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MAP-MOTIVATED CARRIER SYNCHRONIZATION OF GMSK BASED ON THE LAURENT AMP REPRESENTATION[1]

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ABSTRACT

Using the MAP estimation approach to carrier synchronization of digital modulations containing ISI together with a two pulse stream AMP representation of GMSK, it is possible to obtain an optimum closed loop configuration in the same manner as has been previously proposed for other conventional modulations with no ISI. We anticipate that this scheme will outperform other GMSK carrier sync schemes that are derived in a more ad hoc fashion. In view of the fact that the second pulse has significantly less energy than the first, it is also possible that a single pulse stream AMP representation of GMSK is sufficient for satisfactory carrier sync performance thereby reducing the implementation complexity.

INTRODUCTION

More than a decade ago Laurent [1] described an exact representation for constant envelope digital phase modulations, more commonly known as *continuous phase frequency modulation (CPFM)* or simply *continuous phase modulation (CPM)*, in the form of a superposition of a number of time/phase shifted amplitude-modulation pulse (AMP) streams. The number of such streams was dependent on the *partial response* nature of the modulation as described by the duration, L (in symbols), of the frequency pulse that characterizes the CPM. The primary focus of this work was on *binary* modulation[2] because of its relative simplicity of implementation and as such the number of pulse streams in the AMP representation is 2^{L-1}. Laurent's motivation for presenting such a representation was two-fold. First, it allowed for easier evaluation of the autocorrelation and power spectral density (PSD) of such modulations, in particular, simple results were specifically obtained for half-integer index modulations, i.e., ones whose frequency modulation index was of the form $h=n+1/2$, n integer. Second, it allowed for approximation (with reasonably good accuracy) of CPM by a single pulse stream with one optimized pulse shape (called the "main pulse") and as such offered a synthesis means no more complicated, in principle, than *minimum-shift-keying (MSK)*, which itself is a special case of CPM with a rectangular frequency pulse shape and a modulation index $h=0.5$.

Three years later, Kaleh [3] exploited Laurent's representation of CPM to allow for simple implementation of coherent receivers of such modulations, in particular, for the case of *Gaussian MSK (GMSK)*. Two forms of such receivers were considered, namely, a simplification of the optimum maximum-likelihood sequence estimation (MLSE) receiver and a linear MSK-type receiver, both which yielded small degradation relative to the true optimum MLSE receiver. In addition, Kaleh explicitly showed that for GMSK with a bandwidth-bit time product $BT_b = 0.25$ and a $4T_b$-wide approximation of the Gaussian pulse, i.e., $L=4$, a two (rather than $2^{L-1}=8$) pulse stream approximation is for all practical purposes (the fraction of energy in the neglected six pulse streams is 2.63×10^{-5}) exact. The effective pulse shapes on each of the AMP streams have different shapes and are of different durations (one is $3T_b$ wide and one is $5T_b$ wide). As such, both pulse shapes exceed the baud interval and hence each of the AMP pulse streams contains ISI.

In addition to the above advantages of the AMP representation insofar as spectrum evaluation and ideal receiver implementation, there is yet another advantage having to do with carrier synchronization of the receiver. Mengali and Andrea [4] discuss the use of the Laurent representation for CPM primarily in the context of the single pulse stream approximation and as such arrive at decision-directed phase estimation structures that are analogous to those used for MSK.

In this paper, we carry the carrier synchronization problem two steps further with the goal of achieving a more optimum solution. First, we consider the two-pulse stream approximation suggested by Kaleh rather than the single (main) pulse approximation. Second, using the maximum a posteriori (MAP) approach for carrier phase estimation as applied to pulse stream modulations with ISI [5,6], we arrive at a closed loop structure that is not limited to a decision-directed form and moreover accounts for the ISI directly within its implementation. The focus of the presentation here will be on first restating the two pulse

1 This work was performed at the Jet Propulsion Laboratory, California Institute of Technology under a contract with the National Aeronautics and Space Administration.

2 The work was later extended to the M-ary case by Mengali and Morelli [2].

stream AMP representation of GMSK in a form that is amenable to MAP estimation of carrier phase and then presenting the optimum structure.[3]

AMP REPRESENTATION OF GMSK

In what follows it will be convenient to deal with the normalized (unit amplitude) complex envelope of $s(t)$, i.e., the complex baseband signal $\tilde{S}(t)$ defined by the relation

$$\tilde{S}(t)=\exp\{j\phi(t,\boldsymbol{\alpha})\},\quad nT_b \le t \le (n+1)T_b \tag{1}$$

where $\boldsymbol{\alpha}=(\ldots,\alpha_{-2},\alpha_{-1},\alpha_0,\alpha_1,\alpha_2,\ldots)$ is the independent identically distributed (i.i.d.) binary data sequence with each element taking on values ± 1 and

$$\phi(t,\boldsymbol{\alpha})=\pi\sum_{i\le n}\alpha_i q(t-iT_b) \tag{2}$$

is the equivalent phase modulation process with $q(t)=\int_{-\infty}^{t} g(\tau)d\tau$ the normalized phase smoothing response ($g(t)$ is the normalized instantaneous frequency pulse in the zeroth signaling interval) that defines how the underlying phase $\pi\alpha_i$ evolves with time. Assuming that $g(t)$ is L bits in duration, then the above-mentioned normalization is such that

$$q(t)=\begin{cases}0, & t\le 0\\ \dfrac{1}{2}, & t\ge LT_b\end{cases} \tag{3}$$

For GMSK, this normalized phase smoothing response is obtained as,

$$\begin{aligned}q\left(t+\frac{LT_b}{2}\right)=\frac{1}{2}+\frac{1}{2T}&\left[\left(t-\frac{T_b}{2}\right)Q\left(\frac{2\pi B}{\sqrt{\ln 2}}\left(t-\frac{T_b}{2}\right)\right)\right.\\ &\left.-\left(t+\frac{T_b}{2}\right)Q\left(\frac{2\pi B}{\sqrt{\ln 2}}\left(t+\frac{T_b}{2}\right)\right)\right]\\ &-\frac{1}{\sqrt{2\pi}}\left(\frac{\sqrt{\ln 2}}{2\pi B}\right)\left[\exp\left\{-\frac{1}{2}\left[\frac{2\pi B}{\sqrt{\ln 2}}\left(t-\frac{T_b}{2}\right)\right]^2\right\}\right.\\ &\left.-\exp\left\{-\frac{1}{2}\left[\frac{2\pi B}{\sqrt{\ln 2}}\left(t+\frac{T_b}{2}\right)\right]^2\right\}\right],\\ &0\le t\le LT_b\end{aligned} \tag{4}$$

where $Q(x)$ is the Gaussian probability integral, B is the 3 dB bandwidth of the Gaussian transmit filter used to shape the pulse, and the value of L used for the approximation is a function of the BT_b product. For $BT_b = .25$ (typical of current applications in that it represents a good tradeoff between bandwidth compression and error probability performance), a value of $L = 4$ is sufficient, i.e., a total of $2^{L-1}=8$ PAM components is what will be needed to completely represent the signal in AMP form.

3 By optimum we mean that closed loop structure whose error signal is motivated by the derivative of the log-likelihood ratio associated with the MAP estimation of carrier phase.

Define the *generalized* phase pulse function by

$$\Psi(t)=\begin{cases}\pi q(t), & 0\le t\le LT_b\\ \dfrac{\pi}{2}\left[1-2q(t-LT_b)\right], & LT_b\le t\end{cases} \tag{5}$$

which is obtained by taking the nonconstant part of $q(t)$, i.e., the part that exists in the interval $0\le t\le LT$ and reflecting it about the $t=LT$ axis. Thus, in view of (5), $\Psi(t)$ is a waveform that is nonzero in the interval $0\le t\le 2LT_b$ and symmetric around $t=LT_b$. The importance of $\Psi(t)$ is that it allows definition of the following functions which become an integral part of the AMP representation of GMSK:

$$S_0(t)=\sin\Psi(t),\quad S_n(t)=S_0(t+nT_b)=\sin\Psi(t+nT_b) \tag{6}$$

Next define the $2^{L-1}=8$ distinct pulse shapes $C_i(t), i=0,1,\ldots,7$ each of which is a product of the basic generalized pulse shape $S_0(t)$ and $L-1=3$ other time shifts of $S_0(t)$ [1, Eq. (11)]. Finally, then the generic AMP form for the complex envelope of GMSK is [1]

$$\begin{aligned}\tilde{S}(t)&=\sum_{K=0}^{7}\left[\sum_{n=-\infty}^{\infty}e^{j(\pi/2)A_{K,n}}C_K(t-nT_b)\right]\\ &\triangleq\sum_{K=0}^{7}\left[\sum_{n=-\infty}^{\infty}\tilde{a}_{K,n}C_K(t-nT_b)\right]\end{aligned} \tag{7}$$

i.e., a superposition of eight amplitude/phase modulated pulse streams. By virtue of the fact that some of the $C_K(t)$'s extend beyond T_b sec (in particular the first four in the set, i.e., the ones with the most energy), as previously stated the corresponding pulse streams consist of overlapping pulses and hence contribute ISI. Also in (7) $\tilde{a}_{K,n}\triangleq e^{j(\pi/2)A_{K,n}}$ is the equivalent complex (unit amplitude) data symbol for the nth transmitted pulse in the Kth stream whose phase $(\pi/2)A_{K,n}$ depends solely on the past information data sequence $\boldsymbol{\alpha}$[1].

In the AMP representation of (7). the dominant term is the pulse stream corresponding to $C_0(t)$ (for a full response ($L=1$) CPM, e.g., MSK, it would be the only one) since its duration is the longest (at least $2T_b$ longer than any other pulse component) and it also conveys the most significant part of the total energy of the signal. The next most significant term would be the pulse stream corresponding to $C_1(t)$ which contains virtually all the remaining signal energy. Thus, as previously alluded to, it is sufficient to consider only the first two pulse streams in (7) and hence for all practical purposes we may "exactly" describe GMSK by the complex signal

$$\tilde{S}(t)=\sum_{n=-\infty}^{\infty}\tilde{a}_{0,n}C_0(t-nT_b)+\sum_{n=-\infty}^{\infty}\tilde{a}_{1,n}C_1(t-nT_b) \tag{8}$$

where specifically

$$C_0(t)=S_0(t)S_1(t)S_2(t)S_3(t), \qquad 0\le t\le 5T_b$$
$$C_1(t)=S_0(t)S_2(t)S_3(t)S_5(t), \qquad 0\le t\le 3T_b \tag{9}$$

Also, it can be shown that the equivalent complex data symbols satisfy the relations [3]

$$\tilde{a}_{0,n}\triangleq e^{j\frac{\pi}{2}A_{0,n}}=j\alpha_n\tilde{a}_{0,n-1}$$
$$\tilde{a}_{1,n}\triangleq e^{j\frac{\pi}{2}A_{1,n}}=j\alpha_n\tilde{a}_{0,n-2} \tag{10a}$$

which implies

$$\tilde{a}_{0,2n}\in\{j,-j\},\quad \tilde{a}_{0,2n+1}\in\{1,-1\}$$
$$\tilde{a}_{1,2n}\in\{1,-1\},\quad \tilde{a}_{1,2n+1}\in\{j,-j\} \tag{10b}$$

Note that the symbols in the two data streams alternate (from bit to bit) between purely real and purely imaginary unit amplitude values. Thus, in terms of the real (bandpass) GMSK signal we can view it as being composed of the sum of two pulse-shaped offset QPSK-type signals with pulse shapes corresponding to $C_0(t)$ and $C_1(t)$ and I, Q ±1 data symbol ($T_s=2T_b$ in duration) sequences respectively corresponding to

$$a_{0,2n}=\mathrm{Im}\{\tilde{a}_{0,2n}\},\quad b_{0,2n+1}=\mathrm{Re}\{\tilde{a}_{0,2n+1}\}$$
$$a_{1,2n}=\mathrm{Re}\{\tilde{a}_{1,2n}\},\quad b_{1,2n+1}=\mathrm{Im}\{\tilde{a}_{1,2n+1}\} \tag{11}$$

That is,

$$\begin{aligned}
s(t)&=\sqrt{\frac{2E_b}{T_b}}\,\mathrm{Re}\left\{\tilde{S}(t)e^{j\omega_c t}\right\}\\
&=\sqrt{\frac{2E_b}{T_b}}\left[\sum_{n=-\infty}^{\infty}a_{0,2n+1}C_0\left(t-(2n+1)T_b\right)\cos\omega_c t\right.\\
&\quad-\sum_{n=-\infty}^{\infty}b_{0,2n}C_0\left(t-2nT_b\right)\sin\omega_c t\\
&\quad+\sum_{n=-\infty}^{\infty}a_{1,2n}C_1\left(t-2nT_b\right)\cos\omega_c t\\
&\quad\left.-\sum_{n=-\infty}^{\infty}b_{1,2n+1}C_1\left(t-(2n+1)T_b\right)\sin\omega_c t\right]
\end{aligned} \tag{12}$$

Kaleh [3] also shows that the effective data sequences for the two symbol streams as defined in (10) each have uncorrelated symbols and furthermore the two sequences are uncorrelated with each other. It is possible to show a stronger condition on these sequences, namely, that they are each independent identically distributed (i.i.d.) and independent of each other. This property will be important in applying the average likelihood approach for obtaining the carrier phase estimate. To see these independence properties, it is interesting to interpret the equivalent I and Q data sequences in (12) in terms of the differentially encoded version of the true information sequence $\boldsymbol{\alpha}$ [1]. In particular, it is straightforward to show from the properties in (10a) that for the first pulse stream the equivalent I, Q data sequences $\{a_{0,2n+1}\},\{b_{0,2n}\}$ correspond to the odd/even split of the differentially encoded version of $\boldsymbol{\alpha}$ with the additional constraint that every other symbol be inverted. In mathematical terms, if $v_k=\alpha_k v_{k-1}$ represents the differentially encoded version of α_k, then

$$a_{0,2k+1}=(-1)^k v_{2k-1},\quad b_{0,2k}=(-1)^k v_{2k} \tag{13}$$

It should be noted that the relation in (13) is precisely the same equivalence between the frequency modulation representation of MSK and its offset QPSK equivalent. For the second pulse stream, the equivalent I, Q data sequences $\{a_{1,2n+1}\},\{b_{1,2n}\}$ are obtained by first multiplying the differential encoder output by the information bit delayed by one bit interval before performing the odd/even split and alternate symbol inversion, i.e.,

$$a_{1,2k}=(-1)^k v_{2k}\alpha_{k-1}\triangleq(-1)^k w_{2k},$$
$$b_{1,2k+1}=(-1)^k v_{2k+1}\alpha_{k-1}\triangleq(-1)^k w_{2k+1} \tag{14}$$

This equivalence can be seen by rewriting the second relation in (10a) as

$$\begin{aligned}
\tilde{a}_{1,n}=j\alpha_n\tilde{a}_{0,n-2}&=\overbrace{j\alpha_n\tilde{a}_{0,n-1}}^{\tilde{a}_{0,n}}\;\overbrace{\tilde{a}_{0,n-1}}^{\alpha_{n-1}\tilde{a}_{0,n-2}}\;\tilde{a}_{0,n-2}\\
&=\tilde{a}_{0,n}\alpha_{n-1}
\end{aligned} \tag{15}$$

Finally, since for an i.i.d. information sequence $\boldsymbol{\alpha}$ the differentially encoded version of this sequence is also i.i.d. and since multiplication by a one-bit delayed version of the input does not destroy the i.i.d. property, we reach the conclusions regarding the independence properties of the symbol stream sequences given above. Based on the entirety of the above, we conclude that GMSK can be implemented with the superimposed offset QPSK transmitter illustrated in Fig. 1. The two pulse shapes $C_0(t)$ and $C_1(t)$ as defined by (9) together with (4), (5) and (6) are illustrated in Fig. 2.

MAP ESTIMATION OF CARRIER PHASE

Consider now the received signal $y(t)$ composed of the sum of $s(t;\theta)$ and additive white Gaussian noise (AWGN) $n(t)$ where $s(t;\theta)$ is given by (12) with the addition of a uniformly distributed phase θ included in each carrier component. Based on an observation of $y(t)$ over the interval $0\le t\le T_0$ where we arbitrary assume that T_0 is an even integer multiple (say K) of the bit time T_b, we wish to find the MAP estimate of θ, i.e., the value of θ that maximizes the a posteriori probability $p(\theta|y(t))$ or since θ is assumed to be uniformly distributed, the value of θ that maximizes the conditional probability $p(y(t)|\theta)$. For an AWGN channel with single-sided noise power spectral density N_0 watts/Hertz, $p(y(t)|\theta)$ has the form

$$p\left(y(t)|\mathbf{a}_0,\mathbf{b}_0,\mathbf{a}_1,\mathbf{b}_1,\theta\right) = C\exp\left(-\frac{1}{N_0}\int_0^{T_0}\left(y(t)-s(t;\theta)\right)^2 dt\right) \tag{16}$$

where C is a normalization constant and we have added to the conditioning notation the implicit dependence of $s(t;\theta)$ on the i.i.d. I and Q data sequences of the two pulse streams. For a constant envelope (energy) signal such as GMSK, it is sufficient to consider only the term involving the correlation of $y(t)$ and $s(t;\theta)$ and lump the remaining

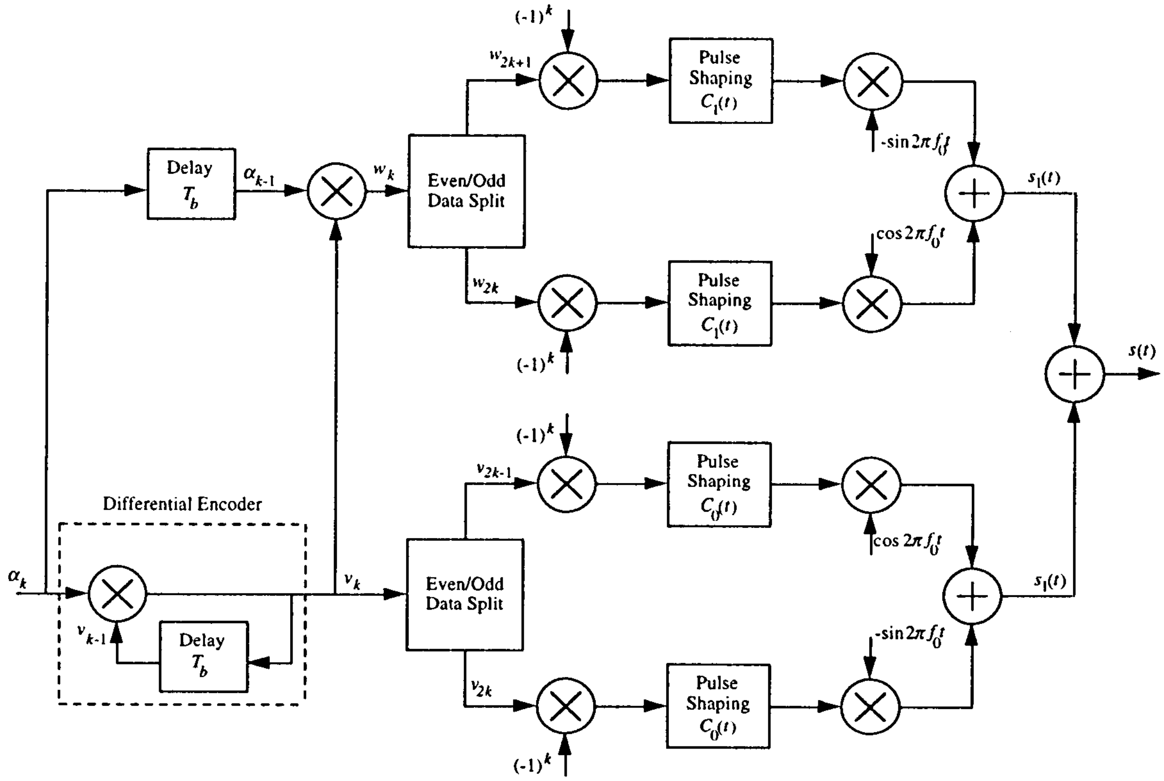

Fig. 1. GMSK Transmitter Implementation Based on AMP Representation

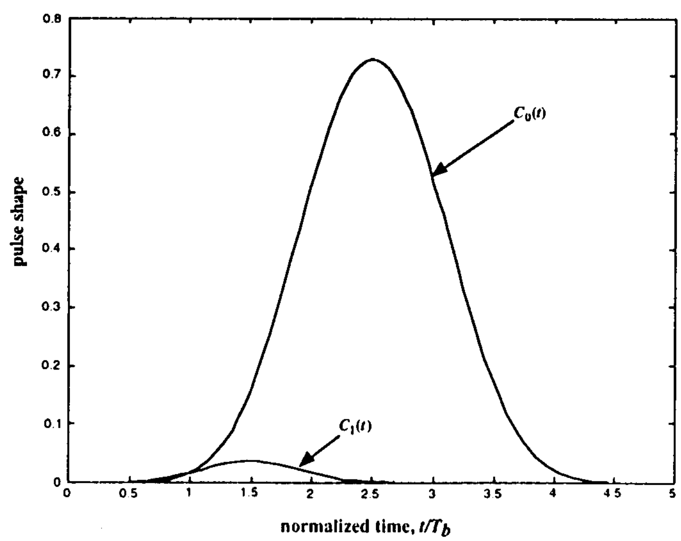

Fig. 2. Pulse Shapes for AMP Representation of GMSK

terms into the normalization constant.[4] Thus, we rewrite (16) as

$$p\left(y(t)\middle|\mathbf{a}_0,\mathbf{b}_0,\mathbf{a}_1,\mathbf{b}_1,\theta\right) = C\exp\left(\frac{2}{N_0}\int_0^{T_0} y(t)s(t;\theta)dt\right) \tag{17}$$

where for convenience we still use C to denote the normalization constant.

Evaluation of (17) for $s(t;\theta)$ corresponding to a single binary pulse stream, e.g., BPSK, with ISI was considered in [5,6]. Extension of the result to an $s(t;\theta)$ corresponding to a single pair of quadrature binary pulse streams (such as QPSK) with identical ISI on the I and Q channels is straightforward and was somewhat discussed in [5]. What we have for the AMP representation of GMSK in (12) is two pairs of offset quadrature binary pulse streams each pair having different amounts of ISI. (Recall that $C_0(t)$ is a pulse of width $5T_b$ and $C_1(t)$ is a completely different pulse of width $3T_b$.) Evaluation of (17) for such a received signal has not been previously considered. Without belaboring the details, following substitution of (12) into (17) and averaging over the four i.i.d. component data sequences $\mathbf{a}_0,\mathbf{b}_0,\mathbf{a}_1,\mathbf{b}_1$, then after considerable manipulation it can be shown that

$$\begin{aligned} p\left(y(t)\middle|\theta\right) = C &\prod_{\substack{k=-3\\k\text{ odd}}}^{K-1}\cosh\{I_c(k,0,\theta)\}\prod_{\substack{k=-4\\k\text{ even}}}^{K-2}\cosh\{I_s(k,0,\theta)\} \\ &\times\prod_{\substack{k=-2\\k\text{ even}}}^{K-2}\cosh\{I_c(k,1,\theta)\}\prod_{\substack{k=-1\\k\text{ odd}}}^{K-1}\cosh\{I_s(k,1,\theta)\} \end{aligned} \tag{18}$$

where

$$\begin{aligned} I_c(k,l,\theta) &\triangleq \frac{2\sqrt{2E_b/T_b}}{N_0}\int_0^{KT_b} r(t)\cos(\omega_c t+\theta)C_l(t-kT_b)dt \\ I_s(k,l,\theta) &\triangleq \frac{2\sqrt{2E_b/T_b}}{N_0}\int_0^{KT_b} r(t)\sin(\omega_c t+\theta)C_l(t-kT_b)dt \end{aligned} \tag{19}$$

Note that because of the presence of ISI in each of the component pulse streams, the arguments of the hyperbolic cosine terms involves integration over the entire observation interval $0\le t\le KT_b$ rather than just integration over a single bit interval as is customary in such problems when ISI is absent. (Actually the finite duration of $C_0(t-kT_b)$ and $C_1(t-kT_b)$ will truncate these integrations to an interval (depending on the value of k) smaller than the observation time interval but still larger than the baud interval.) Finally, the MAP estimate of θ, i.e., θ_{MAP} is the value of θ that maximizes (18).

4 We note that for the general ISI problem as treated in [5,6], the energy-dependent exponential term $\exp\left\{-(1/N_0)\int_0^{T_0} s^2(t;\theta)dt\right\}$ is not constant and in fact depends on the data sequence. However, for the "exact" representation of GMSK by the two pulse stream AMP form, we can make the constant envelope assumption and hence ignore the energy-dependent term.

CLOSED LOOP CARRIER SYNCHRONIZATION OF GMSK

As has been done many times in the past to arrive at closed loop carrier synchronizers based on open loop MAP estimates, one takes the natural logarithm of the likelihood ratio, differentiates it with respect to θ and then uses this as the error signal, $e(\theta)$, in a closed loop configuration. The reasoning behind this approach is that $e(\theta)$ will be equal to zero when $\theta=\theta_{MAP}$ and thus the closed loop will null at the point corresponding to the open loop MAP phase estimate. Thus, proceeding in this fashion, we obtain

$$\begin{aligned} e(\theta)\triangleq\frac{d}{d\theta}\ln p\left(y(t)\middle|\theta\right) = &\sum_{\substack{k=-3\\k\text{ odd}}}^{K-1} I_s(k,0,\theta)\tanh\{I_c(k,0,\theta)\} \\ &-\sum_{\substack{k=-4\\k\text{ even}}}^{K-2} I_c(k,0,\theta)\tanh\{I_s(k,0,\theta)\} \\ &+\sum_{\substack{k=-2\\k\text{ even}}}^{K-2} I_s(k,1,\theta)\tanh\{I_c(k,1,\theta)\} \\ &-\sum_{\substack{k=-1\\k\text{ odd}}}^{K-1} I_c(k,1,\theta)\tanh\{I_s(k,1,\theta)\} \\ \triangleq\; &e_0(\theta)+e_1(\theta) \end{aligned} \tag{20}$$

where we have made use of the fact that from (19) $I_c(k,l,\theta)$ and $I_s(k,l,\theta)$ are derivatives of each other.

The result in (20) suggests a superposition of two loops each contributing a component to the error signal corresponding to associated pulse stream in the two pulse stream AMP representation of GMSK. Fig. 3 illustrates the first of the two loop components (i.e., the one that generates $e_0(\theta)$) that must be superimposed to arrive at the closed loop GMSK carrier synchronizer suggested by the error signal in (20). A similar figure would exist for the second loop component that generates $e_1(\theta)$. We offer this scheme as the "optimum" (in the sense of being MAP-motivated) GMSK carrier synchronizer. As is customary, the "tanh" nonlinearity can be approximated by a linear or hard limiter device for low and high SNR applications, respectively. The rate at which the loop updates its carrier phase estimate can vary from every T_b to every KT_b seconds. In the case of the latter extreme, the observation intervals used for each carrier phase estimate do not overlap and as such the loop represents a sequential block-by-block implementation of the MAP open loop estimator. In the case of the former extreme, the observation intervals used

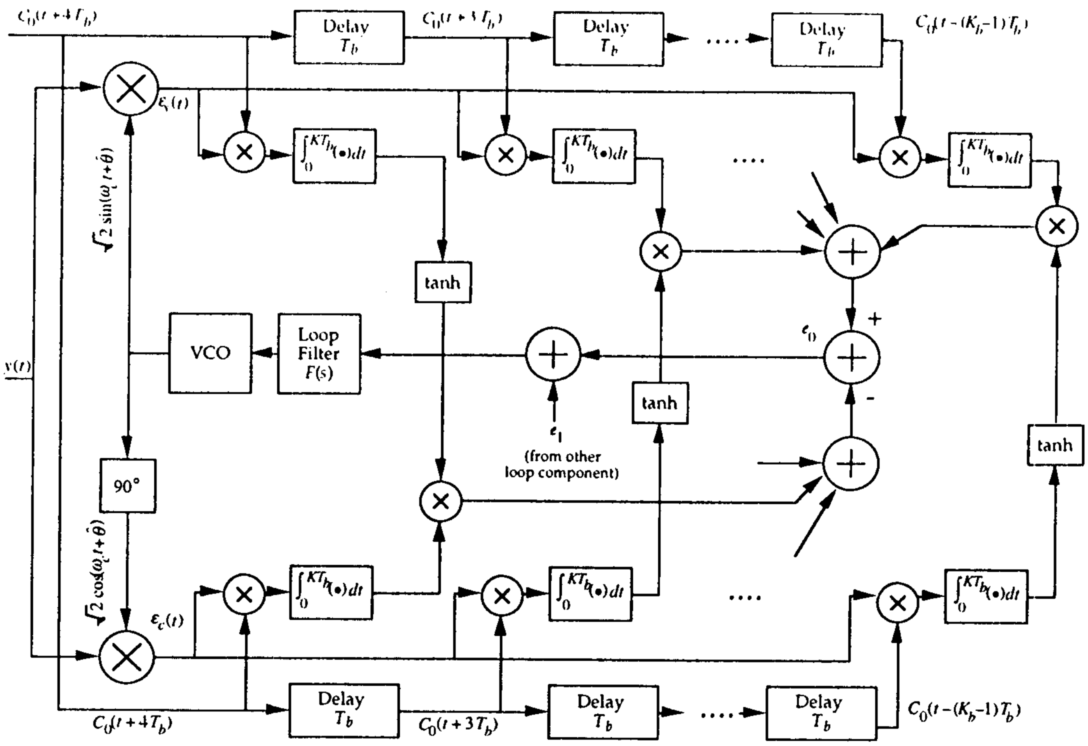

Fig. 3. Block Diagram of Suboptimum ISI-Compensated MAP Estimation Loop for GMSK (First Signal Component)

for each carrier phase estimate overlap by $(K-1)T_b$ seconds and as such the loop represents a sliding window MAP phase estimator.

CONCLUSIONS

Using the MAP estimation approach to carrier synchronization of digital modulations containing ISI together with a two pulse stream AMP representation of GMSK, it is possible to obtain an optimum closed loop configuration in the same manner as has been previously proposed for other conventional modulations with no ISI. We anticipate that this scheme will outperform other GMSK carrier sync schemes that are derived in a more ad hoc fashion. The actual performance of our scheme is currently under investigation and will be reported on in the future. In view of the fact that the second pulse has significantly less energy than the first, the outcome of these evaluations might further demonstrate that a single pulse stream AMP representation of GMSK is sufficient for satisfactory carrier sync performance thereby simplifying the implementation.

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