Reconfigurable Control for the Formation Flying of Multiple Spacecraft

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Abstract

Several results on the reconfigurable control architecture for the formation flying of multiple spacecraft are presented. In this direction, simple control laws are combined with logic-based switching to propose a hybrid control architecture for leader reassignment, leader-following capturing, and dealing with control saturations.

Keywords: Formation flying; Linear matrix inequality; Logic-based switching

1 Introduction

Formation flying (FF) has been identified as an enabling technology for many of the NASA's 21$^{\text{st}}$ century missions, among them, the Space Technology-3 (ST-3) and the Terrestrial Planet Finder (TPF). Formation flying involves flying a group of spacecraft in a particular pattern while maintaining precise (possibly time varying) relative position, velocity, attitude, and angular velocity, with respect to each other [3], [6]. Since traditional spacecraft control is often concerned with measuring and maintaining the same quantities for a single spacecraft with respect to an inertial reference frame, the analogous FF control and estimation problems are often an order of magnitude more challenging than those encountered traditionally for a single spacecraft [1], [4], [8]. In order to make the FF control problems at least *similar* to the single spacecraft case, an approach based on leader-following has been proposed by Wang and Hadaegh [7]. The basic idea in leader-following is to designate a particular frame (or multiple frames) in the formation as the reference frame(s) of interest and measure and control the states of the rest of the formation with respect to them. The purpose of the present paper is to show that linear matrix inequalities (LMIs) [2] can be combined with logic-based switching schemes to propose reconfigurable control architecture for the formation flying of multiple spacecraft.

The outline of the paper is as follows. In §2 the notation used in the paper is presented. Simple control laws for the formation flying control are then derived in §3 based on the leader-following concept. In §4, §5, and §6, the control laws derived in §3 are combined with logic-based switching to propose a hybrid control architecture for leader reassignment, leader-following capturing (defined subsequently), and dealing with control saturations.

2 Notation

Formation flying consists of flying a group of spacecraft in a particular pattern. To be able to express the time evolution of the formation and design the corresponding control laws, it is convenient that a reference frame is attached to each spacecraft. We shall always assume that these reference frames are induced from a dextral set of three orthonormal vectors. Let the formation have n spacecraft labeled as $\mathbf{1}, \mathbf{2}, \ldots, \mathbf{n}$. Let $\mathcal{F}^{\mathbf{i}}$ denote the reference frame attached to the i-th spacecraft; $\mathcal{F}^{\mathbf{I}}$ on the other hand shall designate the inertial reference frame. For the inertia and the mass of the i-th spacecraft we use $I^{\mathbf{i}}$ and $m^{\mathbf{i}}$, respectively. The force and torque acting

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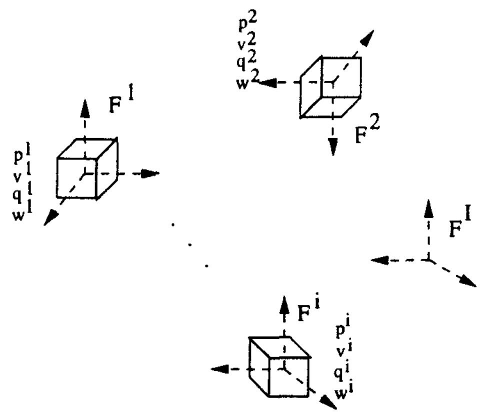

Figure 1: Formation Coordinates

upon $\mathbf{i}$ are denoted by $f^{\mathbf{i}}$ and $\tau^{\mathbf{i}}$; for the mass normalized force we used $u^{\mathbf{i}} := \frac{f^{\mathbf{i}}}{m^{\mathbf{i}}}$. The time derivative with respect to $\mathcal{F}^{\mathbf{i}}$ shall be denoted by $\frac{d}{dt_i}$; likewise, $\frac{d}{dt}$ will be used for the time derivative with respect to $\mathcal{F}^{\mathbf{I}}$. $r^{\mathbf{ij}}$ denotes the position of the origin of $\mathcal{F}^{\mathbf{i}}$ with respect to $\mathcal{F}^{\mathbf{j}}$; $r^{\mathbf{i}}$ is the position of the origin of $\mathcal{F}^{\mathbf{i}}$ with respect to $\mathcal{F}^{\mathbf{I}}$. The desired position of the origin of $\mathcal{F}^{\mathbf{i}}$ with respect to $\mathcal{F}^{\mathbf{j}}$ shall be denoted by $r_d^{\mathbf{ij}}$, and by $r_d^{\mathbf{i}}$ when $\mathbf{j} = \mathbf{I}$. The velocity of the origin of $\mathcal{F}^{\mathbf{i}}$ with respect to $\mathcal{F}^{\mathbf{j}}$, the velocity of the origin of $\mathcal{F}^{\mathbf{i}}$ with respect to $\mathcal{F}^{\mathbf{I}}$, the desired velocity of the origin of $\mathcal{F}^{\mathbf{i}}$ with respect to $\mathcal{F}^{\mathbf{j}}$, and the desired velocity of the origin of $\mathcal{F}^{\mathbf{i}}$ with respect to $\mathcal{F}^{\mathbf{I}}$, shall be denoted by $v^{\mathbf{ij}}$, $v^{\mathbf{i}}$, $v_d^{\mathbf{ij}}$, and $v_d^{\mathbf{i}}$, respectively. The vector $[r^{\mathbf{i}} \ v^{\mathbf{i}}\ ]^T$ shall be referred to as the state of the i-th spacecraft and will be denoted by $x^{\mathbf{i}}$. Similar notations are used for the attitude and the angular velocity of $\mathcal{F}^{\mathbf{i}}$ with respect to $\mathcal{F}^{\mathbf{j}}$: $q^{\mathbf{ij}}$ and $\omega^{\mathbf{ij}}$ are the attitude and the angular velocity of $\mathcal{F}^{\mathbf{i}}$ with respect to $\mathcal{F}^{\mathbf{j}}$ and $q_d^{\mathbf{ij}}$ and $\omega_d^{\mathbf{ij}}$ are the desired angular velocity and attitude of $\mathcal{F}^{\mathbf{i}}$ with respect to $\mathcal{F}^{\mathbf{j}}$ (refer to Figure 1). All other notations are standard: $\Re^n$ denotes the real Euclidean space of dimension n; $||.||_\infty$ and $||.||$ are used for the infinity norm and the 2-norm for vectors and matrices. The cross product matrix induced by the vector $x = [x_1\, x_2\, x_3]^T$ is the matrix,

$$[x] := \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix}$$

3 Simple Control Laws for Leader-following

In this section we go over some simple control laws for formation flying that are derived based on the state feedback synthesis procedure which uses LMIs as its building block. These control laws can be used for the control of the formation pattern under two different measurement scenarios. First, we consider the situation where inertial measurements are available to both the leader(s) and the follower(s); then we comment on the case where the follower(s) measurements are done with respect to its own moving reference frame. In all subsequent sections, we say that $\mathbf{i}$ is the leader of $\mathbf{j}$ if $r_d^{\mathbf{j}}$ is an affine function of $r^{\mathbf{i}}$ which is twice differentiable.

3.1 Inertial Reference Frame Measurements

Let $\mathbf{i}$ be the leader of $\mathbf{j}$ during the time interval $[t_0, t_f]$. The desired position of $\mathbf{j}$ is thus expressed as,

$$r_d^{\mathbf{j}}(t) = r^{\mathbf{i}}(t) + h^{\mathbf{ij}}(t), \quad t_0 \le t \le t_f.$$

The error expression for $\mathbf{j}$ is then simply,

$$e^{\mathbf{j}}(t) = r_d^{\mathbf{i}}(t) - r^{\mathbf{j}}(t) = r^{\mathbf{i}}(t) - r^{\mathbf{j}}(t) + h^{\mathbf{ij}}(t).$$

Assuming that $h^{\mathbf{ij}}$ is twice differentiable on $[t_0, t_f]$, the above expression can be differentiated twice with respect to the inertial reference frame to obtain (recalling that $u^{\mathbf{i}} = \frac{f^{\mathbf{i}}}{m^{\mathbf{i}}}$),

$$\frac{d^2 e^{\mathbf{j}}(t)}{dt^2} = u^{\mathbf{i}}(t) - u^{\mathbf{j}}(t) + \frac{d^2 h^{\mathbf{ij}}(t)}{dt^2}. \qquad (3.1)$$

By letting,

$$u^{\mathbf{j}}(t) = u^{\mathbf{i}} + \frac{d^2 h^{\mathbf{ij}}(t)}{dt^2} + \bar{u}^{\mathbf{j}}(t), \qquad (3.2)$$

one obtains,

$$\frac{d^2 e^{\mathbf{j}}(t)}{dt^2} = -\bar{u}^{\mathbf{j}}(t) \qquad (3.3)$$

(i.e., feedback linearization). The equation (3.3) can be expressed as,

$$\begin{bmatrix} \dot{z}_1(t) \\ \dot{z}_2(t) \end{bmatrix} = \overbrace{\begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}}^{A^{\mathbf{j}}} \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} + \overbrace{\begin{bmatrix} 0 \\ -I \end{bmatrix}}^{B^{\mathbf{j}}} \bar{u}^{\mathbf{j}}(t), (3.4)$$

where $z_1(t) = e^{\mathbf{j}}(t)$, $z_2(t) = \frac{de^{\mathbf{j}}(t)}{dt}$, and the matrices $A^{\mathbf{j}}$ and $B^{\mathbf{j}}$ are defined as suggested by (3.4).

The control design based on the state feedback synthesis for leader-following is thus reduced to finding the term for $\bar{u}^{\mathbf{j}}$ using the following LMI,

$$A^{\mathbf{j}}Q + Q(A^{\mathbf{j}})^T + B^{\mathbf{j}}Y + Y^T(B^{\mathbf{j}})^T < 0, \quad (3.5)$$
$$Q > 0, \quad (3.6)$$

and let $K^{\mathbf{j}} = YQ^{-1}$ [2]. Hence, given that **i** is the leader of **j** the control law for **j** has the form,

$$u^{\mathbf{j}}(t) = u^{\mathbf{i}}(t) + \frac{d^2h^{\mathbf{ij}}(t)}{dt^2} + YQ^{-1}z^{\mathbf{j}}(t), \quad t_0 \le t \le t_f.$$

Employing the above control law by the follower spacecraft **j** guarantees that the origin is the globally asymptotically stable equilibrium of the error function $z(t)$, and thereby, $r^{\mathbf{j}}(t) \to r^{\mathbf{j}}_d(t)$ as $t \to \infty$.

3.2 Moving Reference Frame Measurements

We shall now briefly go over the situation where the measurements are done in the moving frame attached to the follower spacecraft. Feedback linearization is then used to reduce this case to that considered in the previous section.

Again let **i** be the leader of **j** during the time interval $[t_0, t_f]$. Contrary to the case considered previously, we would like to obtain an expression which describes the time evolution of $e^{\mathbf{j}}$ in $\mathcal{F}^{\mathbf{j}}$ (as opposed to $\mathcal{F}^{\mathbf{I}}$). Proceeding from (3.1), one has,

$$\begin{aligned}\frac{d^2e^{\mathbf{j}}(t)}{dt_{\mathbf{j}}^2} + \frac{d\omega^{\mathbf{j}}(t)}{dt_{\mathbf{j}}} \times e^{\mathbf{j}}(t) + 2\omega^{\mathbf{j}}(t) \times \frac{de^{\mathbf{j}}(t)}{dt_{\mathbf{j}}} \\ +\omega^{\mathbf{j}}(t) \times (\omega^{\mathbf{j}}(t) \times e^{\mathbf{j}}(t)) \\ = (u^{\mathbf{i}}(t) - u^{\mathbf{j}}(t)) + \frac{d^2h^{\mathbf{ij}}(t)}{dt^2}; \quad (3.7)\end{aligned}$$

the last term on the right hand side of (3.7) can of course be represented in $\mathcal{F}^{\mathbf{j}}$.

The rate of change of the angular velocity $\omega^{\mathbf{j}}$ with respect to $\mathcal{F}^{\mathbf{j}}$ (or $\mathcal{F}^{\mathbf{I}}$) is related to the applied torque on the spacecraft via the Euler's equation,

$$\frac{d\omega^{\mathbf{j}}(t)}{dt} = (I^{\mathbf{i}})^{-1}(\tau^{\mathbf{j}}(t) - \omega^{\mathbf{j}}(t) \times (I^{\mathbf{i}}\omega^{\mathbf{j}}(t))). \quad (3.8)$$

Let $z_1(t) = e^{\mathbf{j}}(t)$, $z_2(t) = \frac{de^{\mathbf{j}}(t)}{dt_{\mathbf{j}}}$, and $z_3(t) = \omega^{\mathbf{j}}(t)$. The dynamics of **j** can thus be represented as,

$$\begin{aligned}\dot{z}_1(t) &= z_2(t), && (3.9)\\ \dot{z}_2(t) &= -2z_3(t) \times z_2(t) - (I^{\mathbf{i}})^{-1}(\tau^{\mathbf{j}}(t) \\ &\quad - z_3(t) \times I^{\mathbf{i}}z_3(t)) - z_3(t) \times (z_3(t) \times z_1(t)) \\ &\quad + (u^{\mathbf{i}}(t) - u^{\mathbf{j}}(t)) + \frac{d^2h^{\mathbf{ij}}(t)}{dt^2}, && (3.10)\\ \dot{z}_3(t) &= (I^{\mathbf{i}})^{-1}(\tau^{\mathbf{j}}(t) - z_3(t) \times I^{\mathbf{i}}z_3(t)). && (3.11)\end{aligned}$$

Consider two distinct situations.

1. j has constant angular velocity: Consider the case where,

   $$\tau^{\mathbf{j}}(t) = z_3(t) \times I^{\mathbf{i}}z_3(t), \quad (3.12)$$

   i.e., the angular velocity of **j** during the leader-following remains constant. The dynamical equations (3.9)-(3.10) can then be written as,

   $$\dot{z}_1(t) = z_2(t),$$
   $$\dot{z}_2(t) = W_1z_1(t) + W_2z_2(t) + u^{\mathbf{i}}(t) - u^{\mathbf{j}}(t),$$

   where,

   $$W_1 = z_3z_3^T - \|z_3\|_2^2I, \quad \text{and} \quad W_2 = -[2z_3].$$

   Consider again the change of variable of the form $u^{\mathbf{j}}(t) = u^{\mathbf{i}}(t) + \frac{d^2h^{\mathbf{ij}}(t)}{dt^2} + \bar{u}^{\mathbf{j}}(t)$; then,

   $$\begin{bmatrix} \dot{z}_1(t) \\ \dot{z}_2(t) \end{bmatrix} = \overbrace{\begin{bmatrix} 0 & I \\ W_1 & W_2 \end{bmatrix}}^{A^{\mathbf{j}}} \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} + \overbrace{\begin{bmatrix} 0 \\ -I \end{bmatrix}}^{B^{\mathbf{j}}} \bar{u}^{\mathbf{j}}(t).$$

   Define the matrices $A^{\mathbf{j}}$ and $B^{\mathbf{j}}$ as suggested above; we can now proceed to solve the LMI,

   $$A^{\mathbf{j}}Q + Q(A^{\mathbf{j}})^T + B^{\mathbf{j}}Y + Y^T(B^{\mathbf{j}})^T < 0, \quad (3.13)$$
   $$Q > 0, \quad (3.14)$$

   and let,

   $$u^{\mathbf{j}}(t) = u^{\mathbf{i}}(t) + \frac{d^2h^{\mathbf{ij}}(t)}{dt^2} + YQ^{-1}z^{\mathbf{j}}(t), \quad t_0 \le t \le t_f;$$

   note that only the definition of the matrix $A^{\mathbf{j}}$ has been modified from that used previously to reflect the fact that the error vector is now measured in the moving coordinate frame attached to the follower.

2. j has non-constant angular velocity: If the angular velocity of **j** does not remain constant during the leader-following, then we can use feedback linearization to linearize the dynamics in such a

way that the LMI approach above can still be adopted. For this purpose it suffices to let,

$$a^j = -2z_3(t) \times z_2(t) - (I^i)^{-1}(\tau^j(t) - z_3(t) \times I^i z_3(t)) - z_3(t)(z_3(t) \times z_1(t)), \quad (3.15)$$

and let $u^j(t) = u^i(t) + \frac{d^2h^{ij}(t)}{dt^2} + \bar{u}^j(t) + a^j(t)$; as before the expression for $\bar{u}^j(t)$ is found be solving an LMI.

In both scenarios considered above, the control law for the *leader* spacecraft i can also be based on the state feedback synthesis. For this purpose it suffices to let $u^i(t) = YQ^{-1}z(t) + \frac{d^2r_d^i(t)}{dt^2}$ where the matrices Y and Q are found from the LMI (3.5)-(3.6) by letting,

$$A^i = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}, \quad \text{and} \quad B^i = \begin{bmatrix} 0 \\ -I \end{bmatrix};$$

however z is now simply $r_d^i(t) - r^i(t)$.

4 Leadership Re-Assignment

The designation of the leader, aside from its associated hardware and software considerations and the required communication protocol, is rather arbitrary. It is thus of interest to consider a situation where the leader assignments are time varying, and that any subset of the spacecraft in the formation can assume the leadership role.[1]

Suppose that at a particular instance of time, i is the leader of j; in this case the control law of §4 (when inertial measurements are available) can be implemented as,

$$u^i(t) = Kz^i(t) + \frac{d^2r_d^i(t)}{dt^2},$$
$$u^j(t) = Kz^j(t) + u^i(t) + \frac{d^2h^{ji}(t)}{dt^2},$$

where $z^i(t)$ is the state error observed by i at time t. Since $h^{ij}(t) = -h^{ji}(t)$, when the leadership assignment is reversed and j is the leader of i, the control laws can be reconfigured as,

$$u^i(t) = Kz^i(t) + u^j(t) - \frac{d^2h^{ji}(t)}{dt^2},$$
$$u^j(t) = Kz^j(t) + \frac{d^2r_d^j(t)}{dt^2};$$

[1] Leadership reassignment becomes specially relevant when one looks beyond ST-3 and TPF type space interferometry missions to the formation flying of large number of spacecraft.

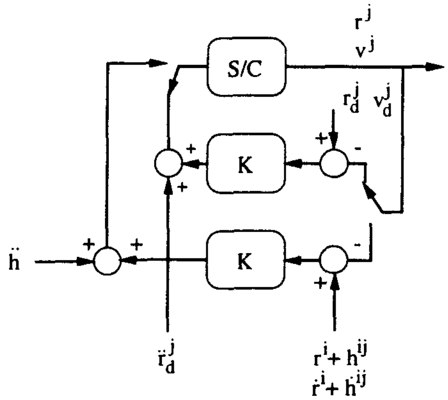

Figure 2: Switching for Leader Reassignment

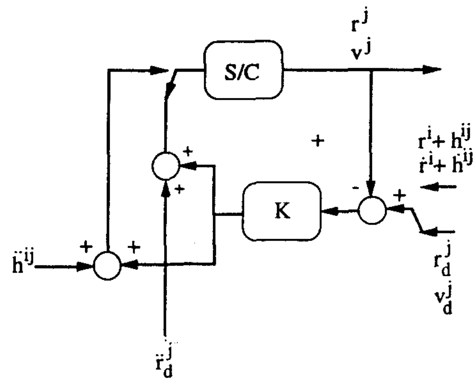

Figure 3: Leader-following capturing

refer to Figure 2

5 Leader-Following Capturing

We consider a situation where a free spacecraft is captured by a leader following scheme. Building on the control laws developed in §4, the corresponding block diagram representing the switching control system can be drawn as in Figure 3. Note that we have considered the situation where the isolated spacecraft is not assigned as a leader; if this is in fact the case, then its control law will not be changed from when the spacecraft was free. However, in this latter situation, the control law for the new followers of the new leader spacecraft changes according to the procedure presented in §5.

6 Control Saturations

We now examine the scenario where the j-th spacecraft, j, following i in a leader-following scheme, is also avoiding control saturation by switching between two or more controllers. In order to simplify the presentation, we shall assume in the rest of this section that $\frac{d^2 h^{\mathbf{ji}}(t)}{dt^2} = 0, \quad t_0 \leq t \leq t_f$. Let m denote the 2-norm of the maximum allowable mass normalized force on each spacecraft in the formation; that is, we require that, $\|u^{\mathbf{j}}(t)\| \leq m, \quad t_0 \leq t \leq t_f$. Now since $u^{\mathbf{j}}(t) = YQ^{-1}z(t) + u^{\mathbf{i}}(t)$ we require that, $\|u^{\mathbf{i}}(t) + YQ^{-1}z(t)\| \leq m, \quad t_0 \leq t \leq t_f$. Note that although $\mathbf{j}$ has no prior knowledge about the values of $u^{\mathbf{i}}$ it has to choose Q and Y such that the control constraint is satisfied. To cope with this lack of knowledge on the values of $u^{\mathbf{i}}(t)$, we proceed to present a controller switching mechanism which satisfies the control constraint, in face of the lack of a priori knowledge of the values of $u^{\mathbf{i}}(t)$ by the follower spacecraft. The only assumption which is required for the proposed approach to work is that, $\|u^{\mathbf{i}}(t)\| < m, \quad t_0 \leq t \leq t_f$. Let us start with the stronger requirement, $\|YQ^{-1}z(t)\| \leq m - \|u^{\mathbf{i}}(t)\| = m^{\mathbf{i}}\ (t), \quad t_0 \leq t \leq t_f$ in order to satisfy the control constraint. Let $\mathcal{E}_{t_0} = \{z : z^T Q_{t_0}^{-1} z \leq 1\}$, where Q_{t_0} is a positive definite matrix which is chosen such that $z(0)$ belongs to $\mathcal{E}_{t_0}$ by solving an LMI, in conjunction with,

$$AQ_{t_0} + Q_{t_0}A + BY_{t_0} + Y_{t_0}^T B^T < 0. \qquad (6.16)$$

For small values of δt, if $x_d^{\mathbf{j}}(t_0) = x_d^{\mathbf{j}}(t_0 + \delta t)$ for $t \in [t_0, t_0 + \delta t]$ and we use the controller $K_{t_0} = Y_{t_0} Q_{t_0}^{-1}$, then it would be the case that $z(t_0 + \delta t) \in \mathcal{E}_{t_0}$. In fact, if $x_d^{\mathbf{j}}(t)$ remains constant, then $z(t) \in \mathcal{E}_{t_0}$ for all $t \in [t_0, t_f]$. In this situation, in order to guarantee that the saturation constraint is not violated, we can augment the above LMIs with another one,

$$\begin{bmatrix} Q_{t_0} & Y_{t_0}^T \\ Y_{t_0} & m^{\mathbf{i}}(t) \end{bmatrix} \geq 0,$$

since [2] $\max_{t \geq 0} \|u^{\mathbf{j}}(t)\| = \max_{t \geq 0} \|Y_{t_0} Q_{t_0}^{-1} z(t)\| \leq \max_{x \in \mathcal{E}_{t_0}} \|Y_{t_0} Q_{t_0}^{-1} z(t)\| \leq \lambda_{\max}(Q_{t_0}^{-1/2} Y_{t_0}^T Y_{t_0} Q_{t_0}^{-1/2})$. The problem is that in general, one cannot guarantee that $z(t_0 + \delta t) \in \mathcal{E}_{t_0}$, nor does the above discussion addresses the situation where $m^{\mathbf{i}}(t)$ does not remain constant. We are thus led to incorporate logic-based switching in conjunction with above LMIs to address both of these scenarios. Let $\underline{m}^{\mathbf{i}} := \min_{t \in [t_0, t_f]} \|m^{\mathbf{i}}(t)\|$; solve the semi-definite program,

$$\min_{Q_{t_0}, Y_{t_0}, \alpha} \alpha$$

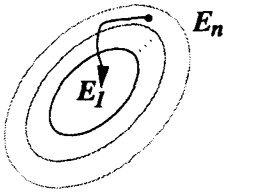

Figure 4: Ellipsoids for Control Switching

$$\begin{gathered} AQ_{t_0} + Q_{t_0}A^T + BY_{t_0} + Y_{t_0}^T B^T < \alpha I, \\ Q_{t_0} > 0, \quad \alpha < 0, \\ \begin{bmatrix} 1 & z^T(0) \\ z(0) & Q_{t_0} \end{bmatrix} \geq 0, \\ \begin{bmatrix} Q_{t_0} & Y_{t_0}^T \\ Y_{t_0} & \underline{m}^{\mathbf{i}} \end{bmatrix} \geq 0, \end{gathered}$$

We then proceed from time t_0 and considered the various scenarios which can occur at time $t_0 + \delta t$:

1. $z(t_0 + \delta t) \in \mathcal{E}_{t_0}$ and $m^{\mathbf{i}}(t)$ has remained constant.
2. $z(t_0 + \delta t) \in \mathcal{E}_{t_0}$, however $m^{\mathbf{i}}$ has changed over the interval $[t_0, t_0 + \delta t]$.
3. $z(t_0 + \delta t) \notin \mathcal{E}_{t_0}$, whether or not $m^{\mathbf{i}}$ has remained constant.

For each scenario above, we were able to show that an LMI can be solved to address the control saturation problem [5]. Moreover, the resulting proposed switching mechanism results in a hybrid dynamical system for which the origin is the globally asymptotically stable equilibrium point.

7 Conclusion

Several reconfigurable control strategies for the formation flying of multiple spacecraft were presented. In this direction, it was demonstrated that by employing feedback linearization and linear matrix inequalities, in conjunction with simple switching mechanisms, various formation control and management issues could be addressed.

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