Radiowave scattering in the outer heliosphere

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Short title: SCATTERING IN THE OUTER HELIOSPHERE
Abstract. The Voyager 1/2 plasma wave instruments have observed low-frequency (1.5-4 kHz) radio waves apparently generated near the heliopause. The waves are found in two distinct bands. Power at the lower frequencies centered on 1.78 kHz, shows no modulation when the spacecraft is rolled about the earth-spacecraft line, indicating that the radiation is isotropic. Power in the higher band centered on 3.11 kHz, shows roll modulation as high as 60%, indicating that the source of the radiation is quite compact, subtending an angle less than 1 radian at the spacecraft. Simple estimates of the scattering of 3.11 kHz radiation from electron density fluctuations indicate that, if the radiation had originated from the distance of the heliopause, the scattering would be so large that no roll modulation should be observed. Here we show that these earlier scattering estimates were too high because they ignored the latitude variation of scattering and the inner scale of the electron density fluctuation spectrum. With these two effects properly included the expected scattering is consistent with the observations and the postulate that the radiation originates from near the nose of the heliosphere.
1. Introduction

A highlight of the Voyager mission’s interplanetary phase is the detection of low-frequency radio waves apparently generated near the boundary between the solar wind and the interstellar plasma [Kurth et al., 1984; Kurth, 1987; Gurnett et al., 1984; Gurnett, 1995; Gurnett, Allendorf, and Kurth, 1998]. These events were seen by both Voyager spacecraft, even though the spacecraft were separated (in 1992) by more than 40 AU, arguing for a source at a large distance from both spacecraft.

The Voyager 1 spacecraft is rolled about the Earth-spacecraft axis regularly to calibrate the magnetometer. The dipole antenna of the low-frequency radio wave detector is oriented perpendicular to the Earth-spacecraft axis. Thus, if a source is not located exactly on the roll axis, its apparent intensity will vary with an amplitude that depends on the location and angular extent of the source. The 1.78 kHz radiation shows no detectable roll modulation at any time, thus its source must be located exactly on the roll axis or it must be essentially isotropic. The 3.11 kHz radio emission showed weak ≤ 1% roll modulation when first detected at about 18 AU from the Sun [Kurth, 1987]. During the second detection in 1992-94, when Voyager 1 was 50-56 AU from the Sun, the modulation amplitude varied from 6% to 61% with no obvious trend [Gurnett, Allendorf, and Kurth, 1998]. This suggests that the 3.11 kHz radiation was almost isotropic at 18 AU, but narrowed to an angular diameter of about 1 radian at 50 AU.

The phase of the roll modulation determines the azimuthal angle from which the radiation originates (with 180° ambiguity). The inferred source azimuth of the early
observations was in the general direction of the nose of the heliosphere, but more recent observations show variation up to 85° from that direction [Gurnett, Allendorf, and Kurth, 1998]. The modulation amplitude depends on both the angular extent of the source and its elevation angle above the plane normal to the roll axis. The two effects cannot be separated but they can be bounded. As discussed in Sections 3 and 5, for a modulation index of 61% the apparent angular diameter of a quasi-Gaussian brightness distribution cannot exceed ≈1.0 radian regardless of its elevation angle, and the elevation angle cannot exceed about 40° even if there were no scattering.

The apparent angular size of the source is broadened by angular scattering from electron density irregularities between the source and the spacecraft. Scattering phenomena have been well-observed and quantitatively-modeled in the inner heliosphere (see [Grall et al., 1997] for references). It is known that the density variance is about 16 times stronger in the equatorial regions than over the solar poles [Coles et al., 1995]. In addition it is known that the density fluctuations are limited by an inner scale that increases approximately linearly with solar distance [Coles and Harmon, 1989]. Neither of these factors were considered in earlier estimates of the scattering of kHz radiation in the outer heliosphere [Cairns, 1995], which resulted in a serious overestimate of the scattering strength.

2. The Scattering Model

The angular distribution of radiation $B(\theta)$ scattered by fluctuations in electron density can be calculated using small-angle forward-scattering theory. In this limit $B(\theta)$
is the Fourier transform of the electric field correlation $\gamma(s) = \langle E(r) E^*(r+s) \rangle / \langle |E|^2 \rangle$, where $r$ and $s$ are vectors in the plane transverse to the propagation direction, $z$. For the electron density spectra considered here, both $B(\theta)$ and $\gamma(s)$ are compact quasi-gaussian functions with half widths (at $e^{-0.5}$) of $\theta_0$ and $s_0$ respectively. The widths are related by $\theta_0 = \lambda/(2\pi s_0)$ where $\lambda$ is the radio wavelength.

The field covariance $\gamma(s)$ is related to the spatial spectrum of electron density $\Phi_{Ne}(q)$ through the wave structure function $D(s)$:

$$\gamma(s) = \exp[-D(s)/2] \text{ where } D(s) = \int_0^L D'(sz/L, z) dz$$

$$D'(s, z) = \frac{\partial D(s, z)}{\partial z} = 4\pi r^2 \lambda^2 \int_0^\infty dq dq_y (1 - \cos(q \cdot s)) \Phi_{Ne}(q, z | q_z = 0)$$

Here $r_e$ is the classical electron radius and $L$ is the source-observer distance. We model the electron density power spectrum as an isotropic powerlaw of the form

$$\Phi_{Ne}(q) = C_n^2 q^{-\alpha-2} \exp[-(ql_i/2)^2],$$

where $C_n^2$ sets the “spectral level”, $\alpha$ is the exponent of the associated structure function at large scales, and $l_i$ is the “inner scale”. The corresponding asymptotic forms of $D'(s)$ are (e.g., [Coles et al., 1987])

$$D'(s, z) = \left(8\pi^2/\alpha 2^\alpha\right) r_e^2 \lambda^2 |\Gamma[1 - \alpha/2]/\Gamma[1 + \alpha/2]| s^\alpha C_n^2(z), \text{ for } s \gg l_i$$

$$D'(s, z) = \left(4\pi^2/2^\alpha\right) r_e^2 \lambda^2 |\Gamma[1 - \alpha/2]| l_i^{-2} s^2 C_n^2(z), \text{ for } s \ll l_i.$$
here, so we have used $\alpha = 5/3$ in the following. Observations close to the sun in the ecliptic plane are in good agreement with $C_n^2 \approx 1.8 \times 10^6 (R/AU)^{-4} m^{-6.67}$ [Armstrong and Woo, 1980]. This model did not include the inner scale, but other observations show that it is consistent with the ion inertial scale and thus varies approximately as $l_i \approx 160 (R/AU)$ km [Coles and Harmon, 1989].

If one uses the [Armstrong and Woo, 1980] model, which neglects the inner scale, and assumes that the kHz radiation comes from the anti-solar direction at a distance of 150 AU, then the computed scattering diameters for 3.11 kHz observations at $R = 18$ and $R = 50$ AU are $2\theta_s \approx 114$ and 18 radians, respectively. Evidently these angles are not consistent with the small angle forward scattering approximation and simply imply that the radiation must be nearly isotropic. Cairns [1995] also used this model to conclude that the calculated scatter-broadening was inconsistent with the observed Voyager roll modulation by a very large factor.

3. Predicted Roll Modulation

The scattering angle computed from the pure powerlaw [Armstrong and Woo, 1980] model, which assumes that $l_i \ll s_0$, implies $s_0 \approx 1.7$ km for 3.11 kHz observations at 50 AU. However the extrapolated inner scale at 50 AU, $\approx 8000$ km, is nearly four orders of magnitude larger. Incorporating this inner scale increases $s_0$ by a factor of 4.2 (to $\approx 7.1$ km) and thus reduces the predicted scattering diameter to $\approx 4.3$ radians. However the [Armstrong and Woo, 1980] model was normalized using data taken in the ecliptic plane, whereas the Voyager 1 data were taken when the spacecraft was at
high ecliptic latitude \((\approx +34 \text{ degrees}, \text{ in 1994})\). The line of sight to the candidate source (interaction region with the interstellar medium, at ecliptic latitude \(\approx +5 \text{ degrees}\)) thus passes substantially or completely through low density polar wind. Based on scattering observations of sources at high ecliptic latitude [Coles et al., 1995], the polar wind has \(C_n^2 \approx 1/16\) of that in the ecliptic. If \(s_0 < l_i\), then \(s_0 \propto (C_n^2)^{0.5}\). Thus \(s_0\) must be increased by a factor of 4. This further reduces the predicted source diameter for 3.11 kHz observations at 50 AU to \(\approx 1.0\) radians. The structure function predicted in this way for the 50 AU observations is given in Figure 1. The dashed line shows the [Armstrong and Woo, 1980] model. The solid line shows our revised model. Thus combined effects of the inner scale and the latitude variation, plus a smaller correction for including the spherical divergence of the incident radiation, reduce the estimated scattering angle by a factor of 18.

The comparison of observations and scattering theory can be made more precisely by calculating the roll modulation that would be observed if the original source is scattered into a quasi-Gaussian brightness distribution centered on a given elevation angle. The solar distance dependence of the observed roll modulation and that theoretically predicted is plotted in Figure 2. One can see that the theory matches the observations rather well in that the observed roll modulation at 3.11 kHz lies in the predicted range. The model correctly predicts no roll modulation would have been detected at 1.78 kHz. However it also predicts that roll modulation should be observed in the 1.78 kHz channel at the present distance of Voyager 1 (72 AU in January 1999).
4. Direction Finding

The predicted angular broadening is a lower bound because we have assumed that the entire path from the source to the spacecraft lies in the low density polar solar wind. Either increased scattering or increased elevation angle will reduce the roll modulation. Therefore the elevation angle prediction that best matches the observation on Figure 2 is an upper bound on the actual elevation angle. Elevation estimates from Figure 2, combined with azimuth measurements of [Gurnett, Allendorf, and Kurth, 1998], are shown in Figure 3. This plot gives the impression of a large angular spread, but some of this is due to the coordinate system. We have overplotted on Figure 3 as small circles, the location of a grid of sources, all at a solar distance of 150 AU, for ecliptic latitudes, b, of -10°, 0°, 10°, 20°, and 30° and ecliptic longitudes at 5° intervals from that of the spacecraft. Using this grid we can translate the source location into ecliptic coordinates as shown in Figure 4.

The measured azimuth's have an inherent 180° ambiguity because of the dipole radiation pattern. The elevation angles are also ambiguous as the roll modulation is invariant under a change of sign in the elevation. For each observation we have chosen the (one of four) location closest to the nose of the heliosphere because it is theoretically plausible that the radiation comes from this direction. Thus from Figure 4 we can say that the observations do not require that the apparent source location be more than about 40° from the nose of the heliosphere.
5. Discussion and Conclusions

The angular broadening predicted by our scattering model for 3.11 kHz radiation at 18 AU is much too large to satisfy the small angle forward scattering approximation, so the model cannot be used to predict the ≤1% roll modulation which is actually observed. In addition we have ignored the effects of reflection from the inner “turning point” where the refractive index goes to zero. This turning point is ≈10 AU in the ecliptic and 5 AU in the polar wind. The corresponding turning points for 1.78 kHz radiation are ≈18 AU in the ecliptic and 9 AU in the polar wind. In addition the ecliptic solar wind contains “global merged interaction regions” of substantially increased density. Thus at 18 AU the spacecraft is near a reflecting ellipsoidal inner boundary (with significant irregularity) and a number of detached reflecting “shards”. In this environment it is not surprising that the radiation would show some residual directivity as indicated by the measured 1% roll modulation. This interpretation is supported by the observation of a 26 day amplitude modulation of the 3.11 kHz radiation that was observed at 18 AU [Kurth, 1987]. It was suggested by the author that the modulation was governed by changes in the local plasma density. During a local density increase the spacecraft will come quite close to the critical surface. Ray paths arriving at that surface at a low incidence angle will turn before reaching the surface, so one might expect the flux to decrease and the roll modulation to increase.

The possibility of observing roll modulation at 1.78 kHz is interesting. If the “isotropization” of the 1.78 kHz were due solely to scattering then roll modulation
should be visible at distances greater than about 65 AU. However the 1.78 kHz radiation could be trapped in a perfectly reflecting cavity because there is also an outer reflecting boundary. If the radiation is trapped in a large, complex cavity then the radiation field anywhere within the cavity will be isotropic, even in the absence of scattering. As the 3.11 kHz radiation shows strong roll modulation it cannot be trapped, therefore the density surrounding the heliosphere must be less than the critical density of 0.12 cm$^{-3}$. However if the density surrounding the heliosphere does not drop below 0.04 cm$^{-3}$ then the 1.78 kHz radiation is trapped. If another burst of kHz radiation is observed we will be able to tell if the 1.78 kHz radiation is indeed trapped and thus either put a lower bound on the density surrounding the heliosphere or reduce the upper bound.

Our calculations confirm that scattering by turbulent density fluctuations in the solar wind has an important effect on the kHz radiation detected by Voyager 1. Scattering calculations are consistent with the observations provided that the scattering model includes the latitude variation and dissipation scale effects which have been measured much nearer the Sun. It is not surprising that the latitude variation should persist out to distances greater than 50 AU since it has been measured at several AU and the flow is quite accurately radial. The dissipation scale is known to increase linearly with distance between about 3 solar radii and 1 AU, but it is less obvious that it should continue to increase linearly out past 50 AU because the exact dissipation mechanism is not known. This strongly suggests that the dissipation is directly related to the ion inertial scale.

Scattering by turbulent density fluctuations would not be expected to shift the
apparent source location significantly because the diameter of the scattering volume is very much greater than the inner scale. However reflections from high density regions could very easily introduce multipath effects which would shift the apparent source location. Some, perhaps all, of the 40° variation in the apparent source location may be due to multipath effects.

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Figure 1. Phase structure function for 3.11 kHz observations at 50AU. Dashed curve: *Armstrong and Woo, 1980* model with $C_n^2$ determined from observations in the ecliptic. Solid curve: revised model with $C_n^2$ appropriate for polar wind turbulence and inner scale determined from observations close to the sun. The unmodified powerlaw predicts a small coherence scale and thus scattering angles more than an order of magnitude larger than observed. The solid curve predicts $s_o \approx 3.0 \times 10^4$ m, and thus scattering diameters $2\theta_o \approx 1$ rad, in good general agreement with the observations.

Figure 2. The observed and predicted roll modulation amplitude, using the *Gurnett et al., 1998* definition, versus the solar distance of the spacecraft. Different symbols are used for the three temporal groups identified by *Gurnett et al., 1998*. Two families of theoretical curves are plotted as solid lines. The leftmost family is for 3.11 kHz. Elevation angles are plotted every 15° as indicated on the 3.11 kHz curves.

Figure 3. Location of the observed 3.11 kHz radiation in (azimuth, elevation) coordinates. The elevation locations are upper bounds. The symbols are the same temporal groups as in Figure 2. The small circles are the locations that would be observed for a grid of sources at a solar distance of 150 AU at the ecliptic latitudes indicated, for ecliptic longitudes at 5° intervals from that of the spacecraft. The nose of the heliosphere is indicated by the circled “N”.
Figure 4. Location of the 1992-1994 observed 3.11 kHz radiation in ecliptic coordinates. The symbols are the same groups as in Figures 2 and 3. The location of the nose of the heliosphere and the spacecraft (circled “S”) are shown. The source elevation is an upper bound, thus the source may be further from the “S” than shown, as indicated by the arrows.
Phase Structure Function for
\( f = 3.11 \text{ kHz; } R = 50 \text{ AU} \)
Figure 3

Elevation (deg)

Azimuth (deg)

b = +30°

b = +20°

b = +10°

b = 0°

b = -10°

N

Legend symbols:

△

◇

○

▲