DEVELOPING FORCE-LIMITED RANDOM VIBRATION TEST SPECIFICATIONS FOR NONLINEAR SYSTEMS USING THE FREQUENCY-SHIFT METHOD

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EXTENDED ABSTRACT

Background

Over a period of years it has been recognized that the classical test approach of using only a motion controlled envelope of input acceleration levels may be too conservative, and may induce excessive vibration responses at equipment resonant frequencies. This problem has been recognized in the literature for some time, and with its identification have emerged proposals for alternative vibration control techniques. Of these techniques, the force limiting approach has shown the most promise and consequently, has been the subject of the most recent developmental work. In particular, the so called dual control method, which involves limiting force levels as well as acceleration levels at the input to the test article, has been very successfully used in the test laboratory to ameliorate overtest conditions on a wide variety of hardware. This technique, first proposed explicitly by Murfin in 1968 and more recently developed by Smallwood (1990) and Scharton (1994, 1995), forms the current state of the art.

In the most recent formulation of the force-limiting problem, the source-load vibratory system is modeled according to the modal- and residual-mass (MRM) two-degree-of-freedom (2DOF) system. In this model, the source and load are not idealized as lumped masses but instead they are treated, more realistically, as distributed masses. For a given mode in such a multi-degree-of-freedom (MDOF) system, there is an effective modal mass that participates in the motion, and a complementary, effective residual mass that does not. At a given mode both the source and load distributed masses can be modeled as discrete effective modal and residual masses connected at the interface, as shown in Fig. 1.

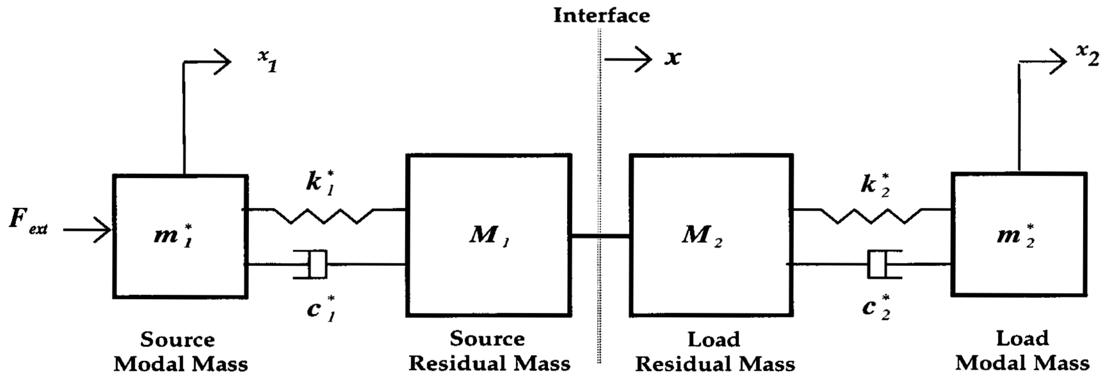

Figure 1. Modal- and Residual-Mass 2DOF Source-Vibratory System

In force-limiting specifications developed according to the so called frequency-shift method, the shaker is limited to deliver a peak interface force equal to the product of the acceleration spectral envelope and the *frequency-shifted values of the load dynamic mass* evaluated at the system *resonant frequencies*. The load dynamic mass, in general a complex quantity, is a frequency response function defined as the force acting on the load divided by its acceleration. Analytically, this may be expressed as

$$|F_c(\omega)| = |m_2(\omega)|_{resonant} |a_c(\omega)|$$

and

$$S_{F_c F_c}(\omega) = |m_2(\omega)|^2_{resonant} S_{a_c a_c}(\omega)$$

for deterministic and random excitations, respectively; the symbol S denotes power spectral density (PSD) of a random process. Both conventional and force-limited vibration testing using the frequency-shift technique assume that the control acceleration spectrum $|a_c(\omega)|$ properly envelopes the interface accelerations $|a(\omega)|_{peak}$ seen in service. The resulting expression for the load dynamic mass evaluated at the system resonances for this MRM 2DOF system are then used to develop the force limits. This limiting technique can accomplish a significant reduction in the interface force, thereby considerably reducing responses at equipment resonances.

New Development

Clearly, nonlinear stiffness and damping effects can commonly occur under normal service conditions. Various materials, some metallic springs for example, obey Hooke's law for small displacements; however, at larger displacements significant departures from linear behavior can occur. Furthermore, it is a well known characteristic of metals under high strains to move from linear to nonlinear elastic behavior and then to nonlinear plastic behavior. Various non-metals, some rubber compounds for example, exhibit distinctly nonlinear elastic behavior even under low strains. Likewise, many structural materials can exhibit highly nonlinear damping behavior, particularly when loaded under high stresses outside of their elastic range. In such cases, energy loss mechanisms such as plastic slip, localized plastic strain, crystal plasticity, cyclic plastic flow, or cyclic deformation predominate. For these materials, the damping term in the force law becomes nonlinear and the corresponding hysteresis loop is nonelliptical.

Incorporating both stiffness and damping nonlinearities into force-limited, random vibration test specifications is attempted in this paper. For this purpose, the technique of statistical linearization is used in conjunction with the frequency shift method in the MRM 2DOF system to derive force-limiting specifications for a nonlinear load mass modeled as a Duffing, Rayleigh damped, and linear plus quadratically damped oscillators, respectively. Generally, statistical linearization is more robust over a wider range of nonlinearities than traditional perturbation methods, and is simpler to apply than Wiener-Volterra series expansions. The equations of motion for a nonlinear MDOF vibratory system can be written most generally in matrix form as

$$\mathbf{M}\,\ddot{\mathbf{x}} + \mathbf{C}\,\dot{\mathbf{x}} + \mathbf{K}\,\mathbf{x} + \Theta(\mathbf{x},\dot{\mathbf{x}},\ddot{\mathbf{x}}) = \mathbf{f},$$

where $\mathbf{M}$ is the system mass matrix, $\mathbf{C}$ is the system damping matrix, $\mathbf{K}$ is the system stiffness matrix, $\mathbf{x}$ is the system displacement vector, $\mathbf{f}$ is the system force vector, and Θ is a vector function containing the system nonlinearities. The crux of the statistical linearization technique lies in reformulating the nonlinear system as the equivalent linear system

$$(\mathbf{M}+\mathbf{M}_e)\,\ddot{\mathbf{x}} + (\mathbf{C}+\mathbf{C}_e)\,\dot{\mathbf{x}} + (\mathbf{K}+\mathbf{K}_e)\,\mathbf{x} = \mathbf{f},$$

where the linearizing matrices $\mathbf{M}_e$, $\mathbf{C}_e$, and $\mathbf{K}_e$ are chosen to minimize a suitable error criteria. If the response is approximated as Gaussian, it can be shown that the optimal choices for the matrices' linearization are given by the expression

$$\begin{bmatrix} \mathbf{k}_i^e \\ \mathbf{c}_i^e \\ \mathbf{m}_i^e \end{bmatrix} = E\left\{\begin{bmatrix} \dfrac{\partial \Theta_i}{\partial \mathbf{x}} \\ \dfrac{\partial \Theta_i}{\partial \dot{\mathbf{x}}} \\ \dfrac{\partial \Theta_i}{\partial \ddot{\mathbf{x}}} \end{bmatrix}\right\},$$

where $\mathbf{m}^e_i$, $\mathbf{c}^e_i$, and $\mathbf{k}^e_i$ are the i^{th} rows of the matrices $\mathbf{M}_e$, $\mathbf{C}_e$, and $\mathbf{K}_e$, respectively; and E denotes the mathematical expectation operator. A thorough treatment of this technique can be found in Roberts and Spanos (1990).

The normalized force-limiting specification for each nonlinear system considered is determined for a range of nonlinear stiffness and damping coefficients and is compared with its linear counterpart over the same range of effective mass parameters. A representative, plot for the Duffing load oscillator in the MRM 2DOF model is displayed in Fig. 2.

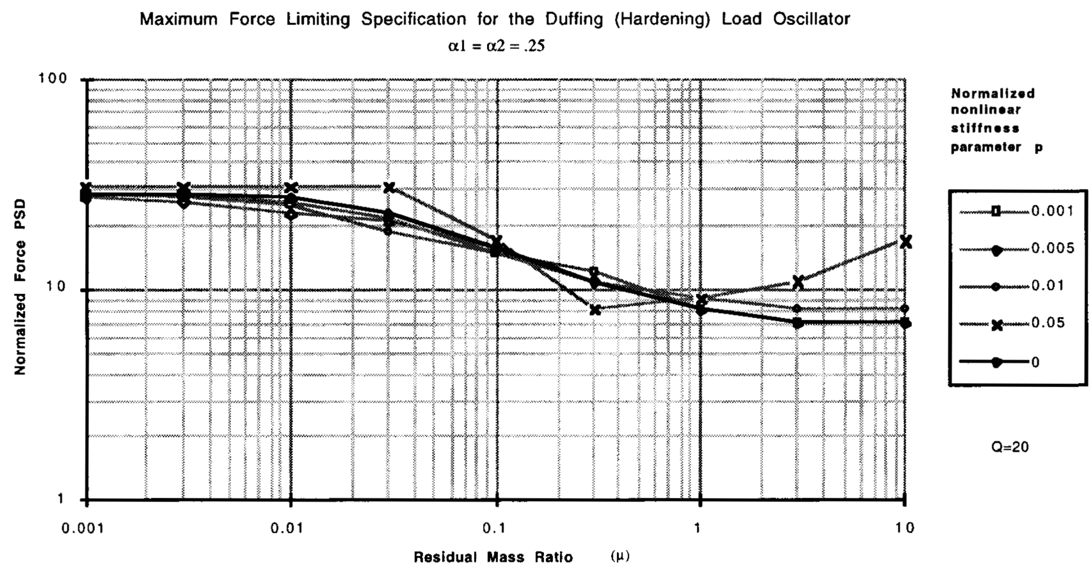

Figure 2. Force Limiting Specification for the Duffing Load Oscillator
$a_1 = a_2 = .25$

Similar results for the Rayleigh damped load oscillator in the MRM 2DOF model are displayed in Fig. 3.

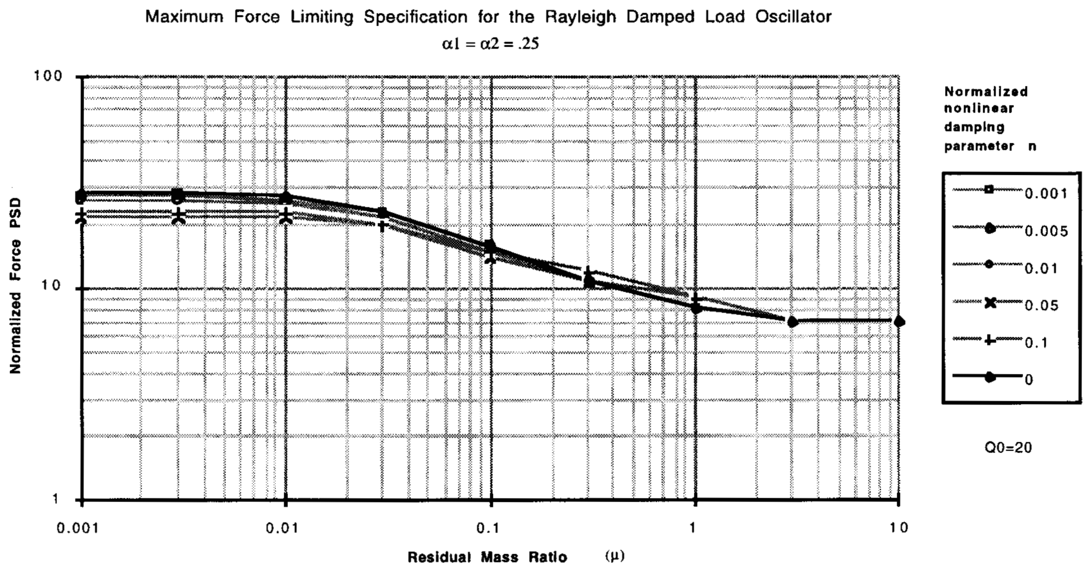

Figure 3. Force Limiting Specification for the Rayleigh Load Oscillator
$a_1 = a_2 = .25$

For each graph, families of curves show the normalized force limit PSD $\left(\dfrac{S_{F_c F_c}(\omega)}{S_{a_c a_c}(\omega)\, M_2^2} \right)$ plotted as a function of the residual mass ratio (μ) for selected values of the damping (ζ_1, ζ_2), source residual mass ratio (α_1), load residual mass ratio (α_2), and the normalized nonlinear stiffness parameter $(\rho\sigma_0^2)$ or normalized nonlinear damping parameter $(\eta\sigma_0^2)$. Detailed definitions of all the parameters appearing in the preceeding figures will be given in the full body of the paper.

It is hoped that this paper will establish a reasonable and practical approach to account for nonlinear effects in equipment where force-limited techniques are used for random vibration testing.

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