Altitude Range Resolution of Differential Absorption
Lidar Ozone Profiles.

Georg Beyerle$^{1,2}$ and I. Stuart McDermid$^1$

(e-mail: beyerle@tmf.jpl.nasa.gov; mcdermid@tmf.jpl.nasa.gov)

(1) Table Mountain Facility
Jet Propulsion Laboratory
California Institute of Technology
P.O. Box 367
Wrightwood
CA 92397, U.S.A.

(2) Alfred Wegener Institute for Polar and Marine Research,
Potsdam
Germany

Submitted to Applied Optics, May 1998
Abstract

A method is described for the empirical determination of altitude-range resolutions of ozone profiles obtained by differential absorption lidar (DIAL) analysis. The algorithm is independent of the implementation of the DIAL analysis, in particular of the type and order of the vertical smoothing filter applied. An interpretation of three definitions of altitude range resolution is given on the basis of simulations carried out using the JPL ozone DIAL analysis program “SO3ANL”. These definitions yield altitude range resolutions differing by up to a factor of 2. It is shown that the altitude resolution calculated by “SO3ANL”, and reported with all JPL lidar ozone profiles, corresponds closely to the full width half maximum of a retrieved ozone profile if an impulse function is used as input ozone profile.
I. INTRODUCTION

Differential absorption lidars (DIAL) have been and continue to be widely used for the remote detection and monitoring of atmospheric trace gases such as ozone e.g., 1-4. They now constitute an integral part of the Network for the Detection of Stratospheric Change (NDSC) for the long-term monitoring of stratospheric ozone. In order to achieve the goals of NDSC the ozone profiles supplied to the NDSC archive have been extensively verified in intercomparison campaigns e.g., 5-7. However, uncertainty exists with respect to the interpretation of altitude resolutions reported by various ozone DIAL instruments.

In the routine DIAL analysis, ozone number densities \( n_o(z) \) are calculated by

\[
n_o(z) = \frac{1}{\sigma_n - \sigma_f} \left[ \frac{1}{2} \frac{d}{dz} \ln \frac{P_f(z)}{P_n(z)} \right] \left[ \alpha_n(z) - \alpha_f(z) \right]
\]

provided that the contribution from aerosol scattering can be neglected 3,8,9. Here, \( z \) is the geometric altitude; for “on”- and “off”-wavelengths \( \lambda_n \) and \( \lambda_f \), \( P_n(z) \) and \( P_f(z) \) are the lidar signal counts at \( \lambda_n \) and \( \lambda_f \), \( \sigma_n \) and \( \sigma_f \) the ozone absorption cross sections, and \( \alpha_n \) and \( \alpha_f \) the extinction coefficients for molecular scattering at \( \lambda_n \) and \( \lambda_f \), respectively.

The evaluation of the term \( d/dz \ln[P_f(z)/P_n(z)] \) is an essential element of the analysis. Generally, differentiation has the effect of applying a high-pass filter to a signal 10. In DIAL analysis this fact represents a problem as the signal counts \( P_n(z) \) at stratospheric altitudes contain high wavenumber noise contributions. A straightforward replacement
would therefore amplify detector and statistical noise contributions more than the low wavenumber signal components due to ozone absorption and render the retrieved ozone profile useless.

Therefore, stratospheric ozone DIAL algorithms, in most cases, employ a derivative smoothing filter which cuts off the high frequency part of \( P_n(z) \).

\[
\frac{d}{dz} \ln \frac{P_f(z)}{P_n(z)} \rightarrow \sum_{j=-N(i)}^{N(i)} c_j \ln \frac{P_f(z_{i,j})}{P_n(z_{i,j})}
\]  

(3)

where \( c_j \) denote the filter coefficients of the derivative filter with order \( M = 2N+1 \). Since signal-to-noise-ratios decrease for increasing altitude, the filter order and the filter coefficients are usually altitude dependent. A variety of different derivative filters have been described in the literature\(^e\,g\,9,11-13\).

Typically, the altitude resolution, \( \delta z(z_i) \), of ozone profiles is defined in terms of the derivative filter. Since various filters are used, values of \( \delta z(z_i) \) from different data analyses are generally not comparable. Here, we propose an empirical approach to determine \( \delta z(z_i) \) which is independent of the particular filter used in the analysis program and can thus be applied to any DIAL analysis. The method is illustrated with simulations performed by “SO3ANL”, the JPL stratospheric ozone DIAL analysis program. “SO3ANL” analyses two pairs of signals, \( P_{n,f}^L \) and \( P_{n,f}^H \). Superscript \( L \) refers to the low sensitivity channels (\( L \)-channels) for
ozone retrieval between 15 and 25 km and superscript \( H \) to high sensitivity channels (\( H \)-channels) for altitudes between 25 and 50 km.

In “SO3ANL” a polynomial derivative filter of degree 1 is used, i.e., the filter coefficients are given by

\[
e_j = \frac{3^j}{N(N+1)(2N+1)} \quad j = -N, \ldots, N
\]

(4)

On the basis of extensive simulations the altitude dependence of the filter order \( M = 2N+1 \) is parameterized as

\[
M_L = 2 \left[ \max(1.9396, 0.1871 \exp(9.1 \times 10^{-5} \text{ m}^{-1} (z - z_s)))^{0.046} \right] + 1
\]

\[
M_H = 2 \left[ \max(1.9396, 0.3741 \exp(9.1 \times 10^{-5} \text{ m}^{-1} (z - z_s)))^{0.046} \right] + 1
\]

for the \( H \) (superscript \( H \)) and \( L \) (superscript \( L \)) channels. \( z_s \) denotes the site altitude and the square brackets indicate rounding to the nearest integer.

**II. ALTITUDE RESOLUTION**

We determined the altitude resolution by performing a DIAL analysis with a simulated raw data profile consisting of 1024 count values corresponding to the 1024 multi-channel-scaler (MCS) bins of the JPL stratospheric ozone lidar data acquisition hardware. The detection channels operate in photon counting mode. Dwell time for each bin is 2 \( \mu \)s (corresponding to a step height of \( \Delta z = 300 \) m). Each simulated signal count profile was analyzed by “SO3ANL” in the same manner as real data.
Two types of synthetic ozone profiles, \( \tilde{n}_{\text{in}} \) and \( \tilde{n}_{\text{in}}^\prime \), were used. One, \( \tilde{n}_{\text{in}} \), is a sine wave with wave number \( k \) and phase \( 0 \leq \phi < 2\pi \),

\[
\tilde{n}_{ji}(z_i) = \frac{A}{2} \sin(2\pi k z_i + \phi) + \frac{A}{2}
\]  
(5)

where \( A = 10^{13} \text{ cm}^{-3} \). Adding the term \( A/2 \) ensures \( \tilde{n}_{ji}(z_i) \geq 0 \). The other, \( \tilde{n}_{\text{in}}^\prime \), is an impulse function

\[
\tilde{n}_{ji}^\prime(z_i) = \begin{cases} 
  A & \text{if } z_i = z_j \\
  0 & \text{else}
\end{cases}
\]  
(6)

For illustration figure 1 shows \( \tilde{n}_{ji}^\prime(z_i) \) for three altitudes, \( z_j = 15 \text{ km} \), \( z_j = 25 \text{ km} \), and \( z_j = 35 \text{ km} \) analyzed as \( L \) channel data (left panel) and \( H \) channel data (right panel). For clarity the figure does not show the maximum of \( \tilde{n}_{ji}^\prime(z_i) \) \((10^{13} \text{ cm}^{-3})\). Arrows indicate the full-width-half-maximum of the results from the \( H \) and \( L \) channel analysis. The evident asymmetry of the \( z_j = 35 \text{ km} \)-profiles is caused by the exponential increase of filter order \( M \) with altitude. We note that in practice at 35 km the \( L \) channel analysis data is not used.

Figure 2 shows \( \tilde{n}_{ji}^\prime(z_i) \) and \( \tilde{n}_{ji}^\prime(z_i) \) for a wavenumber of 3 km\(^{-1}\). Between 41 and 43 km altitude the \( H \) channel data (dashed line, right panel) \( \tilde{n}_{ji}^\prime(z_i) \) exhibits a phase shift of \( \pi \) with respect to \( \tilde{n}_{ji}^\prime(z_i) \). The cause for this phase shift is discussed later.

In the following we investigate three different ways to calculate the altitude resolution \( \delta z \) which are used in DIAL algorithms\(^9\)\(^{12}\)\(^{14}\)\(^{15}\).

1) \( \delta z^{(1)} = 1/k_{1/2} \) where \( k_{1/2} \) is the wave number for which the transfer function drops to 50%, \( H(k_{1/2}) = 1/2 \)
2) $\delta z^{(2)} = M \Delta z$

3) $\delta z^{(3)}$ is given by the full-width-half-maximum (FWHM) of the retrieved profile $\hat{n}_j^\text{out}(z_i)$ if an impulse function $\hat{n}_j^\text{in}(z_i)$ is used as input.

As will be shown below, these three definitions yield results differing by up to a factor of 2.

III. RESULTS AND DISCUSSION

A total of 1651 simulated count profiles were analyzed by "SO3ANL". For clarity no additional noise was added to $P_{n}(z)$. However, as "SO3ANL" expects integer values for the signal counts the simulated values $P_{n}(z)$ were rounded to the nearest integer thereby producing quantization noise. 151 calculations were made using an impulse function with $z_j$, the location of the impulse, between 15 and 60 km and in steps of 300 m. 1500 simulations were performed using sine wave input profiles with wave numbers between $1/300$ m$^{-1}$, $1/600$ m$^{-1}$, $1/900$ m$^{-1}$, etc. decreasing to $1/45000$ m$^{-1}$. For each wave number, 10 profiles with a random phase shift between 0 and $2\pi$ (equation 2) were analyzed.

First we consider the sine input functions $\tilde{n}_j^\text{in}(z_i)$. For $\tilde{n}_j^\text{in}(z_i)$ we define a response function $R$ by dividing the derived and input ozone profile,

$$R(k_j, z_i) = \frac{\tilde{n}_j^\text{out}(z_i)}{\tilde{n}_j^\text{in}(z_i)}$$

(7)

$R$ quantifies how strongly a sine wave ozone profile with wave number $k$ is reduced. For $R(k,z) = 1$ the ozone profile is perfectly reproduced; for $R(k,z) = 0$ the corresponding sine wave component is completely suppressed in the retrieved profile. As an example of the
results from the simulations, the response function for filter order \( N = 19 \) (corresponding to an altitude range 41.7 to 42.6 km, \( H \) channel analysis) is shown in Figure 3. For comparison the theoretically expected result

\[
R(k) = \frac{1}{ik} \sum_{l=-N}^{N} c_l \exp(ilk)
\]  

(8)

obtained from the filter coefficients is plotted as well. Note that \( \Im(R) = 0 \) as \( c_l = -c_{-l} \).

Figure 3 shows that \( R(k) \) is negative (i.e. \( \tilde{n}^\text{in}_j(z_i) \) and \( \tilde{n}^\text{out}_j(z_i) \) are out of phase by \( \pi \)) for wave numbers between 0.25 and 0.43 km\(^{-1}\). Figure 2 illustrates the effect of this phase shift in altitude space. Between 41 and 44 km altitude \( \tilde{n}^\text{out}_j(z_i) \) exhibits an ozone maximum where the input profile \( \tilde{n}^\text{in}_j(z_i) \) has a minimum and vice versa.

We summarize the results of the simulations in Figure 4. It shows the altitude resolutions \( \delta z^{(1)}, \delta z^{(2)}, \) and \( \delta z^{(3)} \) as a function of altitude. For comparison the altitude resolution obtained by “SO3ANL” is plotted as well. We find that altitude resolution reported by “SO3ANL” is in close agreement with \( \delta z^{(3)}(z_i) \). Furthermore, the altitude resolutions \( \delta z^{(1)}(z_i) \) and \( \delta z^{(3)}(z_i) \) differ by about a factor of 2 whereas \( \delta z^{(2)}(z_i) \) is about 80% of \( \delta z^{(1)}(z_i) \).

VI. CONCLUSIONS

There is no unique definition for the altitude resolution of DIAL ozone profiles. The three definitions we investigated are found to yield values differing by up to factor of 2. For \( \delta z^{(1)} \) and \( \delta z^{(3)} \) an empirical approach to determine the altitude resolution can be used which does not depend on the implementation of the DIAL analysis method.
We deliberately avoid discussion of the justifications for or against a certain definition of altitude resolution. The three definitions we studied in this paper all have clear geophysical interpretations. However, we suggest that DIAL ozone data sets should contain information defining the derivation of the altitude resolution reported.

Acknowledgments

Helpful discussions with M. R. Gross, T. J. McGee, J. J. Tsou and P. von der Gathen are gratefully acknowledged. GB thanks the National Research Council for the award of an associateship. The work described in this study was carried out at the Jet Propulsion Laboratory, California Institute of Technology, through an agreement with the National Aeronautics and Space Administration. Contribution XXXX of the Alfred Wegener Institute.
References


FIGURE CAPTIONS

Figure 1.

Input (dotted line) and retrieved output profiles (full line) for $L$ channel (left panel) and $H$ channel (right panel) analysis. The results for three impulse input functions for $z_j = 15\text{km}$, $z_j = 25\text{km}$, and $z_j = 35\text{km}$ are shown. The maximum of the input impulse function is $10^{13}\text{cm}^{-3}$.

Figure 2.

Examples of input (dotted line) and retrieved output profiles (full line) for $L$ channel (left panel) and $H$ channel (right panel) analysis. Here the wave number is $3\text{ km}^{-1}$. Note the attenuation of the sine wave at higher altitudes and the phase shift between input and retrieved profile between $42$ and $44\text{ km}$ in the $H$ channel analysis.

Figure 3.

The response function obtained by the “SO3ANL” simulation for filter order $N = 19$ (circles). The data is derived from the $H$ data channels at altitudes between $41.7$ and $42.6\text{ km}$. For comparison the theoretically expected result is plotted as dashed-dotted line.

Figure 4.

The altitude resolutions $\delta z^{(1)}(z_i)$ (solid line), $\delta z^{(2)}(z_i)$ (dashed-dotted), and $\delta z^{(3)}(z_i)$ (dashed) as a function of altitude for the low sensitivity (top) and high sensitivity detection channels (bottom). The altitude resolution calculated by “SO3ANL” is shown for comparison as dotted lines. The step-like pattern is caused by increases in the filter order.