Bit Geometrically Uniform Encoders and Applications to Serially Concatenated Trellis Coded Modulation

Sergio Benedetto *, Dariush Divsalar **, Roberto Garello *, Guido Montorsi *, Fabrizio Pollara ** 1

*Dipartimento di Elettronica, Politecnico di Torino, Italy  ** Jet Propulsion Laboratory, Caltech, USA

Email benedetto@polito.it  Email dariush@shannon.jpl.nasa.gov

Abstract — A new class of labelings and encoders for which the Uniform Bit Error Property holds is introduced, and an application to the design of good serially concatenated TCM schemes is presented.

I. UNIFORM BIT ERROR PROPERTY

A Euclidean-space constellation has the Uniform Error Property (UEP) if the symbol error probability does not depend on the transmitted signal. UEP proves very useful for code analysis and design. The Hamming space \( H_k \) is the set of all \( 2^k \) binary \( k \)-ples. Given a finite constellation \( S \) with cardinality \( |S| = 2^k \), a binary labeling \( E[S, k] \) for \( S \) is a one-to-one function \( E : S \rightarrow H_k \). The bit error probability with Maximum Likelihood (ML) symbol decoding, when a signal \( s_i \in S \) is transmitted, is:

\[
P_b(e|s_i = s_i) = \sum_{s_j \in S} w_{H}(E(s_i) - E(s_j)) P(s_j|x = s_i) \]

where \( w_{H}(E(c)) = 2 \) for all \( c \in C \) with \( w_{H}(E(c)) = 2 \). According to the definition, the computation of \( d_{b,\text{eff}} \) requires in general testing of all possible pairs \( (c_1, c_2) \). However, if the inner TCM encoder is BGU, we can compute \( d_{b,\text{eff}} \) as \( d_{b,\text{eff}} = \min_{c \in C} w_{H}(E(c)) \) with a great simplification.

II. SERIALLY CONCATENATED TCM

The serial concatenation of an outer binary convolutional encoder with an inner TCM encoder over a multidimensional Euclidean constellation through an interleaver (SCTCM), for brevity, has been introduced in [2]. SCTCM allows to extend the extremely good performance of turbo codes to the case of spectrally efficient coded modulations. In [2] the design approach was based on a “cut-and-try” maximization of the effective free Euclidean distance of the inner TCM recursive encoder, defined as the minimum distance between code sequences generated by information sequences that differ only by two bits: \( d_{b,\text{eff}} = \min_{c_1, c_2} d_{H}(E(c_1), E(c_2)) \). The code (denoted by A in Fig. 1) obtained after an exhaustive search over the class of BGU encoders has \( d_{b,\text{eff}} = 3.76 \), which compares very favorably with the heuristic construction of [2] (code B in Fig. 1), leading to \( d_{b,\text{eff}} = 1.76 \). Analytical upper bounds to the bit error probability, evaluated through an easy (thanks to the BGU property) extension of the technique based on the uniform interleaver (of length 100 and 1000), are reported in Fig. 1 for the two SCTCM schemes.

Fig. 1: Upper bounds to the bit error probability for SCTCM codes A and B of spectral efficiency 2 bps/Hz.

REFERENCES