This paper presents an analysis of the relative motions between formation flying space vehicles in orbit about the Earth and Mars. Formation flying is proposed for: cross calibration of science instruments on follow-on missions and new technology demonstrations, moving from large platforms to several smaller vehicles with distributed instrumentation (virtual platforms) and spaceborne interferometry. In Mars orbit, a small constellation for in-situ navigation is being considered to enable automated operations and to reduce Earth based tracking requirements. To establish achievable control accuracies for these types of missions, dynamics in the orbit design space are examined thoroughly. Aspherical and third body gravity, atmospheric drag, and solar radiation pressure are the primary dynamic forces responsible for producing relative motions. High fidelity models of these forces are used to examine the relative motion characteristics of circular LEO and GEO orbits and circular low and synchronous Mars orbits.

Introduction

Simplest are formations of two space vehicles, separated only in true anomaly (i.e., co-planar with nearly the same altitude and eccentricity). This type of formation is proposed for: follow-on missions requiring sensor cross calibration, new technology validation, and for complementary science via distributed instrumentation on several vehicles.

Navigation operations consist of maintaining a prescribed separation without dedicated satellite-to-

satellite links. This is referred to as non-cooperative formation flying. Navigation functions can be automated or implemented as an autonomous capability by taking advantage of common dynamic force modelling cancellation.

An example of a cross calibration mission is the TOPEX/Poseidon follow-on mission called JASON-1. Formation flying in this case is required to cross calibrate the altimeters that provide the primary science measurements.

A technology validation example is the New Millennium Program’s Earth Orbiter-1 (EO-1) mission. EO-1 will fly one minute (-450 km) behind the Landsat-7 land imaging mission and carry a lighter weight, lower cost version of the Landsat multispectral imaging system. Image co-registration requires control of the mean along track orbit separation to six seconds (~45 km).

Virtual platforms comprised of complementary sensors are under consideration as an alternative to the large EOS platforms. Possible benefits are reduced implementation risk with comparable or improved science return and reduced mission operations costs.

For each of these simple formations, orbit control consists primarily of maintaining the mean semimajor axis, accounting for secular changes caused by atmospheric drag. Occasional inclination control due to lunar perturbations is also required.

More complex is a formation with true anomaly and inclination differences. This type of formation has been proposed for the New Millennium Program’s Deep Space-3 (DS-3) mission.

At geosynchronous altitude, DS-3 consists of three space vehicles (two collectors and a combiner) to obtain interferometric observations over an interval of about one hour per orbit. A unique configuration has been proposed that produces a stable observing geometry over the observation period without the...
need of propulsive maneuvers. Orientation of the vector between the two collectors and a fixed source direction must be initialized and maintained over successive observation intervals. This translates into maintaining primarily the inclination and longitude of ascending node. If the space vehicle physical parameters (i.e., area and mass) differ, additional eccentricity control is required due to solar pressure perturbations.

Finally, a constellation of space vehicles in several orbits differing only in the longitude of the ascending node is considered. Here a Mars constellation is examined. Future Mars exploration will involve many assets such as orbiters, landers, rovers and humans. An in-situ Navigation constellation might improve performance while reducing costs by minimizing Earth based tracking.

A constellation requiring minimal control is desirable since the vehicles initially comprising the constellation may be older assets with little remaining fuel. Understanding the relative motions is useful for designing such a constellation and for developing autonomous navigation algorithms for use on space vehicles with radio tracking links to the constellation.

**Analysis**

Precise motions of each type of formation depend on the dynamic forces acting on each vehicle. Aspherical central body gravity, third body gravity, atmospheric drag, and solar radiation pressure are the dominant forces considered.

**Aspherical Gravity**

Periodic relative motions due to aspherical gravity are characterized for a range of altitudes in Fig. 1. This plot shows the maximum along track periodic excursion for a given mean along track separation. These variations represent the minimum practical limit for along track control since tighter control requires maneuvers at a frequency greater than the orbital period; thus, requiring considerable fuel expense.

Figs. 2a-2c. show the osculating relative motion “mug-shots” of a two LEO space vehicle formation separated by 10 km in true anomaly. The motions are mapped out once every orbit period.

Long period semimajor axis variations exist for synchronous orbits where the orbital period resonates with the central body rotation rate. This gives rise to a “secular looking” along track runoff. Fig. 3. shows the along track time variation of Earth and Mars synchronous formations.

**Atmospheric Drag**

Fig. 4 presents the along track variations with the addition of atmospheric drag to the aspherical forces. Differences in the space vehicle ballistic coefficients (i.e., area-to-mass ratios) produce a secular runoff in the along track. Each vehicle experiences nearly the same atmosphere but by projecting a different area and possessing a different mass the separation commences.

For the simple two space vehicle formations, the drag forces acting on each vehicle differ only by the constant ballistic coefficients ratio. So as revealed in Fig. 4, the along track separation varies quadratically and can be represented as follows:

\[ \Delta AT = \Delta AT_0 - 2kda_0 t + k\alpha(R-1) t^2 \]  

1
where: \( \Delta AT = \) along track separation  
\( k = \) orbit constant = \( 3V^2/4a \)  
\( V = \) mean circular velocity  
\( a = \) mean semimajor axis  
\( \Delta a = \) initial mean semimajor axis difference  
\( R = \) ballistic coefficient ratio  
\( = (area/mass)_{v1} / (area/mass)_{v2} \)  
\( t = \) time

Controlling this secular runoff is the primary navigation task for LEO formations.

**Solar Radiation Pressure**

When atmospheric drag effects are significant the contribution of solar radiation pressure to formation degradation is not easily separated. Solar pressure forces ultimately produce systematic eccentricity variations. These are usually small and can be controlled when performing atmospheric drag maneuvers. That is, selectively choosing the true anomaly of a drag maintenance maneuver can accommodate eccentricity variations.

For interferometry formations in GEO, solar pressure effects are significant. Fig. 5 depicts the the formation proposed for DS-3. The vectors between the collectors and the combiner must be kept nearly equal (±30cm). Differences in the solar ballistic coefficients of the collectors and combiner produce separations that must be controlled. Fig. 6 shows the divergence for solar ballistic coefficient differences of one and five percent.

**Third Body Gravity**

In LEO, relative motions due to third body forces are much smaller than those produced by aspherical gravity. However, at GEO these forces are significant for an interferometry formation. From Fig. 5, another control requirement for the DS-3 mission is to maintain orthogonality (±20mdeg) between the baseline of the two collector vehicles and a fixed source direction. Fig. 7 shows that daily maneuvers would be required to compensate for the perturbations caused primarily by the Moon.

**Conclusions**

Precise relative motions of space vehicle formations depend on the dynamic forces acting on each vehicle. This paper has examined the pertinent forces to determine their effects on the relative motion of circular LEO and GEO orbits and circular low and synchronous Mars orbits. The relative motions are graphically displayed and navigation control identified for each formation type.

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**References**


Fig.5 - DS-3 Interferometry Mission
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Fig. 2a - View from Side

Radial (m)

Along Track (m)

Cross Track (m)

Fig. 2b - View from Behind

Radial (m)

Cross Track (m)

Along Track (m)

Aspherical Gravity Only
Fig. 3 - Aspherical Gravity Only (Resonant Effects)

Fig. 4 - Aspherical Gravity + Atmospheric Drag Effect
Fig. 6 - Solar Pressure Effects on Interferometry Formation Relative Baselines

Fig. 7 - Third Body Effects on Interferometry Formation Baseline Orientation