fringe phase is estimated as

$$\phi = \tan^{-1} \frac{\widehat{Y}}{\widehat{X}}, \tag{15}$$

[2]With step, rather than fringe-scanning modulation, the leading coefficient of Eqs. 13, 16, and 20 would be 4.0.

where we make no attempt to be rigorous with respect to phase offset. These estimates can be made for each channel: we typically use the suffix *wl* to refer to the white-light channel, viz. $V^2_{\rm wl} = V^2_0$. For the spectrometer channels $1 \ldots R$, we also compute a composite spectrometer visibility $V^2_{\rm spec}$ as

$$V^2_{\rm spec} = \frac{\pi^2}{2} \frac{\sum_i \langle\langle \widehat{\widehat{\rm NUM}_i} \rangle\rangle W_i}{\sum_i \langle\langle N_i \rangle\rangle^2 W_i}. \tag{16}$$

The range of the summation covers channels $1 \ldots R$, or a subset (for example, $2 \ldots (R-1)$, which excludes the lower-flux channels at the band edges). The weights W_i can be uniform, but are typically computed as $W_i = N_i^2/\sigma^4_{i,\rm cds}$, which are proportional to $1/\sigma^2_{V^2}$, as discussed below. This composite estimator provides an improved signal-to-noise ratio, and is useful for compact sources where visibility changes with wavelength are smaller than the estimator noise. However, it still retains the wide fringe envelope (and thus decreased sensitivity to visibility errors caused by fringe-tracking errors) corresponding to the narrow spectral channels of the spectrometer; the use of the weights is useful for accommodating occasional spectrometer pixels with large read noises. For consistency, when the composite visibility is used for science, a composite wavelength computed with the same weighting is also employed. At the block level, the SNR of the V^2 estimates is usually sufficiently high that the final V^2 estimate for the scan is calculated as a simple average of the block V^2 values, rather than carrying numerator and denominator separately.

6. Coherent estimators

We refer to the previous estimators as incoherent, in that NUM, the sum of the square of the fringe quadratures, is computed and summed over the 10-ms samples; these are generally our default estimators. Alternatively, we can coadd the fringe phasors over multiple samples before computing NUM and related quantities, providing improved signal-to-noise ratio (Shao & Colavita 1992a), but at the expense of some atmospheric bias. To coadd the fringe phasors requires a phase reference, for which we use the white-light phase $\phi_{\rm wl} = \phi_0$.

We can compute a coherent visibility as follows: the white-light phase is scaled by the wavelength ratio between the white-light and channel of interest to yield $\theta_i = \phi_{\rm wl}\lambda_{\rm wl}/\lambda_i$. The fringe quadratures are derotated and averaged as

$$(\widehat{X}_i)_{\rm coh} = \langle \widehat{X}_i \cos\theta_i - \widehat{Y}_i \sin\theta_i \rangle \quad (17)$$

$$(\widehat{Y}_i)_{\rm coh} = \langle \widehat{X}_i \sin\theta_i + \widehat{Y}_i \cos\theta_i \rangle, \quad (18)$$

At PTI, the coadd time is typically one 0.5-s frame, although this is convenient rather than fundamental. A coherent value of NUM is computed as

$$(\widehat{\rm NUM})_{\rm coh} = (\widehat{X})^2_{\rm coh} + (\widehat{Y})^2_{\rm coh} - (B^{\rm pn} + B^{\rm rn})/L, \quad (19)$$

where L is the number of samples in the coadded frame, which reduces the ac bias correction to account for the reduced noise in the coadded quantities. From $(\widehat{\rm NUM})_{\rm coh}$, and $(\widehat{N})_{\rm coh} = \langle \widehat{N} \rangle$, the coherent V^2 can be estimated as

$$(V^2)_{\rm coh} = \frac{\pi^2}{2} \frac{\langle\langle(\widehat{\rm NUM})_{\rm coh}\rangle\rangle}{\langle\langle(\widehat{N})_{\rm coh}\rangle\rangle^2}. \quad (20)$$

A composite V^2 for the spectrometer channels can also be computed as for the incoherent channels, similar to Eq. 16.

Given that the white-light channel has a high SNR, as required for real-time tracking, the coherent white-light V^2 is not an improved estimator because of coherence losses which occur in the phase-referencing process. However, it is valuable as an estimator of at least part of this coherence loss. We can estimate the coherence loss $\Gamma^{\rm a}$ as

$$\Gamma^{\rm a}_i \simeq (V^2_{\rm wl})_{\rm coh}/V^2_{\rm wl}, \quad (21)$$

and we usually divide the coherent spectrometer V^2 values through by this value as a partial correction. To be more exact, one can account for the wavelength difference between the white-light and spectrometer channels by scaling the correction with wavelength as

$$\Gamma_i^{\mathrm{a}} = \exp\left(\left(\frac{\lambda_{\mathrm{wl}}}{\lambda_i}\right)^2 \ln\left(\frac{(V_{\mathrm{wl}}^2)_{\mathrm{coh}}}{V_{\mathrm{wl}}^2}\right)\right), \tag{22}$$

which assumes a simple exponential form for the coherence loss. We note that there are additional coherence losses in phase referencing, some of which are discussed in Sec. 8.2.

7. SNR of the V^2 estimators

The "detection" noise on the V^2 estimator attributable to photon and read noise (as opposed to noise attributable to atmospheric turbulence) is readily calculated. As is usual, we model only noise on NUM, given by Eq. 4, and ignore the smaller noise in N that normalizes NUM in calculating V^2 (Tango & Twiss 1980). The quadratures X and Y are each comprised of two correlated-double-sample reads, so that the variances of X and Y are given as $\sigma_X^2 = \sigma_Y^2 = 2\sigma_{\mathrm{cds}}^2$. For additive Gaussian noise, $\sigma_{X^2}^2 = 2\sigma_X^4 = 8\sigma_{\mathrm{cds}}^4$, and similarly for Y, yielding $\sigma_{\mathrm{NUM}} = 4\sigma_{\mathrm{cds}}^2$. Thus, the standard deviation of the (incoherent) V^2 estimate in the read-noise limit is

$$\sigma_{V^2} = \frac{2\pi^2}{\sqrt{M}}\left(\frac{\sigma_{\mathrm{cds}}}{N}\right)^2, N \ll N_{\mathrm{rn}}, \tag{23}$$

where M is the total number of samples, both temporal and spectral, in the estimate, and is thus applicable to both single-channel and composite (with equal weights) visibility estimates.[3] For arbitrary photon fluxes, read noise can be incorporated into the standard

[3]With step modulation, the leading coefficient of Eqs. 23, 26, 27, and 28 would be 16.0, with similar changes to Eq. 24.

(4-bin) photon-counting result (Tango & Twiss 1980), yielding

$$\sigma^2_{V^2} = \frac{\pi^4}{4MN^4}\left(N^2 + \frac{4}{\pi^2}N^3V^2 + 16\sigma^4_{\rm cds}\right). \tag{24}$$

Thus the read-noise limit applies when $N \ll N_{\rm rn}$, where

$$N_{\rm rn} = \min\left(4\sigma^2_{\rm cds}, 3.4V^{-2/3}\sigma^{4/3}_{\rm cds}\right). \tag{25}$$

A numerical example is illustrative. For the case of a read noise of 16 e^- per pixel, 125 s of data at 10 ms per sample, and 5 spectrometer channels in the spectral composite, a standard deviation of 0.02 requires 32 photons per channel per sample.

For the coherent estimators, the standard deviation is similar. Assume as above that M is the total number of 10-ms samples in the estimate, but that they are first coadded to frames of length L before calculating NUM. In this case

$$\sigma_{(V^2)_{\rm coh}} = \frac{2\pi^2}{\sqrt{ML}}\left(\frac{\sigma_{\rm cds}}{N}\right)^2, N \ll N_{\rm rn}, \tag{26}$$

Thus, the required photon flux for a given accuracy scales as $L^{-1/4}$. With $L = 50$ and the parameters above, an accuracy of 0.02 now requires 12 photons per channel per sample, although, as discussed above, the coherent estimate is more susceptible to systematic biases.

7.1. SNR for bias estimation

Strictly speaking, the above analysis is somewhat simplistic, as it assumes that bias correction adds no additional noise. For low light levels, the largest errors in bias correction are attributable to estimation of the ac biases $B^{\rm pn}$ and $B^{\rm rn}$; errors in their estimation are additive with the noise on NUM as calculated above. However, as the ac biases are computed from NUM measured under known conditions (Sec. 4), the V^2 errors due to imperfect bias estimates can be computed using the expressions above. Thus, for incoherent quantities, the (incoherent) V^2 error due to imperfect bias subtraction is given by

$$\epsilon_{V^2} \simeq 2\pi^2 \frac{2\pi^2}{\sqrt{M_b}} \left(\frac{\sigma_{\rm cds}}{N}\right)^2, N \ll N_{\rm rn}, \tag{27}$$

where M_b is the number of samples used in estimating the ac bias. This expression is strictly accurate only for the read-noise bias $B^{\rm rn}$, or when both ac bias terms are computed from a foreground calibration. However at low photon fluxes, where bias errors are most significant, the read-noise term is dominant and the above expression is a good approximation.

For the coherent V^2, the situation is somewhat better, as the errors in the ac biases are reduced by the number of samples in the coherent average, per Eq. 19. For the ac biases computed incoherently, and applied via Eq. 19, the applicable expression is

$$\epsilon_{(V^2)_{\rm coh}} \simeq \frac{2\pi^2}{\sqrt{M_b L}} \left(\frac{\sigma_{\rm cds}}{N}\right)^2, N \ll N_{\rm rn}, \tag{28}$$

subject to the same caveats at Eq. 27. By way of comparison, if the biases were computed "coherently", i.e., from measured values of $(NUM)_{\rm coh}$, then Eq. 28 would have the same dependence on L as Eq. 26.

Thus, the total "detection" noise on V^2 is the quadrature sum of σ^2 and ϵ^2, and the contribution due to bias estimation can be important. This contribution is generally not important on bright sources where the noise on V^2 is dominated by atmospheric effects. On fainter targets, the relative bias noise can be decreased by incorporating additional calibration data (for example, using background calibrations from a larger time window about the science scan, rather than just its explicitly-associated background), although eventual nonstationarity of the underlying statistics presents a practical limit.

While calibration errors at low light levels are dominated by the ac terms, the errors attributable to the dc bias terms are easily computed: for both incoherent and coherent estimators, the errors $\epsilon^X_{V^2}$, $\epsilon^Y_{V^2}$, and $\epsilon^N_{V^2}$ associated with B^X, B^Y, and B^N are given by

$$\epsilon^X_{V^2} = \epsilon^Y_{V^2} = \frac{\sqrt{2}\pi V}{\sqrt{M_b}} \left(\frac{\sigma_{\rm cds}}{N}\right), N \ll N_{\rm rn} \tag{29}$$

$$\epsilon_{V^2}^{N} = \frac{2V^2}{\sqrt{M_b}}\left(\frac{\sigma_{\rm cds}}{N}\right), N \ll N_{\rm rn}. \tag{30}$$

8. Data quality measures

Inter-block fluctuations of estimated quantities are useful to estimate internal errors. However additional data quality measures are available.

8.1. Lock time

PTI uses a multi-stage algorithm for fringe acquisition and track (Colavita et al. 1988). Essentially, the average SNR must exceed a given threshold for the system to enter the "locked" state; loss of lock and reacquisition occurs if the SNR falls below a second threshold. Fringe data is only recorded when locked; to account for the time delay caused by the memory of the averaging filter in detecting loss of lock, data at the end of a lock is automatically expunged. Thus, with multiple locks, the elapsed time to collect a fixed amount of data in order to complete a scan is increased.

Two heuristic data-quality measures are the fraction of lock time to elapsed time, and the number of separate locks that make up the total data on a scan. For bright stars and good seeing, each scan is comprised of just several long locks. For very faint stars, or with poor seeing, each scan can be comprised of many short locks, reflecting the inability of the system to consistently track the fringe. While visibility can be estimated in all cases, the data quality in the latter case will be inferior. Typically, this poorer data quality is evident in the inter-block fluctuations, in which case the lock-time metric is only advisory.

8.2. Jitter

We can estimate a first-difference phase jitter $\sigma_{\Delta\phi}$ as

$$\sigma_{\Delta\phi}^2 = \langle\langle(\phi_{\rm wl}(n) - \phi_{\rm wl}(n-1))^2\rangle\rangle, \tag{31}$$

where $\phi_{\rm wl}$ is computed from the 10-ms samples. While this quantity is not unbiased with respect to detection noise, successful fringe tracking typically requires an SNR > 5, so that the detection bias on $\sigma_{\Delta\phi}^2$ should be < 0.08.

With an ideal instrument, $\sigma_{\Delta\phi}$ is related to the atmospheric coherence time. Coherence time can be defined in various ways (Buscher 1994). Let $\tau_{0,i}$ denote the structure-function definition of coherence time, viz. that sample spacing for which the phase difference between samples is one radian rms. The structure function depends on time as $D_i(t) = (t/\tau_{0,i})^{5/3}$. For $i = 1$, representing contributions from a single point on the wavefront (the usual adaptive-optics definition), $\tau_{0,1} = 0.314 r_0/W$ for coherence diameter r_0 and constant wind speed W; for $i = 2$, applicable to interferometry, $\tau_{0,2} = 0.207 r_0/W$. Let $T_{0,i}$ to denote the variance definition of coherence time, viz. that time interval for which the phase fluctuations about the interval mean are one radian rms. It is given by $T_{0,1} = 1.235 r_0/W$ and $T_{0,2} = 0.815 r_0/W$, with time evolution $\sigma_i^2 = (T/T_{0,i})^{5/3}$.

Thus, for an ideal instrument, the coherence time $T_{0,2}$ can be estimated as

$$T_{0,2} = 3.94 t/\sigma_{\Delta\phi}^{6/5}, \tag{32}$$

where t is the sample spacing. Fringe motion during the sample integration time blurs the fringe, reducing the visibility. For rapid (with respect to underlying phase motion) fringe scanning, the coherence reduction is related to the high-pass fluctuations about the interval mean, $(\sigma_\phi)_{\rm hp}$, as $\Gamma^b = \exp(-(\sigma_\phi)_{\rm hp}^2)$, or given the coherence definitions above, $\Gamma^b = \exp(-(T/T_{0,2})^{5/3})$. We can write this in terms of the phase jitter as

$$\Gamma^b = \exp\left(-C_\Gamma \sigma_{\Delta\phi}^2\right), \tag{33}$$

with the coefficient C_Γ given by

$$C_\Gamma = \left(\frac{T}{3.94t}\right)^{5/3} . \quad (34)$$

For $T = 6.75$ ms (for the white-light pixel) and $t = 10$ ms, $C_\Gamma = 0.053$.

A more careful calculation of C_Γ can be done for this case (Appendix A), accounting for the finite integration time required to measure $\phi_{\rm wl}$, which yields $C_\Gamma = 0.057$. A similar calculation can be done under the assumption that all phase noise is caused by narrow-band vibrations with frequency $\ll 1/t$; in this case, $C_\Gamma = 0.038$.

When we apply this correction, we usually err on the side of undercorrection by adopting a modest leading coefficient of 0.04. In general, an empirical visibility-reduction coefficient can be adopted from fits to the measured data applicable to the actual atmospheric realization and instrumental configuration. However, for data calibrated with spatially- and temporally-local calibrators (and especially if the calibrators are of similar brightness to the target), the reduction in visibility due to the above temporal effects will be mostly common mode and divide out of the normalized visibility. In this case, the value of the jitter is useful as a measure of the seeing, and indirectly of the data quality. Finally, we note that the coherent V^2 estimates on PTI often exhibit coherence losses larger than predicted from the models above. These may be attributable to different apodizations of the starlight pupil between the spectrometer and white-light sides of the beamsplitter. In particular, the single-mode fiber preceding the spectrometer imposes a Gaussian apodization on the pupil, while the white-light channel—with no explicit spatial filter—imposes a more uniform pupil weighting. These different apodizations will result in slightly different instantaneous phases between the two beamsplitter outputs, and thus a coherence loss when phase referencing the spectrometer channels to the white-light phase.

8.3. Ratio Correction

PTI uses a single-mode fiber after beam combination to spatially filter the spectrometer channels. Spatial filtering increases the raw visibility and reduces the concomitant noise attributable to spatial effects; temporal effects must still be calibrated. As spatial filtering by the fiber essentially rejects light which would not interfere coherently, there is induced scintillation, which has a second-order effect on visibility. With simultaneous intensity measurements of each arm in a fully single-mode combiner (Foresto 1994), an essentially perfect correction for this effect is possible, but it can be shown (Shaklan, Colavita, & Shao 1992) that measurement of only the average intensity ratio between the two arms is adequate. If we denote this ratio as R_{12}, then the correction for the induced scintillation is

$$S_{12} = \frac{(1 + R_{12})^2}{4R_{12}}. \tag{35}$$

As discussed in Sec. 4, the combination of the foreground and ratio measurements allows estimation of S_{12} for each scan.

Currently, strict application of the ratio correction at PTI has been unsatisfactory, and we generally do not apply it. We attribute this to two effects. One is that given noisy values of R_{12}, S_{12} is a biased estimator, and will tend to over-correct the visibility. The second is that the measurements of the ratio are not truly simultaneous with the scan. Thus, seeing nonstationarity will affect the estimate. Also, there is a selection effect as fringe data is only recorded when locked, while the flux calibrations are contiguous.

Even without the ratio correction, the spatially-filtered data yields significantly-improved visibility estimates. However, the ratio correction has been useful as an additional indicator of data quality. For example, at high zenith angles, asymmetric (due to misalignment) vignetting in the system will increase S_{12}. But as with jitter, vignetting is tied to sky position, and spatially-local calibration will ameliorate most of the systematic visibility effects.

9. Conclusion

The use of array detectors on PTI requires attention to bias correction in fringe-parameter estimators, especially energy measures like V^2 which use squared quantities. Observations with PTI incorporate nightly and on-going bias calibrations, which can be used to compute optimal bias corrections. In addition to statistical noise in the estimators themselves, noise in the bias terms plays a role in the overall data quality. Inter-block fluctuations of estimated quantities are useful to estimate internal errors. Auxiliary data quality metrics include the tracking jitter and the ratio-correction estimate, which can be used for open-loop corrections or as independent data quality measures.

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A. Temporal coherence calibration using the phase jitter

We start with the assumption that the coherence reduction on V^2 can be written as

$$\Gamma = \exp\left(-(\sigma_\phi)^2_{\mathrm{hp}}\right), \tag{A1}$$

where $(\sigma_\phi)_{\mathrm{hp}}$ is the high-pass phase jitter. This is strictly true for the case where the fringe scanning is much faster than any frequencies of interest, although the results are similar with a slower scan. The high-pass jitter in Eq. A1 is given by the frequency-domain integral

$$\sigma^2_{\mathrm{hp}} = \int \mathrm{d}f W(f)(1 - \mathrm{sinc}^2(\pi f T)), \tag{A2}$$

where $W(f)$ is the phase power spectrum, $1 - \mathrm{sinc}^2()$ is a high-pass filter, and T is the sample integration time. A similar spectral representation exists for the phase jitter (Eq. 31):

$$\sigma^2_{\Delta\phi} = \int \mathrm{d}f W(f) \mathrm{sinc}^2(\pi f T) 4 \sin^2(\pi f t), \tag{A3}$$

where $\mathrm{sinc}^2()$ accounts for averaging over the sample integration time, while $\sin^2()$ is a high-pass corresponding to a sample spacing of t.

For $f < 1/T$, the filter function in the integral for σ^2_{hp} is $H_{\mathrm{hp}}(f) \simeq \frac{1}{3}\pi^2 T^2 f^2$, while for $f < 1/t$, the filter function in the integral for $\sigma^2_{\Delta\phi}$ is $H_{\Delta\phi}(f) \simeq 4\pi^2 t^2 f^2$. The ratio of the filter functions is

$$C_\Gamma = \frac{H_{\mathrm{hp}}(f)}{H_{\Delta\phi}(f)} = \frac{1}{12}\left(\frac{T}{t}\right)^2 . \tag{A4}$$

With $T = 6.75$ ms (for the white-light pixel) and $t = 10$ ms, we calculate $C_\Gamma = 0.038$. Thus, for narrowband low-frequency noise, we can write the visibility reduction directly in terms of the first-difference variance as

$$\Gamma^{\mathrm{b}} = \exp\left(-C_\Gamma \sigma^2_{\Delta\phi}\right) . \tag{A5}$$

This same formulation applies for other noise models. For $W(f)$ given by an atmospheric power spectrum, nominally $W(f) \propto f^{-8/3}$ (assuming a low fringe-tracker bandwidth), it is necessary to compute the integrals numerically. For power laws of the form $f^{-\alpha}$, some representative results for $T/t = 0.675$ are

α	C_Γ
8/3	0.057
2.5	0.070
7/3	0.088
2.0	0.145

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