SIM vs. SOS: A Space Interferometry Trade Study

Richard L. Baron[1] Mark H. Milman[2] and Kim M. Aaron[3]

Jet Propulsion Laboratory, California Institute of Technology, 4800 Oak Grove Dr., Pasadena CA. 91109-8099

ABSTRACT

This study was undertaken at the Jet Propulsion Laboratory to identify salient features of two competing instrument designs and to select the design that best meets the goals of the Space Interferometry Mission. Features were examined in terms of meeting performance, cost, schedule and risk requirements.

The study included the spacecraft, the space environment, metrology considerations, stabilization of optics with temperature, spacecraft structure, complexity, and end-to-end testing among other items.

The most significant determinant was the fundamental implementation of the instrument's metrology system. The impact on the testbed program associated with the mission was considered the second most important issue. An error propagation formalism was developed to address various instrument geometries examined as part of this study. The formalism propagates metrology errors from the gauge readings through to the angle on the sky (the desired measurement of the interferometer). An introduction to the formalism is presented.

Keywords: trade-study, Space Interferometry Mission (SIM), interferometry, micro-arcsecond, Jet Propulsion Laboratory, Son of SIM, (SOS), delay line, Laser Metrology.

INTRODUCTION

The Space Interferometry Mission is proceeding from inception through development to flight in a series of steps. Within these series of steps, definition of the instrument architecture occurs. Two instrument architectures appeared to have nearly equal strengths. A trade study was used to select the one that appeared to best satisfy the mission requirements.

TRADE METHODOLOGY

[1]rlb: M/S: 306-388, Fax 818 393-9471, Tel 818 354-8677, email: rlb@huey.jpl.nasa.gov

[2]MHM: M/S: 198-138, Fax 818 393-4440, Tel 818 354-3627,email: milman@csi.jpl.nasa.gov

[3]KMA: M/S: 301-370, Fax 818 393-4878, Tel 818 354-2816, email: kim@squid.jpl.nasa.gov

A trade study begins with an assessment of how well each of the design alternatives meets the "system/mission goals" in terms of: effectiveness; cost, schedule and risk.[1] Effectiveness was defined as the ability: to make an angular measurement on the sky with a 4 μ-arcsec precision (single measurement); to observe a Vm = 20 source in 15 hr with a 4 μ-arcsec precision limited by photon noise; of the instrument to continue working for minimum of 5 years; and to cover the "UV" plane uniformly with 400 points in an imaging mode. Cost was capped at $450 M for phase C/D. The schedule had to meet a June 2005 launch date and the design must incorporate no single string failure modes (except structure and cabling).
The trade study was carried out as a Tiger Team effort over approximately four months.

STUDY ASSUMPTIONS

The basic assumptions of the study were: Astrometry was to be carried out in a 'point and stare' mode of observation; the instrument would have at least a 10-meter baseline. Further assumptions were applied to each conceptual architecture. The SIM Classic architecture would be composed of: 6 siderostats, 6 beam compressors, a switchyard, 3 delay lines, 3 beam combiners and an external truss (One or two additional siderostats would be needed to meet single string failure criteria). The SOS architecture would be composed of: 6 beam compressors, no switchyard, 3 beam combiners and 3 delay lines (An additional set of 2 beam compressors, 1 beam combiner and a delay line were expected to be needed to meet single string failure criteria). Both architectures must be capable of imaging though this function is secondary to astrometry in the mission. Nulling capabilities/differences of the two architectures were expected not to be a major discriminator. Last, angle feed forward implementation was considered similar for both architectures.

TRADE ISSUES (SIM classic vs SOS)

Architecture selection criteria were derived from a broad spectrum of engineering judgment and recent experience with the SIM classic design. These criteria were reviewed and used to guide the study. The following is a list of the criteria deemed most important to this study.

Flowdown/up of requirements

A number of requirements can be derived from higher or lower level requirements, some of these are: Architectural advantages in terms of efficiency of measurements on the sky (grid simulations); Constraints on selection of standards, fields-of-regard and interleaving of measurements for specific science goals; Constraints from overlapping vs non-overlapping fields-of-regard of the beam compressors; Observing mode differences and their contribution to measurement accuracy;

Instrument Operation

Acquisition time and complexity of procedures; Calibration, alignment and control of siderostats vs hexapods; Beam shear control.

Thermal performance

Structural thermal performance- siderostat bay vs hexapod bay; Optics- siderostat and beam compressor in an open bay vs beam compressor on a hexapod in a relatively closed pod; Thermal time constants inherent to each architecture and ease/difficulty of thermal control.

Metrology considerations

Some of these items are: Number of laser gauges 48 vs 10 laser gauges; Laser power required; Relative number of metrology beam launchers and dither hardware mechanisms; The requirement for a large metrology boom with attendant deployment concerns versus no boom; The requirement to operate precision metrology gauges on the boom; Solar glint as a function of expected relative boom and sun positions.
Further considerations are: The impact of collinearity of baseline errors and metrology triangulation errors as a function of architecture; The number of control loops and metrology feedforward requirements; The requirements on spacecraft slew, pointing and stabilization capabilities.

Overall complexity of architecture

Two axis siderostat vs hexapod mechanisms; controllable degrees-of-freedom within the designs; sensor/actuator suites required to meet performance requirements; Sunshade considerations for the beam compressors.

Data reduction and scientific error sources (excluding grid closure): non-collinearity of baselines; errors associated with hexapod vs siderostats positioning.

Testing philosophy and methodology: Component testing of structures, optics (quality, precision, complexity needed, changes over temperature, time), mechanisms (E.g., testing of hexapod with optics vs testing of a siderostat and bay optics); Ground testing of subsystems, systems and the completed spacecraft for performance that will ensure a 4 micro-arcsec science result.
The need for a large and possibly complex vacuum chamber; complex vibration isolation and control hardware; feasibility of productive thermal tests; and the cost, schedule and design of a Pseudo Star needed for Flight Article testing.

Redundancy reliability and fail soft operational modes

Some of the items in this design comparison are: The 2 tilt and 3 alignment actuators of the siderostat vs 6 hexapod actuators with possible redundant Degrees of Freedom; The use of guide/finder cameras with interferometer imagers as partial metrology/encoder backup (for feed forward). Heritage of design was also considered.

Management of risk

A primary reason to do a trade study prior to extensive funding of a project is increase the chances that the instrument will perform as envisioned. This management of risk has therefore been considered in terms of the two architectures. By explicitly noting this function, an attempt has been made to identify the steps to take to reduce the risk. Two of these are, an educational program defining program risks and the use of outside consultants to identify risks within the details of the study.

METROLOGY: A FUNDAMENTAL ISSUE

SIM classic architecture- Uses one multiple corner cube fiducial per siderostat for a total of 6 (may have 2 extra siderostats for redundancy). Each cube has one internal and 4 external metrology beam/s registered on it.

SOS (Son of SIM) architecture- Uses two multiple corner cube fiducials. It is shared by all interferometers. Each corner cube registers one external and 4 internal metrology beams. Representative metrology beam differences are shown below: The internal metrology remains at one beam per interferometer. Each beam splits to feed the two arms of the interferometer.
The results of an error propagation analysis played an essential role in validating the SOS conceptual design. Though the analysis did not show an order of magnitude difference between the two designs, it served to significantly clarify the differences thus making an informed decision possible. This analysis is presented below.

Metrology Error Propagation Analysis

The basic astrometric measurement equation is

$$d = \langle s, b \rangle, \tag{1}$$

where d is the optical pathlength delay that is measured by the interferometer, s is the normal to the wavefront of the starlight and b is the interferometer baseline vector. One of the important mission objectives is to accurately determine the directions to a grid of stars, improving *a priori* knowledge of these relative directions by nearly three orders of magnitude. Because the accuracy with which these objectives are to be met is so great, not only is the star direction vector unknown, but the interferometer baseline vector must also be estimated in an *a posteriori* manner since the knowledge provided by on–board attitude determination is several orders of magnitude inadequate. A consequence of this is that in order to generate a consistent set of equations from (1), multiple measurements of each star with different baseline orientations must be made, and dually multiple star measurements must be made with each baseline. The concept of a tile measurement refers to this latter requirement[2,3].
The general operation of the SIM instrument to synthesize the measurement d requires a combination of internal metrology measurements to determine the distance the starlight travels through the two arms of the interferometer and a measurement of the white light fringe to pinpoint the zero fringe position. Viewing dim objects, as most of the science targets are, requires a non–negligible integration time to measure the zero fringe position. The baseline vector cannot be considered stationary over this time period as its absolute length and orientation are time–varying, and the spacecraft's attitude control system bandwidth is several orders of magnitude to slow to meet the required stability. To circumvent this problem changes in the baseline orientation are estimated by two other interferometers. The interferometers that produce the attitude information are referred to as guide interferometers. In addition, changes in the absolute length of the baseline must be monitored, since this is also a time varying quantity subject to dynamic and quasistatic distortions due to onboard disturbances and thermal loads. How the system operates will be explained in greater detail in the following paragraphs.
Consider the idealized interferometer below. Here we assume the optical axes, OA and OB, of both telescopes are aligned with the starlight direction vector s. The planes Π_A and Π_B are two planes of equal phase for the planar wavefront of the starlight. x_A and x_B denote the intersection of these planes with OA and OB, respectively. Thus s is orthogonal to both Π_A and Π_B. The light of the two interferometer arms (call them the A and B arms), combine at z. Let $l_A(x_A)$ and $l_B(x_B)$ denote the (internal) optical pathlength through the two arms of the system from x_A and

x_B to z, respectively. Let T_A and T_B be the total pathlengths of the starlight through the A and B arms to z. The role of internal metrology is to measure the distances $l_A(x_A)$ and $l_B(x_B)$. The white light fringes are used to make the measurement of $T_A - T_B$. Now define

$$e(x_A) = T_A - l_A(x_A); \quad e(x_B) = T_B - l_B(x_B). \tag{2}$$

Although we cannot measure $e(x_A)$ or $e(x_B)$, their difference can be measured since $(T_A - T_B)$ is determined by the white light fringes and $l_A(x_A)$ and $l_B(x_B)$ are independently measured by internal metrology. Hence, we have the indirect measurement

$$e(x_A) - e(x_B) = (T_A - T_B) - (l_A(x_A) - l_B(x_B)), \tag{3}$$

Referring to the figure, $e(x_A) - e(x_B)$ is the delay measurement in the basic astrometric equation, and the baseline vector b is $x_A - x_B$:

$$e(x_A) - e(x_B) = < s, x_A - x_B > . \tag{4}$$

It is important to emphasize that the fiducial points x_A and x_B do not necessarily have to lie on the optical axis of the individual telescopes. Technically, they can be defined almost anywhere. Here's the argument and assumptions that make this work. Let $x_{A'} \in \Pi_A$ and $x_{B'} \in \Pi_B$. Assume the optical system is perfect so that

$$l_A(x_{A'}) = l_A(x_A), \quad l_B(x_{B'}) = l_B(x_B) \tag{5}$$

Hence,

$$e(x_A) = T_A - l_A(x_{A'}) \quad \text{and} \quad e(x_B) = T_B - l_B(x_{B'}). \tag{6}$$

Now using the fact that

$$< s, x_A - x_{A'} > = < s, x_B - x_{B'} > = 0 \tag{7}$$

for all $x_{A'} \in \Pi_A$ and $x_{B'} \in \Pi_B$, it follows that

$$< s, x_B - x_A > = < s, x_{B'} - x_B > . \tag{8}$$

Consequently from (4)

$$< s, x_{B'} - x_{A'} > = e(x_B) - e(x_A) \tag{9}$$

so long as $x_{A'} \in \Pi_A$ and $x_{B'} \in \Pi_B$.

Thus the vector between the fiducials defines the baseline, regardless of where it is situated. The underlying assumption that must be satisfied, however, is that the starlight pathlength from each fiducial to the beam combiner can be measured. Practically speaking this puts a requirement on the colinearity of the metrology path and the starlight path.

In the SOS architecture the science interferometer and the two guide interferometers share the same fiducial points x_A and x_B; hence, there is a single baseline vector for all three interferometers. This differs from the SIM CLASSIC design in which each interferometer has its own baseline. This leads to one of the major simplifications of the SOS design.

2. Error Analysis in Making an Astrometric Measurement. An important comparison study between SOS and SIM was to determine how internal and external metrology errors propagate through the two architectures. The major difference is how external metrology factors into the astrometric error.

An idealized astrometric measurement of the form (1) is used to conduct this analysis. The most salient features of the problem that must be retained in the model are the time–varying nature of the interferometer baseline and the need for a finite integration time over which the metrology and fringe measurements are averaged to complete an astrometric observation.

Let $b(t) = x_A(t) - x_B(t)$ denote the time–varying science interferometer baseline vector (which is common to all three interferometers in the SOS design). Let s denote the direction vector to the science target star. The instantaneous delay equation is

$$d(t) = < s, b(t) >, \tag{10}$$

and the time averaged delay equation has the form

$$\bar{d} = < s, \bar{b} >, \tag{11}$$

where the overbar notation denotes a time–averaged quantity. Equation (11) is the kernel of the astrometric analysis. The quantity $\bar{d}$ is what is measured by the instrument. It is computed by taking the average of the science interferometer internal metrology measurements over the observation together with the position of the white light zero fringe.

It is one of the main caveats of SIM and SOS that a single baseline vector be used for observations of multiple science target stars, for otherwise an inconsistent set of equations result. In the course of the error analysis, we will also show the rudiments of how such a single baseline vector can be produced from $\bar{b}$.

First we model the evolution of the baseline vector as a function of time. The only changes in the baseline vector that affect the delay measurement are rotations and changes in length. Thus we can write

$$b(t) = (1 + \epsilon(t))U(t)b(0). \tag{12}$$

Here $b(0)$ is the baseline vector at the beginning of the observation, $\epsilon(t)$ is proportional to the change in length, and $U(t)$ is a rotation matrix. Then to first order $b(t)$ evolves as

$$b(t) = b(0) + \omega(t) \times b(0) + \epsilon(t)b(0), \tag{13}$$

where we have used the linear approximation (valid for small rotations), $U(t) \approx I + \omega(t) \times$. Thus

$$\bar{b} = b(0) + \bar{\omega} \times b(0) + \bar{\epsilon} b(0).$$

Now $\bar{\epsilon}|b(0)|$ is obtained by averaging the external metrology measurements. It remains to determine $\bar{\omega}$. This is where the guide star information is used. In the SOS configuration (10) is valid for each of the guide interferometers, viz.

$$d_i(t) = < s_i, b(t) >,$$

where d_i is the delay measurement made with guide interferometer i and s_i is the direction vector to guide star i. (SIM requires a different baseline vector for each interferometer.) For the purpose of the error propagation analysis it may be assumed that the guide star directions are known and that $b(0)$ is known. In actuality $b(0)$ must be estimated in an *a posteriori* manner. Substituting (13) into the above we obtain the equations

$$d_i(t) = < s_i, b(0) + \omega(t) \times b(0) + \epsilon(t)b(0) >, \quad i = 1, 2. \tag{15}$$

Only the component of $\omega(t)$ that is orthogonal to $b(0)$ contributes to the delay measurement. As $\epsilon(t)|b(0)|$ is measured by the external metrology subsystem, (15) generates two equations (one for each guide interferometer) in two unknowns (the two components of $\omega(t)$ orthogonal to $b(0)$).
The estimate of $\omega(t)$ is derived from (15) in the following manner. Define the matrix

$$T = \begin{pmatrix} s_1 \times b(0) \\ s_2 \times b(0) \end{pmatrix}. \tag{16}$$

The least squares estimate of $\omega(t)$ is

$$\hat{\omega}(t) = -T^{\dagger}[d_g - S_g(1+\epsilon)b(0)], \tag{17}$$

where $T^{\dagger}$ denotes the pseudoinverse of T, $d_g = [d_1 \quad d_2]^T$ is the vector of measured pathlength differences for the guide interferometers, and S_g is the matrix formed from the guide star positions

$$S_g = \begin{pmatrix} s_1^T \\ s_2^T \end{pmatrix}. \tag{18}$$

When there is no noise in the measurements, the error in the least squares estimate is the unobservable roll component of $\omega(t)$ about the interferometer baseline. In the SOS architecture this does not contribute any error, however in the SIM architecture a second order error is incurred. A colinearity requirement on the three SIM interferometer baselines is necessary to control this error source.
Now suppose there are errors in the internal and external metrology measurements. With the expressions above we can trace their impact on the resulting astrometric equation. Let $d^e(t)$, $d_g^e(t)$, and $\epsilon^e(t)$ denote the science interferometer internal metrology measurement error, the guide interferometer internal metrology measurement error, and the external metrology measurement error, respectively. Equation (11) is written as

$$\bar{d} = < s, b(0) > + < s, \bar{\omega} \times b(0) + \bar{\epsilon} b(0) > . \tag{19}$$

Including the metrology measurement error leads to the equation

$$\bar{d} + \bar{d}^e = < s, b(0) > + < s, [\bar{\omega} + T^{\dagger}[d_g - S_g(1+\epsilon)b(0)]] \times b(0) + (\bar{\epsilon} + \bar{\epsilon}^e)b(0) > . \tag{20}$$

Thus the measurement error introduces a pathlength error P_{SOS} of

$$P_{SOS} = \bar{d}^e + < s, T^{\dagger}[d_g - S_g(1+\epsilon)b(0)] \times b(0) + \bar{\epsilon}^e b(0) >, \tag{21}$$

is introduced.
Note from the form of the astrometric equation actually produced by the instrument (e.q. (20)), that multiple science star measurements can be made using the same baseline vector $b(0)$ so long as the guide interferometers are locked onto the same stars, i.e., with the guides locked the science interferometer can shift to any other star in its field of view.
Let the variances of the random variables $\bar{\epsilon}^e|b(0)|$, $\bar{d}_g^e$ and $\bar{d}^e$ be σ_{ext}^2, $2\sigma_{guide}^2$, and σ_{sci}, respectively. Then the total variance of the delay measurement can be written as $V_{SOS} = E(P_{SOS}^2)$,

$$V_{SOS} = \sigma_{guide}^2 |T^{\dagger}(s \times b(0))|^2 + \frac{\sigma_{ext}^2}{|b(0)|^2}[S_g^T T^{\dagger T}(s \times b(0)) + s]^T b(0) b(0)^T [S_g^T T^{\dagger T}(s \times b(0)) + s] + \sigma_{sci}^2. \tag{22}$$

The term $T^\dagger(s \times b(0))$ has a major influence on the magnitude of the error. As can be seen, this term is a function of the geometry of the science and guide stars, and the baseline vector. Roughly speaking, this error is minimized when the elevation angle of the science star above the baseline is between the elevation angles of the guide stars.
Expression (22) is very similar to the noise variance for SIM. The SIM expression from[4] is

$$V_{SIM} = \sigma^2_{guide}|T^\dagger(s \times b(0))|^2 + \sigma^2_{ext}[S^T T^{\dagger T}(s \times b(0)) + \Pi^T s]^T Q [S^T T^{\dagger T}(s \times b(0)) + \Pi^T s] + \sigma^2_{sci}. \quad (23)$$

Here S is the 2×9 matrix

$$S = \begin{pmatrix} s_1 & 0_{1\times 3} & 0_{1\times 3} \\ 0_{1\times 3} & s_2 & 0_{1\times 3} \end{pmatrix}, \quad (3.3)$$

Π is a 3×9 matrix that projects a 9–vector onto its last three coordinates, and Q is the 9×9 covariance matrix of the 3–D errors of the 3 SIM interferometer baselines due to 1–D external metrology errors[4]. Assuming all 1–D metrology errors are equal, the difference in error propagation between the SOS and SIM architectures is in the middle terms of (22) and (23) that propagate the external metrology errors. The middle term in (22) involves the matrix $b(0)b(0)^T/|b(0)|^2$, while the corresponding term in (23) is the covariance matrix Q. The extent to which the SOS architecture is superior to the SIM architecture is a function of the relative importance of the propagation of the external metrology errors with respect to the other error sources. Analytical and numerical comparisons between V_{SOS} and V_{SIM} were made in[6]. Here we will briefly recount this study and its major conclusion.

4. Quantitative Results. To derive comparisons between the two architectures, numerical experiments were performed using the variance expressions (22) and (23). Guide and science star locations were selected with good geometries so that these parameters would not skew the results. The weighting of the rms error of the 1–D metrology measurements involving corner cubes corresponding to the science interferometer for SIM was varied using factors of 1, 2, 5, and 10. The corresponding metrology errors for SOS were twice these numbers. The reasoning here is that there is a greater rotation of the corner cubes in the SOS design, and thus corner cube imperfections produce a greater error than with SIM. While these factors only affect the term σ^2_{sci} in (22), they impact both Q and σ^2_{sci} in (23). Thus setting all other 1–D metrology errors to unity, and introducing the weighting term Factor=1,2,5,10, we have $\sigma^2_{sci} = (2 \times \text{Factor})^2$ in (22), $\sigma^2_{sci} = (\text{Factor})^2$ in (23), and the external metrology covariance matrix Q in (3.2) has the general form

$$Q = Q_0 + (\text{Factor})^2 \times Q_1, \quad (25)$$

where Q_0 is the covariance contribution excluding errors made by metrology measurements to guide interferometer corner cubes and Q_1 is the covariance contribution containing only measurement errors to science interferometer corner cubes.
The outcome of numerical studies involving various values for the Factor term, ranging from 1 to 10, and over several star geometries, was that the resulting rms error in the SOS design was consistently better by about a factor of two (rms). The reason for this is that the external metrology propagation error associated with the science interferometer corner cube positions in the SIM design remains a larger error than the internal pathlength science interferometer error for SOS, even though the latter is weighted by a factor of two greater (rms). Earlier numerical studies[5], that did not include the greater sensitivity of SOS design to science interferometer corner cube errors, would have concluded that SOS was a factor of 6 (rms) better than SIM.

Although these results clearly favored the SOS design, they were not in themselves the key discriminator in switching designs.

SUMMARY/CONCLUSION

After an intense 4 month Tiger Team Study the issues were presented to a select review board composed of project management, line management and outside members. Two significant and distinguishing issues were identified-

- ◇ a- Metrology system complexity (favors SOS)
  b- Testbed changes required (favors SIM classic)
- ◇ These led to a decision process as follows:
- ◇ a) ⇒ less risk with SOS
- ◇ b) ⇒ assuming the project can deal with the testbed re-direction, this negative for SOS is mitigated.

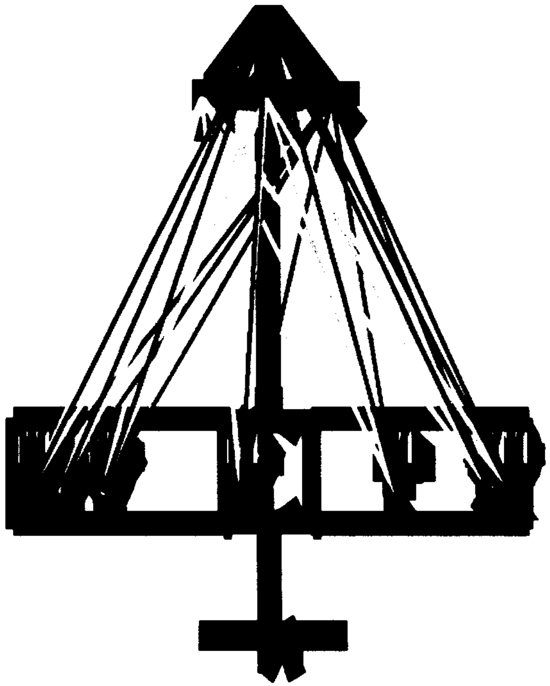
In conclusion, a new architecture has been chosen for the mission.

ACKNOWLEDGMENT

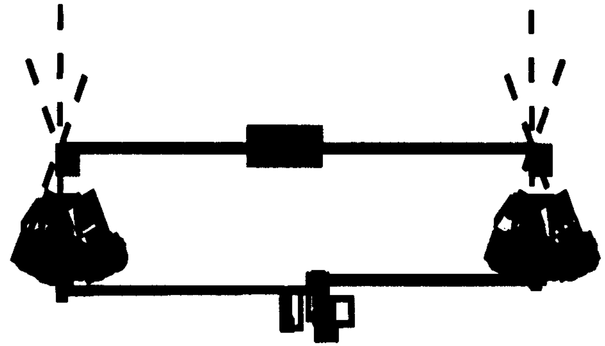
This work was carried out by the Jet Propulsion Laboratory, California Institute of Technology under contract with the National Aeronautics and Space Administration.
The authors would like to thank the SIM Program management for their support of this work, especially James Marr IV.

REFERENCES

[1] R. Shisko and P.E. Chamberlain, *NASA Systems Engineering Handbook SP-610*, June 1995.
[2] A. Boden, SIM astrometric grid simulation development and performance assessment, IOM ABODE9702031, Feb. 1997.
[3] M. Milman, Geometric simulation of SIM, IOM 3455–97-032.
[4] M. Milman, Feedforward signal error due to metrology errors, IOM 3454–97-015.
[5] M. D. Rayman, M. M. Colavita, R. N. Moster, J. Yu, et al, Orbiting Stellar Interferometer (OSI): FY 92 Study Team Progress Report, JPL D–10375, Dec., 1992.
[6] M. Milman, A First Order Look at SOS Astrometry, Dec., 1997.



SIMclassic design



SOS design

Figure 1.

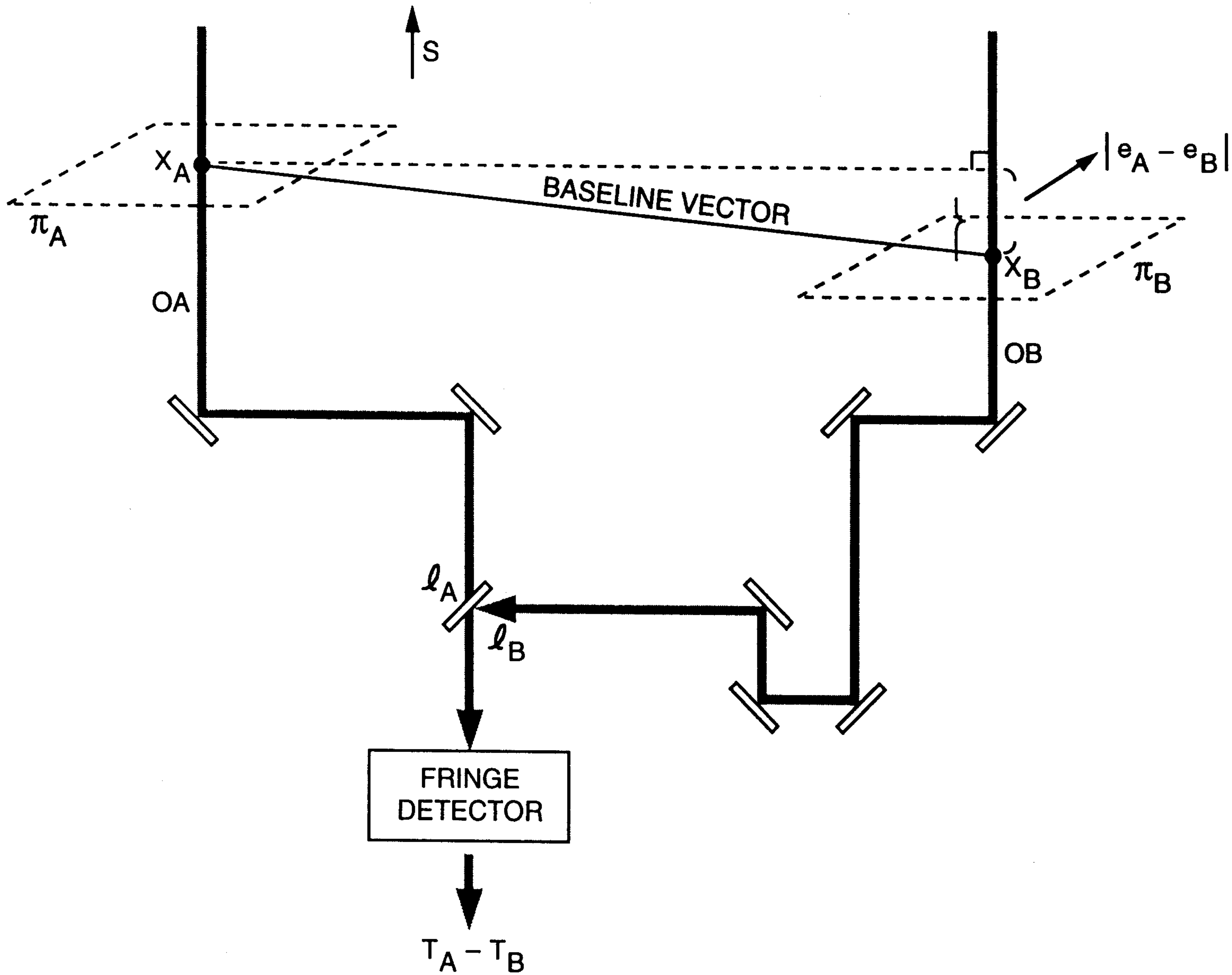

S
X_A
BASELINE VECTOR
$|e_A - e_B|$
π_A
X_B
π_B
OA
OB
ℓ_A
ℓ_B
FRINGE DETECTOR
$T_A - T_B$

Figure 2.