Microtorus: A High Finesse Microcavity with Whispering Gallery Modes

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Microsphere -- a low-loss photon trap, efficient optical cavity

Whispering-gallery modes - closed circular waves under total internal reflection
(Term by J.W.S.Rayleigh, analogy to acoustic modes in the gallery of St Paul cathedral)

(MUST BE) Sustained in any axisymmetric dielectric body with \( R \gg \lambda \)
low material loss (transparent material, e.g. fiber grade silica)
low bending loss (\( R \gg \lambda \))
low scattering loss (TIR always under grazing incidence
+ molecular-size surface roughness)

\[
\text{Quality-factor } Q = \frac{\lambda}{\Delta \lambda_{\text{RES}}} \quad \text{up to } \sim 10^{10}
\]
\[
\text{Photon lifetime } \tau = \frac{\lambda Q}{2\pi c} \quad \text{up to } \sim 3 \mu s
\]

(cavity ringdown time)


Visualization of WG mode field by residual scattering in silica microsphere, \( \text{V.S.Ilichenko et al, Opt.Commun. 113, p.133(1994)} \)
Why spheres?

- low material loss (transparent material)
- low bending loss (high-contrast boundary)

LOW SCATTERING LOSS (TIR always under grazing incidence) $\Theta \rightarrow \pi/2$; compare to disks/ rings:

$$\frac{I_R}{I_I} = e^{-\left(\frac{4\pi \sigma}{\lambda} \cos \Theta\right)^2}$$  (J.W.S.Rayleigh)

EVEN WITH MOLECULAR ROUGHNESS $\sigma$, ONLY CURVATURE CONFINEMENT ALLOWS $Q$ LIMITED BY MATERIAL ATTENUATION:

$10^8...10^{10}$ in spheres vs. $10^3...10^5$ in microrings !!

Drawback: “too many modes” compared to planar rings!
Spectrum of microspheres: Families of non-degenerate $TE(TM)_{lmq}$ modes.

"Small" FSR \[ \nu_{lmq} - \nu_{l,m-1,q} \sim \nu \frac{\varepsilon^2}{2l} \] - few GHz with typical $\varepsilon^2 \sim (1-3) \times 10^{-2}$.

"Big" FSR \[ \nu_{lmq} - \nu_{l-1,mq} = \frac{c}{2\pi n a} (t_{l,q} - t_{l-1,q}) \sim \nu / l \] - few hundred GHz (few nm)

Input power 7.5...8.3mW; maximum transmission at resonance ~23.5% (fiber-to-fiber loss 6.3dB); $Q_{load} > 3 \times 10^7$ at 1550nm; sphere diameter 405\,\mu m. Unloaded $Q_o \approx 1.2 \times 10^8$ ($Opt. Lett. 24$, 723 (1999))
**Novel geometry: a highly oblate spheroid, or microtorus**

Near the symmetry plane (at the location of WG modes), toroidal surface of outer diameter $D$ and cross-section diameter $d$ coincides with that of the osculating oblate spheroid with large semiaxis $a = D/2$ and small semiaxis $b = \frac{1}{2} \sqrt{Dd}$.
Names

[F. seesaw]: an apparatus or structure end is counterbalanced by the other on or by weights

và-sàz [ME, fr. esp, base, fr. bainein

a: the bottom of support: FOUNDA-


tion of a wall, pier, or separate architectural part of a complete side or face of a

which an altitude can

on which the figure

a bodily organ by

other more central

2 a: a main in-

latex \rightarrow b: a

ingredient (as of a

ental part of some-

the lower part of a

point or line from which a start is made

b: a line in a survey which serves as

(Webster's New College dictionary, G & C Merriam Co. Springfiled, Mass., 1975, p.92)
Calculation of the spectrum of the dielectric spheroid

is not a trivial problem, even numerically. In “quasiclassical” approximation with assumptions:

1) a WG mode is a closed circular beam supported by TIR, 2) optical field tunnels outside at the depth \( 1/k\sqrt{n^2 - 1} \), and 3) the tangential component of \( E \) (TE-mode), or normal of \( D \) (TM-mode) is continuous at the boundary. Eigenfrequencies of high-order WG modes \( l \gg 1; l \approx m \) in dielectric sphere can be approximated via solutions of scalar wave equation with zero boundary conditions, because most of the energy is concentrated in one component of the field \( E_\theta \) for TE-mode and \( E_r \) for TM-mode.

Based on above considerations, let us estimate WG mode eigenfrequencies in oblate spheroids of large semiaxis \( a \), small semiaxis \( b \), and eccentricity \( \varepsilon = \sqrt{1 - b^2/a^2} \). Since WG modes are localized the “equatorial” plane, we shall approximate the radial distribution by cylindrical Bessel function \( J_m(n\tilde{k}_{mq}r) \) with \( n\tilde{k}_{mq}a = na\sqrt{k_{mq}^2 - k_\perp^2} \approx T_{mq} \) where \( J_m(T_{mq}) = 0 \) and \( k_\perp \) is the wavenumber for quasiclassical solution for angular spheroidal functions. For our purposes a rough approximation is enough:

\[
k_{mq}^2 \approx \frac{2(l-m)+1}{a^2\sqrt{1-\varepsilon^2}}m;
\]

more rigorous consideration can follow the approach given in [I.V.Komarov, L.I.Ponosov, S.Yu.Slavianov, Spheroidal and Coulomb Spheroidal Functions, Moscow, Nauka (1976) (in Russian)]. Taking into account that \( T_{mq} \approx \tau_q - (l-m + 1/2) \), we finally obtain the following approximation:

\[
nk_{lmq}a - \frac{\chi}{\sqrt{n^2 - 1}} \approx \tau_q + \frac{2(l-m)+1}{2}\left(\frac{1}{\sqrt{1-\varepsilon^2}} - 1\right)
\]

\( \tau_q \) -- \( q \)-th zero of the spherical Bessel function of the order \( l \); \( \chi = n \) for TE-mode, \( \chi = l/n \) for TM-mode.


2. Discrepancy with numerical calculations is \(<5\%\) in prediction of “small” FSR and \(<0.1\%\) of absolute frequencies, even with \( \varepsilon^2 \sim 0.8 \), even small \( l = 100 \)

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Schematic of the experimental setup

to obtain wide range (~900GHz, or 7.2nm) high-resolution spectra of WG modes in microcavity
Spectrum of TE whispering-gallery modes in spheroidal dielectric microcavity

\( D = 2a = 165\,\mu m; \, d = 42\,\mu m; \, 2b = 83\,\mu m. \) Free spectral range (between largest peaks 1 and 2) 383.7GHz (3.06nm) near central wavelength 1550nm. Individual resonance bandwidth 23MHz (loaded Q = 8.5\times10^6). Finesse \( F = 1.7\times10^4 \)
CONCLUSIONS

1. It seems indeed we can combine small size, ultra-high-Q with “nice” FP-like spectrum: true finesse $10^4 \ldots 10^6$ becomes available in microcavities as opposed to supermirror FPs

2. Host of potential applications is significant