A Reduced Complexity Highly Power/Bandwidth Efficient
Coded FQPSK System with Iterative Decoding

Marvin K. Simon, Dariush Divsalar
Jet Propulsion Laboratory, 4800 Oak Grove Drive, Pasadena, CA 91109
California Institute of Technology, 1200 E. California Blvd., Pasadena, CA 91125

Abstract

Based on a representation of FQPSK as a trellis-coded modulation, this paper investigates the potential improvement in power efficiency obtained from the application of simple (small number of states) outer codes to form a concatenated coding arrangement with iterative decoding. Several possible configurations for the concatenation are suggested and specific numerical results are presented for one of these in order to demonstrate the large coding gains that are achievable even when using a reduced complexity FQPSK receiver. The end result of these investigations is a system which has application in scenarios requiring a high degree of both power and bandwidth efficiency.
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Introduction

Feher-patented QPSK (FQPSK) [1-5] whose generic form finds its roots in cross-correlated PSK (XPSK) [6] is a highly spectral efficient modulation that is derived from staggered offset quadrature raised cosine (SQORC) [7] modulation but unlike the latter maintains a nearly constant envelope\(^1\) by manipulating the pulse shapes of the inphase (I) and quadrature (Q) baseband signals via a cross-correlation mapping. To achieve additional spectral efficiency and true constant envelope, filtered (with a proprietary design) and hard-limited variants of FQPSK referred to as FQPSK-B [1,8] have been implemented in commercial hardware and achieve bandwidth efficiencies comparable to Gaussian Minimum-Shift-Keying (GMSK) [9], a modulation used in the Global System for Mobile (GSM) cellular mobile communication system [10] currently deployed in Europe. Because of its demonstrated ability to achieve high data rates (FQPSK-B modems operating at a data rate of 20 Mbps are currently available as an off-the-shelf product from several commercial vendors), FQPSK-B has been recommended for adoption as a standard by the Consultative Committee on Space Data Systems (CCSDS) and the Inter Range Instrumentation Group (IRIG) [11]. FQPSK-B has also been adopted by the US DoD joint services Advanced Range Telemetry (ARTM) program as their Tier I modulation for missile, aircraft, and range applications to replace existing PCM/FM systems.

As is true for most bandwidth efficient modulation (BEM) schemes, the price paid for a high degree of spectral efficiency is a degradation in bit error rate (BER) performance. FQPSK (or FQPSK-B) is no exception to this rule. Specifically, using the traditional (symbol-by-symbol detection) FQPSK receiver, the SNR degradation relative to the highly bandwidth inefficient binary phase-shift-keying (BPSK) is already 1.4 dB at a BER of 10\(^{-3}\). Recently, a new interpretation of FQPSK as a cross-correlated trellis-coded quadrature modulation (XTCQM) [12] was introduced in [13] and by virtue of replacing the traditional receiver with a trellis (Viterbi) demodulator allowed for the possibility of achieving a significant improvement in

\(^1\)The maximum fluctuation in the envelope of the modulated signal is 0.18 dB.
power efficiency. While the full benefit of the power efficiency improvement was obtained using the optimum receiver, i.e., one that implements a 16-state trellis [13], a reduced complexity Viterbi receiver with only 2 states per I and Q channel has been shown [14] to yield FQPSK-B performance within a few tenths of a dB of the optimum one.

While the previous two contributions [13,14] are quite significant in their own right in both enhancing the BER performance and simplifying the receiver complexity, the desire to achieve additional power efficiency while at the same time maintaining the high degree of bandwidth efficiency inherent in the modulation motivated the authors to investigate the application of error correction coding to FQPSK. Here again, as in [14], the motivation was accompanied by the desire to maintain a simple receiver architecture. As such we investigated the coupling (through an interleaver) of very simple short constraint length codes with the convolutional coding inherent in the FQPSK modulation itself [13] to determine how much further SNR improvement is achievable. This paper reports the results of the somewhat dramatic findings of these investigations.

The Trellis-Coded Interpretation of FQPSK and its Reception

As implied above, in its generic (unfiltered) form, FQPSK is conceptually the same as XPSK introduced in 1983 by Kato and Feher [6]. This technique was in turn a modification of the previously introduced (by Feher et al [15]) interference and jitter free QPSK (IJF-QPSK) with the express purpose of reducing the 3 dB envelope fluctuation characteristic of IJF-QPSK to 0 dB (or as close to that as possible) thus making it appear constant envelope. It is further noted that using a constant waveshape for the even pulse and a sinusoidal waveshape for the odd pulse, which was the case considered in [6], IJF-QPSK becomes identical to the SQORC scheme introduced by Austin and Chang [7]. The means by which Kato and Feher achieved their 3 dB envelope reduction was the introduction of an intentional but controlled amount of cross-correlation between the I and Q channels. This cross-correlation operation was applied to the IJF-QPSK (SQORC) baseband signal prior to its modulation

\[^2\]Needless to say there is an inevitable bandwidth expansion that occurs proportional to the inverse of the code rate. The hope is to produce a coding (power) gain that more than compensates for this bandwidth expansion.

\[^3\]The reduction of the envelope from 3 dB to 0 dB occurs only at the uniform sampling instants on the I and Q channels, i.e., in between these sampling instants the envelope has a small amount of fluctuation as previously stated. For this reason XPSK (or FQPSK) is referred to as being "pseudo"- or "quasi"-constant envelope.
onto the I and Q carriers (see Fig. 1). Specifically, this operation was described by mapping in each half symbol the 16 possible combinations of I and Q channel waveforms present in the SQORC signal into a new\(^4\) set of 16 waveform combinations chosen in such a way that the cross-correlator output is time continuous and has unit (normalized) envelope at all I and Q uniform sampling instants. By virtue of the fact that the cross-correlation mapping is based on a half symbol characterization of the SQORC signal, there is no guarantee that the slope of the cross-correlator output waveform is continuous at the half-symbol transition points. In fact, it has been shown [13] that for a random data input sequence such a discontinuity in slope occurs one quarter of the time.

By restructuring the cross-correlation mapping into a symbol-by-symbol (rather than half symbol-by-half symbol) representation as proposed in [13], the slope discontinuity referred to above is placed in evidence and at the same suggests a means to eliminate it.\(^5\) This representation also has the advantage that it can be described directly in terms of the data transitions on the I and Q channels and thus the combination of IIF encoder and cross-correlator in Fig. 1 can be replaced simply by a single modified cross-correlator (to be discussed shortly).

A further and more important advantage of the reformulation as a symbol-by-symbol mapping is the ability to design a receiver for FQPSK (or enhanced FQPSK) that specifically exploits the correlation introduced into the modulation scheme to significantly improve power efficiency or equivalently, BER performance. Such a receiver which takes a form analogous to those used for trellis-coded modulations will yield significant performance improvement over receivers that employ symbol-by-symbol detection thereby ignoring the inherent memory of the modulation.

Fig. 2 is an illustration of the trellis-coded interpretation of FQPSK. The input data bit stream is split into time-aligned I and Q symbol streams each at half the bit rate, i.e., \(1/T_s = 1/2T_b\). Each of these symbol streams is passed through rate 1/3 convolutional encoders (note that the I and Q channel encoders are different). The output bits of these encoders can be considered to be grouped into 3 categories – those that only influence the choice of the signal in the same channel, those that only

\(^4\)Of the 16 possible cross-correlator output combinations, only 12 of them are in fact new, i.e., for 4 of the input I and Q combinations, the cross-correlator outputs the identical combination.

\(^5\)Since the rate at which the spectral sidelobes of a modulation's power spectral density (PSD) roll off with frequency is related to the smoothness of the underlying waveforms, one would anticipate that elimination of the slope discontinuity would enhance spectral efficiency. Indeed this has been shown to be the case [13] and results in what is referred to there as enhanced FQPSK.
influence the choice of the signal in the other channel, and those that influence the
choice of the signal in both channels (hence the notion of cross-correlation mapping).
For example, of the three bits that eminate from the I encoder in Fig. 2, \( I_z \) is used to
determine the signal transmitted on the I channel, \( Q_0 \) is used to determine the signal
transmitted on the Q channel and \( I_z = Q_i \) is used to determine both the signals
transmitted on the I and Q channels. Letting \( d_{in} \) and \( d_{qn} \) respectively denote the (+1,−1)
I and Q data symbols in the \( n \)th transmission interval and \( D_{in} \equiv (1−d_{in}) / 2 \) and
\( D_{qn} \equiv (1−d_{qn}) / 2 \) their (0,1) equivalents, then the mappings appropriate to the I and Q
encoders of Fig. 2 are

\[
\begin{align*}
I_0 &= D_{qn} \oplus D_{q,n-1}, \quad Q_0 = D_{l,n+1} \oplus D_{ln} \\
I_1 &= D_{q,n-1} \oplus D_{q,n-2}, \quad Q_1 = D_{ln} \oplus D_{l,n-1} = I_2 \\
I_2 &= D_{ln} \oplus D_{l,n-1}, \quad Q_2 = D_{qn} \oplus D_{q,n-1} = I_0 \\
I_3 &= D_{ln}, \quad Q_3 = D_{qn}
\end{align*}
\]

(1)

Corresponding to the four I channel coded bits (two from the I encoder output and two
from the Q encoder output) and likewise for the four Q channel coded bits a pair of
binary coded decimal (BCD) indices \((i,j)\) is defined in accordance with

\[
\begin{align*}
i &= I_1 \times 2^3 + I_2 \times 2^2 + I_3 \times 2^1 + I_0 \times 2^0 \\
j &= Q_0 \times 2^3 + Q_2 \times 2^2 + Q_3 \times 2^1 + Q_0 \times 2^0
\end{align*}
\]

(2)

These indices range over the set of integers between 0 and 15 and are used to select the
baseband signals \( s_i(t) \) and \( s_j(t) \) to be transmitted on the I and Q channels, respectively.
The set of waveforms (of duration \( T_s \)) from which \( s_i(t) \) and \( s_j(t) \) are selected in each
transmission (symbol) interval are illustrated in Fig. 3 and defined as follows:

\[
s_1(t) = \begin{cases} 
A, & -T_s / 2 \leq t \leq 0 \\
1 - (1 - A) \cos^2 \frac{\pi t}{T_s}, & 0 \leq t \leq T_s / 2 
\end{cases}
\]

\[
s_9(t) = -s_1(t)
\]

\[
s_2(t) = \begin{cases} 
1 - (1 - A) \cos^2 \frac{\pi t}{T_s}, & -T_s / 2 \leq t \leq 0 \\
A, & 0 \leq t \leq T_s / 2 
\end{cases}
\]

\[
s_10(t) = -s_2(t)
\]

\[
s_3(t) = 1 - (1 - A) \cos^2 \frac{\pi t}{T_s}, -T_s / 2 \leq t \leq T_s / 2
\]

\[
s_{11}(t) = -s_3(t)
\]

\[
s_4(t) = A \sin \frac{\pi t}{T_s}, -T_s / 2 \leq t \leq T_s / 2
\]

\[
s_{12}(t) = -s_4(t)
\]
\[
\begin{align*}
  s_5(t) &= \begin{cases} 
  A \sin \frac{\pi t}{T_s}, & -T_s/2 \leq t \leq 0 \\
  \sin \frac{\pi t}{T_s}, & 0 \leq t \leq T_s/2
  \end{cases}, \\
  s_6(t) &= \begin{cases} 
  \sin \frac{\pi t}{T_s}, & -T_s/2 \leq t \leq 0 \\
  A \sin \frac{\pi t}{T_s}, & 0 \leq t \leq T_s/2
  \end{cases}, \\
  s_7(t) &= \sin \frac{\pi t}{T_s}, \quad -T_s/2 \leq t \leq T_s/2
\end{align*}
\]

Note that for any value of \( A \) other than unity, e.g., \( A = 1/\sqrt{2} \) (the value suggested in [6] as leading to minimum envelope fluctuation), \( s_5(t) \) and \( s_6(t) \) as well as their negatives \( s_{13}(t) \) and \( s_{14}(t) \) will have a discontinuous slope at their midpoints (i.e., at \( t = 0 \)) whereas the remaining twelve waveforms all have a continuous slope throughout their defining interval. This confirms statements made earlier and also suggests the further enhancement in spectral efficiency discussed in [13] obtained by redefining \( s_5(t), s_6(t), s_{13}(t), s_{14}(t) \) so as not to have a midpoint slope discontinuity.\(^6\)

Also, all sixteen waveforms in (3) have zero slope at their endpoints and thus concatenation of any pair of these as results when data is sequentially transmitted will similarly not result in a slope discontinuity.

Finally, the I and Q baseband signals, \( s_i(t) \) and \( s_q(t) \), are offset (by half a symbol) relative to one another, modulated onto quadrature carriers and then summed for transmission over the RF channel.

The above trellis-coded characterization of FQPSK is in principle an \( M \)-ary signaling scheme, i.e., a given pair of I and Q data symbols results in the transmission of a given pair of I and Q waveforms in each signaling interval. However, because of the presence of the I and Q encoders and the cross-correlation mapping, restrictions are placed on the allowable sequences of waveforms that can be transmitted in each of these channels, e.g., the I and Q waveform sequences will always be continuous. It is these restrictions on the transitional behavior of the

\(^6\) Since our interest in this paper is focussed on improving the power efficiency of the scheme we shall not pursue the distinction between FQPSK and enhanced FQPSK since as shown in [13] the two schemes have the same SNR behavior (as measured by minimum squared Euclidean distance).
transmitted signal that result in the narrow spectrum characteristic of FQPSK. More important, however, is the fact that the trellis-coded structure of the transmitter suggests that the optimum receiver of FQPSK should be a form of trellis demodulator. In [13] it was shown that such an optimum receiver (see Fig. 4) consists of a bank of 16 biased matched filters7 followed by a 16 state trellis demodulator. The simulated BER performance of FQPSK using the receiver in Fig. 4 was presented in [13] and is illustrated here in Fig. 5 along with that obtained by using a conventional OQPSK receiver. Also shown for purpose of comparison is the BER performance of conventional uncoded QPSK.

In a desire to reduce the complexity of the 16-state receiver of Fig. 4 with the hope of not sacrificing significant power efficiency, two suboptimum configurations were considered. The first and least complex is the average matched filter receiver [13] which replaces the bank of sixteen matched filters by a single filter matched to the average of the sixteen waveforms in Fig. 3 followed by symbol-by-symbol detection. Unfortunately, this memoryless receiver gives a performance only slightly better than that obtained from the conventional OQPSK receiver which is also memoryless.

In an attempt to bridge the gap between the 16-state optimum receiver and the oversimplified averaged matched filter receiver, a reduced complexity FQPSK Viterbi receiver was considered in [14] based on the following considerations. Recognizing the similarity in the shape properties, (e.g., “odd” vs “even”, “flat” vs. “sinusoidal”) of certain members of the waveform set in Fig. 3, the sixteen waveforms of this set were separated into four different groups (in reality two groups and their negatives) as follows: \( s_0(t) \) through \( s_7(t) \), \( s_8(t) \) through \( s_{15}(t) \). Hypothetically, if one was to form the average of the waveforms in each group (see Fig. 6), i.e.,

\[
q_0(t) = \sum_{i=0}^{3} s_i(t), \quad q_i(t) = \sum_{i=4}^{7} s_i(t),
\]

\[
q_2(t) = \sum_{i=8}^{11} s_i(t) = -q_0(t), \quad q_3(t) = \sum_{i=12}^{15} s_i(t) = -q_i(t)
\]

and replace the waveform assignments of the group members in Fig. 3 by their corresponding average waveform, e.g., \( s_0(t),s_1(t),s_2(t),s_3(t) \) all now become \( q_0(t) \), \( s_4(t),s_5(t),s_6(t),s_7(t) \) all now become \( q_1(t) \), etc., then because of the relation between the I and Q coded bits and the BCD mapping in Fig. 2, the cross-correlation between the I

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7The necessity of a bias associated with the matched filters comes about because of the fact that the members of the \( M \)-ary signaling set defined in (3) do not all have equal energy.
and Q channels disappears. To see how this comes about we note that what ordinarlily distinguishes the four waveforms in each group from each other are the two least significant bits, namely, $I_0, I_1$ and $Q_0, Q_1$. Since by sending only one waveform per group, i.e., $q_0(t), q_1(t), q_2(t)$ or $q_3(t)$, we no longer need to make this distinction, then the BCD mapper requires only the two most significant bits, i.e., $I_2, I_3$ and $Q_2, Q_3$ to specify the transmitted waveform pair. By inspection of Fig. 2, we see that this is tantamount to the I channel signal being chosen based only the I encoder output bits and the Q channel signal being chosen based only the Q encoder output bits, i.e., the cross-correlation of the encoder outputs in choosing the I and Q waveforms disappears. Assuming this to be the case, then the trellis coded structure of the modulation decouples into two independent I and Q 2-state trellises (see Fig. 7) whereupon the transmitter simplifies to the structure in Fig. 8 with the corresponding optimum receiver as in Fig. 9. (We have relabelled the I and Q pairs of bits with a prime notation so that for convenience of notation we can continue to index the transmitted signals between 0 and 4). By independent we mean that the I and Q decisions are no longer produced jointly but rather separately by individual Viterbi algorithms (VA's) acting on the energy-biased correlations derived from the I and Q demodulated signals, respectively. This trellis structure of the simplified Viterbi receiver as well as the simplified transmitter of Fig. 8 are identical to that of so-called trellis-coded OQPSK discussed in [16] but with a different waveform assignment. The metrics used for the simplified Viterbi receiver in Fig. 9 are the same as those for the full-blown receiver of Fig. 3 except that the matched filter biases are now computed from the energies of the waveform averages in (4) rather than from the energies of the individual waveforms of (3). Finally, since the true trellis-coded structure of FQPSK entails transmitting a set of sixteen different waveforms [as described in (3)], using the above simplified receiver, which is based on the hypothetical mapping to only the four signals in (4), for demodulating FQPSK represents a mismatched condition and as such is suboptimum.

Using Signal Processing WorkSystem (SPW) computer simulations, the BER performance of FQPSK using the simplified Viterbi receiver is superimposed here on the results in Fig. 5. Where for the full-blown 16-state Viterbi receiver the truncation path length (decoding depth) was chosen equal to 50 bits, for the simplified receiver a truncation path length of only 10 bits is needed thereby significantly reducing the decoding delay. Comparing the simplified Viterbi receiver performance with that of the full-blown receiver the $E_b/N_0$ loss was 0.25 dB at a BER of $10^{-3}$ and 0.3 dB at a BER of $10^{-5}$. These small degradations are far compensated by the significant reduction in receiver complexity.
Error Correction Coded FQPSK with Iterative Decoding

Motivated by the potentially large coding gain achievable with iterative decoding of concatenated codes using a soft input-soft output (SISO) a posteriori probability (APP) algorithm, the authors set out to apply this technique to FQPSK which by virtue of its inherent coding supplies the inner code. To keep matters simple, the investigations considered applying the outer code to an FQPSK system using the simplified receiver of Fig. 9 where the 2-state Viterbi algorithms are now replaced with 2-state SISO max-log algorithms [17] which themselves are equivalent to modified soft output Viterbi algorithms (SOVA's) [18]. In order to have a coding gain resulting from the interleaving/deinterleaving process between the inner and outer codes, we must first remap the I and Q FQPSK nonrecursive inner codes to recursive types [19]. The particular remapping of the input bits and trellis code states of the I and Q inner codes to change the input-output relation required by the concatenated coding scheme are shown in Figs. 10a and 10b, respectively. This remapping is aimed at providing recursiveness for the parts of the FQPSK encoders that are matched to the reduced 2-state SOVA's for the inner code. Furthermore, such a remapping method will provide the maximum coding gain for the concatenated coded FQPSK system.

It is important to note that with the remapped encoders of Fig. 10, in the absence of an outer code, a particular random input bit sequence will now result in a different pair of transmitted I and Q baseband waveforms. However, since the remapping does not change the allowable FQPSK encoder output sequences, the envelope and spectral characteristics of the resulting modulated signal would be identical to the FQPSK signal produced by the transmitter of Fig. 2. If now in the presence of the outer code we assume that the interleaver size is sufficiently large that the random interleaver approximately outputs a random uncorrelated sequence, then the same conclusion regarding the envelope and spectral properties of the transmitted signal in the concatenated coding case once again applies. For example, it is known that for many optimum convolutional codes (used here as an outer code), a random input bit sequence produces an uncorrelated output sequence.

With regard to the manner in which the outer code is applied to the FQPSK modulator/demodulator, we propose three different concatenation schemes each of

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8To see this, note that if once again one were to postulate a hypothetical modulation scheme with a mapping to the signals in (4), then the encoders of Fig. 10 would simplify in an analogous manner to the way in which the encoders of Fig. 2 simplified to those in Fig. 8 and the associated 2-state trellis would be appear as in Fig. 7 with the input bits on the two transitions emanating from the "1" state reversed.
which incorporates iterative decoding. The transmitter/receiver combinations for each of these schemes are illustrated in Figs. 11a, b, and c. As in Fig. 9, the energy-biased matched filter bank provides the four required branch metrics (per I and Q channel) for the simplified 2-state SISO FQPSK decoders. The FQPSK SISO decoders in turn provide extrinsics (added-value reliabilities) associated with the FQPSK encoder input bits to the outer decoders through deinterleaver(s). The SISO output decoder(s) provide enhanced versions of the received extrinsics using the code constraint as output extrinsics which through interleaver(s) are fed back to the inputs of the SISO FQPSK demodulators. This process repeats (iterates) several times. At the end of the last iteration, the output of the SISO outer decoders which correspond to the input bits to the outer encoders are hard-limited to produce decisions on these bits. To reduce the complexity of the SISO's for both the FQPSK demodulators and the outer decoders, we used max-log versions of these SISO's which, as previously stated, are equivalent to modified versions of the original SOVA algorithm proposed by Hagenauer and Hoeher [20].

As a start, the concatenated coding scheme of Fig. 11a was used with a simple rate 1/2 repetition outer code followed by a block interleaver of size $N$ applied to the I channel encoder and, for simplicity of implementation, no outer code and interleaver applied to the Q channel encoder.\footnote{Since as we shall see momentarily the receiver corresponding to this scenario was simulated to only measure BER on the I channel, because of the previously discussed reasoning that resulted in the independence of the I and Q trellises in Fig. 8, outer coding on the Q channel would result in no additional coding gain. However, to match the increase in the symbol rate on the I channel caused by the inclusion of a rate 1/2 repeat code, the symbol rate on the Q channel was also doubled.} At the receiver the I channel 2-state modified SOVA generates the soft output extrinsics required for iterative decoding. These soft output extrinsics after being deinterleaved are provided to the SISO outer decoder which for a repetition code consists of simply swapping the order of successive pairs of bits (see Fig. 12). The average BER of the I channel data was then determined by computer simulation. Since the rate 1/2 repeat code after sufficiently large interleaving looks like a random data sequence at twice the data rate then in so far as the output of the FQPSK modulator is concerned, it has the appearance of an FPQSK signal but with twice the bandwidth.

Superimposed on the results in Fig. 5 is the BER performance of the above concatenated coding scheme with $L = 5$ iterations and interleaver block sizes of $N = 2048$ and $N = 16384$ bits. Using the same rate 1/2 repetition code, similar performance would be expected from the concatenating coding schemes in Figs. 11b and c. We observe from these results that even with such a very simple outer code,
at a BER of $10^{-5}$ an improvement of 3.75 dB relative to the performance of the simplified FQPSK receiver without outer coding is obtained for $N = 2048$ whereas $N = 16384$ yields an improvement of 4.5 dB.

The next case considered was a rate $1/2$ optimum 4-state outer code with minimum free distance $d_f = 5$. The interleaving gain for this case is asymptotically proportional to $N^{-3}$ versus $N^{-1}$ for the previous case as in turbo codes. The BER performance of this case is also illustrated in Fig. 5 for $L = 5$ iterations and interleaver block sizes of $N = 2048$ and $N = 16384$ bits. At the same BER of $10^{-5}$, the improvement relative to that of the simplified FQPSK receiver without outer coding has now increased to 7.25 dB for $N = 2048$ and 7.7 dB for $N = 16384$.

**Conclusions**

We have shown that by applying simple (small number of states) outer codes to a FQPSK modulation to form a concatenated coding arrangement with iterative decoding, it is possible to achieve a highly power and bandwidth efficient system. Several possible configurations for the concatenation were suggested and specific numerical results were presented for one of these in order to demonstrate the large coding gains that are potentially achievable even when using a reduced complexity FQPSK receiver. In view of the very positive results reported here, further investigation of the other concatenated coding schemes is warranted as well as consideration of higher rate codes to reduce the bandwidth expansion proportional to the inverse of the code rate.

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**References**

1. K. Feher et al, U.S. Patents: 4,567,602; 4,339,724; 4,644,565; 5,784,402; 5,491,457. Canadian Patents: 1,211,517; 1,130,871; 1,265,851.


The diagram provides a conceptual block diagram of FQPSK (XPSK) modulation. The NRZ Data Stream is first converted to a serial to parallel format. The data stream is then processed through a SQORC (Symbol-by-Symbol Orthogonal Rayleigh Channel) Modulator, followed by an IJF (Intersymbol Interference Free) Encoder. The resulting signals, $S_I(t)$ and $S_Q(t)$, are then multiplied by $90^\circ$ and $0^\circ$ phases, respectively, and combined to form the FQPSK (XPSK) Signal.

*Note that what is referred to as an "Intersymbol Interference/Jitter-Free (IJF) Encoder" is in fact a waveform mapping function without any error-correcting capability.

The choose statements for $S_I(t)$ and $S_Q(t)$ are:

- $S_I(t) = s_e(t)$ if $d_{I,n-1} = 1, d_{I,n} = 1$
- $S_I(t) = -s_e(t)$ if $d_{I,n-1} = -1, d_{I,n} = -1$
- $S_I(t) = s_o(t)$ if $d_{I,n-1} = -1, d_{I,n} = 1$
- $S_I(t) = -s_o(t)$ if $d_{I,n-1} = 1, d_{I,n} = -1$

Similarly for $S_Q(t)$.

Fig. 1. Conceptual Block Diagram of FQPSK (XPSK)
Fig. 2. Alternative Implementation of FQPSK Baseband Signals
\[ s_0(t) = -s_8(t) \]

\[ s_1(t) = -s_9(t) \]

\[ s_2(t) = -s_{10}(t) \]

\[ s_3(t) = -s_{11}(t) \]

\[ s_4(t) = -s_{12}(t) \]

\[ s_5(t) = -s_{13}(t) \]

\[ s_6(t) = -s_{14}(t) \]

\[ s_7(t) = -s_{15}(t) \]

\[ (A = \frac{1}{\sqrt{2}} \text{ for "constant" envelope}) \]

Fig. 3 FQPSK Full Symbol Waveforms
Fig. 4 Optimum Trellis-Coded Receiver for FQPSK
Fig. 5 A Comparison of the BEP Performances of Various Uncoded and Coded FQPSK Systems
Fig. 6 Averaged Waveforms for Simplified Viterbi Receiver

Fig. 7 Trellis Diagram for Simplified FQPSK Viterbi Receiver
Fig. 8 Simplified Implementation of "FQPSK" Baseband Signals
Fig. 9 Simplified FQPSK-B Viterbi Receiver
Fig. 10a. Original and Remapped I Encoder and Trellis
Fig. 10b. Original and Remapped Q Encoder and Trellis
$$\Pi = \text{Interleaver}$$
$$\Pi^{-1} = \text{Deinterleaver}$$

Fig. 11. Several Transmitter/Receiver Combinations for Coded FQPSK with Iterative Decoding
Fig. 12. SISO Outer Decoder for Rate 1/2 Repetition Code