

USING THE CASIMIR FORCE TO MEASURE THE GRAVITATIONAL CONSTANT

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Abstract

I show that the dynamics of two coupled torsion pendulums is drastically affected by their mutual Casimir surface interaction if the masses involved are relatively close to each other ($\sim 0.1-10 \mu\text{m}$). The effect is directly related to the ratio of the masses used to the Planck mass. This system could be used to obtain an improved value of the Newtonian constant of gravitation G and it represents an accessible environment where to study the simultaneous effects of gravitation and quantum field theory.

Introduction

Despite the formidable increase both in sophistication of the instrumentation and in its detailed characterization since the publication of Cavendish's seminal paper exactly 200 years ago [1], the inconsistencies among different measurements of the Newtonian gravitational constant G are still mysteriously much larger than those of other typical natural constants. As the causes of such disagreements are still the subject of debate, much work has been devoted to not only better understand existing experimental strategies, but also to devise new ones.

The rationale to consider experiments making use of appropriate quantum processes to measure G is at least twofold. From the experimental standpoint, we have the exceptional record of verification of quantum mechanics on any scale and the high accuracy with which natural constants in the quantum domain are known. On the theoretical side, strategies of this kind may allow one to shed light on quantum gravity.

Traditionally, the search for environments where gravitation and quantum field theory play significant, simultaneous roles has been relegated to regions of space-time unamenable to direct experimentation. Some experiments based on supercooled atom interferometry have achieved impressive results in the measurement of the local value of the gravitational acceleration g . This approach ("measure g to measure G "), however, cannot result in any significant progress in our knowledge of the Newtonian constant of

gravitation, as the mass of the Earth cannot be determined with accuracy higher than that of G itself.

In this paper, I propose carrying out a direct measurement of G by means of a system of two identical torsion pendulums coupled by both their mutual gravitational interaction and by the Casimir (retarded van der Waals) surface force between two very close masses. The advantage of such an approach lies with the possibility to use a well reproducible and theoretically understood non-gravitational force to obtain a value of G less sensitive to the metrology of the masses involved. Also, I show that the behavior of this system of coupled oscillators is critically determined by the ratio of the torsion pendulum masses to the Planck mass. This represents an example of a classical system whose dynamics cannot be described without relativistic quantum field theory in the presence of gravitation.

Theoretical and Experimental Considerations

As is well known, a typical measurement of G is performed by studying either the static or the dynamic effect a well characterized perturbing mass causes on an otherwise ideally isolated system, such as a torsion pendulum. Evidently, such a procedure can yield G to within a precision not higher than that within which the field source itself is known. That is, this approach can at the most yield Gm , where m is the perturbing mass.

In principle, this problem can be mitigated by letting both the measuring

system and the perturbing mass interact under the action of a well characterized non-gravitational force. The overall dynamics of the measuring system and of the perturbing mass can then be analyzed to determine their inertial masses. From this point of view, the measuring system becomes the combination of two objects moving under the action of both gravitational and non-gravitational internal forces. In what follows, we shall investigate the use of the Casimir force between two surfaces to provide the needed non-gravitational interaction.

The motivation for this choice is of both practical and theoretical nature. On the one hand, very recent experiments on the Casimir force, carried out with a torsion pendulum in static deflection mode, have yielded results in striking agreement with quantum field theory predictions. Also, as the nature of this force is quite fundamental, our ability to model its effects rests upon well-understood theoretical principles.

Several recent developments indicate that accounting for Casimir forces may not only be ultimately rewarding, but in several cases absolutely necessary to develop a correct modeling of the instrumentation used. This is the case, for instance, in the optimization of experiments aimed at constraining new, hypothetical long-range interactions.

Generally speaking, one may mention the effort to design new instrumentation to test the law of gravity on ever shorter (sub-centimeter) scales. At the same time, there exists great interest in probing the Casimir force as

predicted by quantum field theory in regimes where the retardation is dominant, that is, on longer and longer scales (~ 1 cm). As these efforts continue, experimenters from these two areas are bound to meet on an exciting middle ground where both gravitation and quantum field theory play an equally important role.

The idea of measuring G by using two physical pendulums coupled only by their mutual gravitational interaction is not new. However, no results from this approach appear to have ever been published in the literature after the procedure was first described. The original setup includes two physical pendulums oscillating in the gravitational field of the Earth and coupled only by their mutual gravitational interaction. One pendulum is displaced and released from rest, while the other one starts also at rest from its vertical position of stable equilibrium. The value of G is then obtained from a direct measurement of the displacement of the latter pendulum as a function of time. As the coupling of the two pendulums is purely gravitational, this method can again only yield a value of Gm .

In what follows, I propose the use of two coupled torsion pendulums. The reasons are rather standard, and relate, for instance, to the fact that much lower natural frequencies of oscillation can be achieved in this way, resulting in increased values of the observables. The treatment shown, however, can easily be adapted to any other system of similarly coupled oscillators.

In order to investigate the feasibility of measuring G from the dynamics

of the two coupled torsion pendulums, we must obtain an estimate of the relative importance of gravitational and Casimir forces under typical laboratory conditions.

In his original paper, Casimir obtained an expression for the force per unit area between two neutral, perfectly conducting parallel surfaces at a distance s from each other. This force, due to the effect on the vacuum energy of the quantized electromagnetic field between two boundaries, is $F_{\text{Cas,PP}}(s) = (\pi^2/240)(\hbar c/s^4)$.

However, as it is extremely difficult in practice to deal with two surfaces which must be both extremely close and perfectly parallel, the typical experimental arrangement has been that of using a spherical surface and a plane []. If the surfaces are very close compared to their Gaussian curvature (“gently curved surfaces”), one can use the Proximity Force Theorem (PFT) to obtain a very good approximation of the surface force [].

In our case, use of the PFT to approximate the (attractive) Casimir force between two identical spherical surfaces yields:

$$F_{\text{Cas,ss}} \approx \frac{\pi^2}{740} \hbar c \frac{R}{s^3}. \quad (1)$$

Let us now consider two homogeneous spheres having the same mass and radius, m and R , respectively. Let the distance of closest approach between their surfaces be $s \ll R$. The distance s at which the Casimir force and the gravitational force are equal in magnitude can be approximately obtained by

setting $F_{\text{grav}} = F_{\text{Cas,ss}}$. This yields

$$s \approx \left(\frac{\pi^2 \hbar c R^3}{740 G m^2} \right)^{1/3} = \left[\frac{\pi^2}{740} \left(\frac{M^*}{m} \right)^2 \right]^{1/3} R, \quad (2)$$

where, interestingly, M^* is the Planck mass ($M^* = 2.18 \times 10^{-5}$ g). For two spheres with a mass of 10 g each and the approximate density of tungsten (5 g/cm³), or $R \approx .78$ cm, this distance is $s \approx 0.4$ μm , well typical of present-day Casimir force experimentation. This result justifies our expectation that the effect of the non-gravitational coupling can in fact be measured.

Casimir Force-Coupled Torsion Pendulums

Let us consider two ideal torsion pendulums each made up of two identical homogeneous, spheres of gravitational and inertial masses m_g and m_i , respectively, and of radius R whose centers are at a distance L from the attachment point of a suspension rod to a torsion string of constant κ (Fig. 1). Let us use the coordinates θ_1 and ϕ_2 to describe the position of the two pendulums, counted from the x -axis counter-clockwise and clockwise respectively. We shall assume that all motion takes place in the (x, y) plane and indicate the distance between the two attachment points O_1 and O_2 as D and the distance vector from the center of the first to the center of the second mass as \mathbf{r}_{12} . As a first approximation, we shall neglect all gravitational effects of the counterweights and rods. The equations of motion for these two rigid bodies

are then:

$$\ddot{\theta}_1 = -\frac{\kappa}{I}\theta_1 + \frac{L}{Ir_{12}}(F_g + F_C) [L \sin(\theta_1 + \phi_2) - D \sin \phi_1] , \quad (3)$$

$$\ddot{\phi}_2 = -\frac{\kappa}{I}\phi_2 + \frac{L}{Ir_{12}}(F_g + F_C) [L \sin(\theta_1 + \phi_2) - D \sin \phi_2] , \quad (4)$$

where I is the moment of inertia of each pendulum, and F_g and F_C are the magnitudes of the gravitational and Casimir forces, respectively. Notice that, because of the spherical symmetry of the masses, the Casimir force can be treated as a central force.

Linearization of the above system of coupled differential equations for θ_1 , $\phi_2 \ll 1$ rad ensures consistency with the requirement from the PFT that the two surfaces be relatively close at all times. A standard solution yields the following frequencies for the two normal modes of oscillation:

$$\Omega_1^2 = \omega_0^2 + \alpha(D - 2L) , \quad (5)$$

$$\Omega_2^2 = \omega_0^2 + \alpha D , \quad (6)$$

where $\omega_0^2 = \kappa/I$ and the parameter α is defined in terms of the equilibrium distance of the centers of the two spheres, $r_{12}(\theta_1 = 0, \phi_2 = 0) = D - 2L$ and the distance of closest approach of the two surfaces, $\delta = r_{12}(0, 0) - 2R = D - 2L - 2R$, as:

$$\alpha \equiv \frac{Gm_g^2L}{Ir_{12}^3(0, 0)} \left[1 + \frac{\pi^3}{720} \left(\frac{M^*}{m_g} \right)^2 \frac{r_{12}^2(0, 0)R}{\delta^3} \right] . \quad (7)$$

One can thus write the following solution corresponding to the initial condi-

tions $\theta_1(0) = \theta_{1,0}$, $\dot{\theta}_1(0) = 0$, $\phi_2(0) = 0$, and $\dot{\phi}_2(0) = 0$:

$$\theta_1(t) = \theta_{1,0} \cos(\omega_{\text{mod}}t) \cos \omega_{\text{av}}t, \quad (8)$$

$$\phi_2(t) = \theta_{1,0} \sin(\omega_{\text{mod}}t) \sin \omega_{\text{av}}t, \quad (9)$$

where the average and modulation frequencies are defined as $\omega_{\text{av}} \equiv \frac{1}{2}(\Omega_1 + \Omega_2)$ and $\omega_{\text{mod}} \equiv \frac{1}{2}|\Omega_1 - \Omega_2|$, respectively. Given the relative weakness of the coupling, $\omega_{\text{mod}} \ll \omega_0$ and $\omega_{\text{mod}}t \ll 1$ rad for all times of practical interest. Thus, the above result for $\phi_2(t)$ can be approximated as:

$$\phi_2(t) \approx \theta_{1,0}(\omega_{\text{mod}}t) \sin \omega_{\text{av}}t = \theta_{1,0} \frac{\alpha L}{\omega_0} t \sin \omega_{\text{av}}t. \quad (10)$$

By assuming the moment of inertia of each pendulum to be $I = 2m(\frac{2}{5}R^2 + L^2)$, and by expanding the expression for the parameter α to first order in δ/R , we find the following expression for $\phi_2(t)$:

$$\phi_2(t) \approx \frac{1}{1 + \frac{2}{5} \left(\frac{R}{L}\right)^2} \frac{\theta_{1,0}}{16\omega_0} \frac{Gm_g}{R^3} \left(1 - \frac{3}{2} \frac{\delta}{R}\right) \left[1 + \frac{\pi^3}{180} \frac{M^{*2}}{m_i m_g} \left(1 + \frac{\delta}{R}\right) \left(\frac{R}{\delta}\right)^3\right] t \sin \omega_{\text{av}}t. \quad (11)$$

Discussion and Conclusions

By assuming a torsion fiber with $\kappa \approx 10^{-8}$ kg·m²/s² for a torsion pendulum with two 10 g-masses attached at the end of massless rods 10-cm long, we find a natural frequency $\omega_0 \approx 7.1 \times 10^{-3}$ s⁻¹, or a period $T_0 \approx 15$ min. If the two nearest masses are at a distance $\delta = 1.0$ μm at equilibrium, and $\theta_{1,0} = 1$ deg, the above solution becomes $\phi_2(t) \approx 2.2 \times 10^{-7}$ rad/s $[1 + .39] t \sin \omega_{\text{av}}t$.