Ionosphere Delay Calibration and Calibration Errors for Satellite Navigation of Aircraft

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ABSTRACT

The Federal Aviation Administration (FAA) is implementing a satellite-based navigation system for aircraft using the Global Positioning System (GPS). Positioning accuracy of a few meters will be achieved by broadcasting corrections to the direct GPS signal. These corrections are derived using the wide-area augmentation system (WAAS), which includes a ground network of at least 24 GPS receivers across the Continental US (CONUS). WAAS will provide real-time total electron content (TEC) measurements that can be mapped to fixed grid points using a real-time mapping algorithm. These TECs will be converted into vertical delay corrections for the GPS L1 frequency and broadcast to users every five minutes via geosynchronous satellite. Users will convert these delays to slant calibrations along their own lines-of-sight (LOS) to GPS satellites.

Uncertainties in the delay calibrations will also be broadcast, allowing users to estimate the uncertainty of their position. To maintain user safety without reverting to excessive safety margins an empirical model of user calibration errors has been developed. WAAS performance depends on factors that include geographic location (errors increase near WAAS borders), and ionospheric conditions, such as the enhanced spatial electron density gradients found during ionospheric storms.

1. INTRODUCTION

The FAA has determined that GPS is capable of serving as the primary navigation tool for all commercial and general aviation in the United States. To have the greatest value this navigation capability must provide sufficient accuracy to perform precision approaches to airfields, as well as waypoint navigation. It must also provide this accuracy reliably.

GPS navigation fixes are produced via signal travel time calculations, and the largest error sources for civilian users are transmitter clock uncertainties and ionospheric delays. The FAA’s response is WAAS, a network of GPS receivers distributed over 24 sites in the CONUS. One purpose is to provide a “differential GPS” service, which can derive and broadcast adjustments to overcome uncertainties in the transmitter clocks. These are deliberately introduced by the Department of Defense for national security reasons. WAAS also provides a resource for observing ionospheric conditions over the CONUS and calibrating ionospheric delays for broadcast – the subject of this paper.

GPS space vehicles (SVs) transmit signals at two frequencies, L1 (1575.42 MHz) and L2 (1227.6 MHz), a combination of which give the ionospheric delay. However, the code needed to extract ranges, and thus positions, from the signals is only available to civilian users on L1. Dual-frequency “codeless” receivers are able to track the signal carrier wave at both frequencies, but these are more complex and expensive than single-frequency receivers, and are more difficult to operate in a dynamic flight environment. Ionospheric corrections will instead be provided by dual-frequency receivers at permanent WAAS reference sites (WRSs). Each WRS contributes ionospheric delay
measurements every second, which are used to update delay correction values at a series of ionospheric grid points (IGPs) across the CONUS every five minutes. These grid delays will then be transmitted to aircraft via geostationary satellite. Pilots use them by interpolating the values to their position, and applying an obliquity factor to reproduce the delay values along the slant propagation paths to SVs in their field of view.

Just as important is information on the accuracy of the delay calibrations, which will be broadcast along with the calibrations themselves. These Grid Ionosphere Vertical Errors (GIVEs) must successfully balance two opposing requirements: underestimating the calibration error could lead to a dangerous overconfidence in an aircraft’s position accuracy, while an excessively conservative error will make precision navigation impossible in all but the most ideal conditions. It is important to recognize that the GIVE is a mechanism for bounding the errors in the slant path corrections finally computed by users, and thus incorporates uncertainties introduced by a variety of factors, such as data coverage and modelling approximations.

Although ionospheric delays and errors are considered here in the context of aircraft navigation, the WAAS corrections will be broadcast openly. It becomes obvious, then, that WAAS will also be used as a source of corrections by a wide variety opportunistic users in scientific and commercial fields outside aviation.

2. **WAAS Ionosphere Calibrations**

The techniques used to derive WAAS ionospheric delays have grown directly from ionospheric TEC mapping procedures developed as an ionospheric science and space weather resource, and for other calibration purposes, such as spacecraft tracking and single-frequency ocean altimetry. The start point for these methods are LOS delay/TEC measurements derived from observations made by regional (in the case of WAAS) or global dual-frequency GPS receiver networks. These values are scaled to vertical using an obliquity function and the horizontal positions of the resulting vertical delays are defined by where the LOS pierces a model of the ionosphere. The WAAS specification for the user algorithm defines this model as having all electrons confined to a thin shell at a fixed altitude. For other applications, such as global ionospheric maps, other representations, such as a tapered slab model, can be used instead. Fig. 1 shows a map of ionospheric pierce points (PPs) resulting from five minutes of data from the FAA’s National Satellite Testbed (NSTB) network. This is used in place of WAAS here as the WRS receivers are undergoing testing at this time.

The irregularly distributed WRS delay measurements are incorporated into a dynamic mapping procedure. We estimate the vertical TEC at a set of triangular grid points that are fixed in space but allowed to vary in time. A bilinear distance-weighted interpolation algorithm is used to fit the grid point delays to discrete batches of data, typically five-minutes duration. The TEC model is written as follows:

\[ I_{rs}(t_i) = M(E) \sum_{i=1}^{3} W_i(\phi_{pp}, \lambda_{pp}) V_i \]

![Figure 1. Ionospheric pierce points for five minute measurements made by the NSTB network of receivers.](image)
where \( L_{rs}(t) \) is the ionospheric delay between receiver \( r \) and \( SV \) \( s \) at time \( t \), \( M(E) \) is the obliquity function, \( W_{i}(\phi_{pp}, \lambda_{pp}) \) is a distance weighting factor which depends on the position of the PP within a grid tile, and \( V_{i} \) is the delay value at triangular grid point \( i \).

Fitting data to this model is done using a Kalman filter algorithm, which optimally solves for all values \( V_{i} \). Rather than using a physical or empirical ionospheric model, as new measurements become available the filter continuously updates values \( V_{i} \) modelled as stochastic processes, in which the new values are correlated with their previous ones. This solution is produced in a solar-geomagnetic reference frame as the ionosphere varies slowly in this frame. Translation to the Earth-fixed WAAS IGPs is performed as the final step.

Calculations made by Raytheon, the WAAS prime contractor, indicate that WAAS requirements can be met if the vertical delay corrections are accurate to 60 cm, \( 1\sigma \) (3.7 TEC units; 1 TEC unit (TECU) = \( 10^{16} \) electrons/m\(^2\)). To meet the goal of providing a precision approach service GIVE values must be less than 2 meters (12.2 TECU) a substantial fraction of the time (> 90%). To ensure safety, the GIVE should also bound the true user calibration errors over 99.9% of the time.

3. IONOSPHERIC CALIBRATION ERRORS

An error in the ionospheric calibration will translate directly to a navigation error. So that aircraft can calculate the precision with which GPS defines their position a delay error value for each IGP, the GIVE, will also be broadcast.

In general, an error value is derived from a statistical calculation involving the original measurements, but there are several factors that make this unsuitable for GIVE derivation on its own. For GIVE the robustness of such a calculation depends on data availability around the IGPs. Although this is sufficient in the central part of the CONUS it is generally reduced near coasts and land borders, for simple geometric reasons. It is therefore prudent to increase the uncertainty of the corrections where data are sparse. In the current implementation the GIVE thus depends on the distribution of WRS PPs as well as the spread of delay measurements about the fit.

The other factors in the delay calculation and user reconstruction that also increase the GIVE concern possible discrepancies between the ionospheric representation used and its actual behavior, and are discussed below. The GIVE produced is ultimately a means of bounding the user slant delay errors, rather than simply a vertical delay error.

3.1 Statistical Error

The Kalman filter is able to propagate the measurement noise through to the estimated parameters, providing statistical errors in the vertical delay values thus derived. The square-root variance of the vertical delay estimate, \( \sigma_{SE} \), depends on the quantity and spatial distribution of the data, and thus increases at the boundaries of the WAAS coverage. The \( \chi^{2} \) ("chi-squared") goodness-of-fit metric is also computed, to help in circumstances where the estimate of \( \sigma_{SE} \) provided by the filter is poor. The final expression for the statistical error term is thus

\[
\text{GIVE}_{SE} = 3.3 \alpha \sigma_{SE} \sqrt{\chi^{2}}
\]

where \( \alpha \) is a scaling factor used to account for non-Gaussian statistics. The factor of 3.3 is introduced as the GIVE is designed to bound 99.9% of errors, i.e. \( 3.29\sigma \). \( \chi^{2} \) is given by
\[
\chi^2 = \frac{1}{N} \sum_{i=1}^{N} \frac{(\text{Fit}_i - \text{Data}_i)^2}{\sigma_i^2}
\]

where \(\text{Data}_i\) is a WRS delay measurement with noise \(\sigma_i\), and \(\text{Fit}_i\) is the slant delay at the corresponding measurement PP computed by a user. The lowest value of \(\chi^2\) used is unity to prevent underestimating the statistical error when the data and fit are in good agreement due to happenstance.

\(GIVE_{SE}\) concerns the fitting procedure used to produce the vertical TEC maps, and does not reproduce errors incurred as users take the broadcast corrections and apply them to slant raypaths. Additional terms are required to reproduce these.

### 3.2 Spatial Decorrelation

The spatial variability, or decorrelation, of the ionosphere becomes important when WRS measurements and user PPs are widely separated. We therefore calculate an additional GIVE term that depends on the distance between the user PP and the WRS measurements. Since the locations of user PPs are unknown during GIVE generation, we compute a “worst-case” distance \(D_{\text{MAX}}\) for a user in the region surrounding an IGP. This GIVE term is thus

\[
GIVE_{\text{DEC}} = \beta V_{\text{MAX}} D_{\text{MAX}}
\]

where \(V_{\text{MAX}}\) is the largest vertical ionospheric delay in the four quadrants surrounding the IGP, included since we expect that spatial decorrelation errors increase with the overall level of the ionosphere. \(\beta\) is a scaling factor based on estimates of decorrelation derived from previous experience at JPL.

### 3.3 Gradient

WRS measurements are mapped to vertical, and IGP calibration delays to slant paths, using the obliquity function, \(M(E)\), which is based on the thin shell ionospheric model. \(M(E)\) depends on the elevation of a slant LOS, but is independent of the azimuth. Calibration errors for users will therefore arise that depend, in part, on the local horizontal electron density gradients found in the real ionosphere that cause azimuthal delay variation. Such effects are accounted for by the GIVE term

\[
GIVE_{\text{GRAD}} = \gamma V_{\text{MAX}}
\]

where \(V_{\text{MAX}}\) is an estimate of the maximum delay gradient surrounding the IGP, and \(\gamma\) is a scaling factor based on studies of obliquity function errors made at JPL. The gradient estimate is derived from the structure of the vertical delay map itself, and may not capture small-scale spatial structures.

### 3.4 Combined GIVE

Currently used values of the scaling factors are given in Table 1. The individual terms above are combined in root-sum-square fashion to form the total GIVE, \(GIVE_{\text{TOT}}\), and then rounded up to one of a discrete set of values available for transmission by WAAS.

\[
GIVE_{\text{TOT}} = \sqrt{GIVE_{\text{FE}}^2 + GIVE_{\text{DEC}}^2 + GIVE_{\text{GRAD}}^2}
\]
This method of combining GIVE terms is appropriate when the different error contributions are statistically uncorrelated. We have found that a direct summing of the different terms tends to produce an overly conservative GIVE.

<table>
<thead>
<tr>
<th>GIVE parameter</th>
<th>Value</th>
<th>Comments</th>
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<tbody>
<tr>
<td>α</td>
<td>1.4</td>
<td>&gt;1 as errors are non-Gaussian</td>
</tr>
<tr>
<td>β</td>
<td>1.0 x 10⁻⁷/meter</td>
<td>Increases GIVE by 5% at 500km</td>
</tr>
<tr>
<td>γ</td>
<td>1.0 x 10⁶ meters</td>
<td>Increases GIVE by 0.5m (3.1 TEC units) when gradient is 0.5cm/km (max. daytime value)</td>
</tr>
</tbody>
</table>

Table 1. GIVE parameter values.

4. ALGORITHM TESTING

The main features of the ionospheric delay calibrations and errors are illustrated here by some test results using data from the NSTB (fig. 2). Independent data for comparison is provided by a separate network of receivers across the CONUS, also shown in fig. 2. This data benefits from post-processing and is therefore generally considered to provide results closer to actual user conditions than the NSTB data.

Results are shown for three different levels of ionospheric and geomagnetic activity in August 1998, summarized in Table 2. Note that although high Ap indices correspond with disturbed ionospheric conditions in these examples, this is not always the case.

<table>
<thead>
<tr>
<th>Activity level</th>
<th>Ap</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quiet</td>
<td>1</td>
<td>August 25th, 1998</td>
</tr>
<tr>
<td>Moderate</td>
<td>64</td>
<td>August 6th, 1998</td>
</tr>
<tr>
<td>Severe</td>
<td>144</td>
<td>August 27th, 1998</td>
</tr>
</tbody>
</table>

Table 2. Dates and ionospheric/geomagnetic conditions analyzed.

4.1 Quiet Conditions

Contour plots showing snapshots of night and daytime delay corrections and GIVEs on August 25, 1998, a “quiet ionosphere” day, are show in figs. 3 - 6. As would be expected, delays are greater during the daytime. The GIVEs are also increased during the daytime, and generally increase toward the boundaries of the WAAS coverage, i.e. the CONUS. They can increase rapidly outside this region, e.g. fig. 5.

Recalling that optimization of the GIVE is a tug-of-war between conservatively estimated errors and the desire to improve precision, fig. 7 shows how this is resolved in this case. It shows a comparison between all user errors computed from the GIVEs for the day, and the corresponding “true” errors, where the latter are the differences between the NSTB-derived corrections and
the independent data. Points above the x=y line indicate that the GIVE has successfully bounded the true error. Points well above the x=y line indicate GIVEs that were too conservative.

The GIVE failed to bound the true error on 11 occasions out of 11,815, a high degree of reliability. Fig. 7 shows that 92.9% of the GIVEs were ≤ 2m, which is not overly conservative.
4.2 Moderate Conditions

On August 6, 1998, the ionosphere was moderately disturbed, a condition that arises roughly every two weeks in northern mid-latitudes. Delay corrections and GIVEs were similar to those for quiet conditions and are therefore not displayed here. This time the GIVE failed to bound the true error in 7 cases out of 13,494, while 93.3% of the GIVEs were ≤ 2m.

4.3 Severe Conditions

An intense geomagnetic storm occurred on August 27, 1998, where the Ap index reached over 140. Such storms will affect the ionosphere a couple of times a year, and a snapshot of conditions is displayed in figs. 9 and 10. On this occasion the main electron enhancement occurred early on the 27th, so nighttime plots are shown. Delay corrections and GIVEs are slightly higher over most of the CONUS than for the quiet ionosphere day, with gradients increasing rapidly toward the south.

![Delay Correction Map](image1)

![GIVE Map](image2)

Figure 9. Ionospheric delay correction map for disturbed conditions.

Figure 10. GIVE map for disturbed ionosphere conditions.

The most significant feature for a WAAS user is the increase in incidents of error underestimation by the GIVE; 92 out of 13,197 measurements (0.7%) (fig. 8). On a few occasions this discrepancy exceeded 1m. A second feature is the increase in the average GIVE; 16.6% were ≥ 2m, while the “true” error remained below 2m in 99.4% of cases. This indicates that, despite the increase in underestimates, more conservative GIVEs were generally produced. This would decrease availability of precision approach coverage.

A detailed analysis of this day reveals that most underestimates occurred near a narrow but deep trough in the ionosphere. Its width was below the resolution of the WRS distribution, so that user LOSs transiting the region measured delays significantly lower than the WAAS corrections. Severe ionospheric storms cause electron depletions and enhancements which vary in size and location across the mid-latitudes, so this trough can be considered typical of the features that WAAS must contend with. We are developing algorithms applied to the reference measurements so that trough features can be detected and used to further increase the GIVE in the specific regions they occur.

5. CONCLUSIONS

A wide area ionospheric correction technique developed by JPL has been applied to the FAA’s WAAS delay correction scheme. Under normal ionospheric conditions delay values and gradients increase towards the south, as observed using a CONUS receiver array and an independent
network. Delay error values do not necessarily increase with the delay, being more dependent on network coverage.

The values that bound the user slant errors, the GIVEs, have been derived using an expression containing three terms: 1) the statistical error from the measurement system; 2) spatial decorrelation of the ionosphere; 3) errors from converting the vertical corrections to slant paths. Testing shows that the GIVE successfully bounds users errors in the great majority (>99.9%) of cases under quiet and moderately disturbed ionospheric conditions. Under severely disturbed conditions the GIVE also bounds user errors in most (99.3%) cases. However, there are more incidences where the GIVE was significantly overestimated, which would reduce the availability of the precision navigation capability that WAAS is intended to provide.

Preliminary analysis indicates that differences between quiet and severely disturbed performance is due to ionospheric depletions and enhancements that are smaller than the spatial scale of the WAAS correction grid. Continuing development of the delay correction calibration will therefore involve a study of performance near small-scale ionospheric features found during storms.

A detailed investigation of the three GIVE terms and their dependencies is also warranted, as improving the accuracy of the GIVE directly increases the availability of precision navigation. For example, the statistical error term depends on the noise characteristics of the receivers used, while the other terms depend in part on the tuning of the Kalman filter. Further studies performed on a daily basis are planned, to tune GIVE parameters and assess algorithm performance under a wider variety of conditions.

ACKNOWLEDGEMENTS

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