Ichiro Fukumori*, Tong Lee, Dimitris Menemenlis, Lee-Lueng Fu,
Benny Cheng, Benyang Tang, Zhangfan Xing, and Ralf Giering
Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA

1. INTRODUCTION

An assimilation effort is undertaken with the goal of understanding seasonal-to-interannual changes of the tropical Pacific Ocean from the late '80s to the present, encompassing the WOCE and TOPEX/POSEIDON (T/P) periods. The model-data synthesis by assimilation will maximize utilization of extant observations and yield complete and optimal descriptions of the global ocean circulation. The effort is based on a dual approach that consists of Kalman filtering and the adjoint method. The approach capitalizes on the respective advantages of the two methods such as formal state error estimates and computational efficiency of the approximate Kalman filter on the one hand and the rigorous and more complete optimization of the adjoint method on the other. The effort is part of the Consortium for Estimating the Circulation and Climate of the Ocean (ECCO), and aims to lay the foundation for routine analysis of global ocean circulation.

2. MODEL

The model is based on the parallelized version of the MIT general circulation model (Marshall et al., 1997). To best resolve upper ocean processes of the tropical oceans, the model employs a telescoping horizontal grid with 1/3° latitudinal resolution within 10° of the equator that gradually increases to a uniform 1° resolution at 20° latitude and beyond. Vertically, the model consists of 46-layers, with thickness ranging from 10m within 150m of the surface to 400m near the bottom. The model domain is global from 80°S to 80°N so as to minimize uncertainties associated with open boundary conditions. The model employs advanced mixing schemes including the eddy parameterization of Gent and McWilliams (1990) and the KPP mixed layer formulation of Large et al. (1994).

---
* *Corresponding author address:* Ichiro Fukumori, JPL MS 300-323, Pasadena, CA 91109; e-mail: if@pacific.jpl.nasa.gov.

3. "CELLULAR" KALMAN FILTER

The main computational difficulty of applying Kalman filtering to oceanographic and atmospheric data assimilation is in evaluating the enormously large model state error covariance matrix. The reduced-state approximation (e.g., Fukumori and Malanotte-Rizzoli, 1995; Pham et al., 1999) significantly decreases these requirements by approximating the error covariance with fewer degrees of freedom than that of the models themselves while retaining the dominant model error processes. Here, we extend this approach by recognizing the limited spatial extent and/or the near independence of many error processes, and evaluate these individual error components separately. For instance, meso-scale errors near the Kuroshio region are likely independent of equivalent errors in the Gulf Stream region. Then, one may consider evaluating the meso-scale errors of the Kuroshio and Gulf Stream regions ("cells") separately. Enormous computational savings can be achieved by such cellular approach by virtue of each cell's smaller dimension. For one-dimensional problems, an example of such approximation can be visualized as in Figure 1, where overlapping block diagonal matrices approximates the overall error covariance matrix. The overlaps are introduced to best approximate the errors in the gray region that is not well accounted for by the blocks with solid boundaries. The overall matrix is positive definite so long as individual blocks are positive definite.

Figure 1: Schematic of approximating a covariance matrix by a sum of block diagonal matrices.

4. ESTIMATION BY KALMAN FILTER

A cellular Kalman filter is being implemented to our model to establish a baseline assimilation of satellite altimetry. A preliminary version of this filter consists of a global barotropic filter and a regional baroclinic filter of the tropical Pacific

Ocean. Errors associated with the barotropic component are dominated by high-frequency variabilities at high latitudes, in particular the Southern Ocean, and are largely independent of large-scale baroclinic uncertainties that are large at lower latitudes and of lower frequencies (Fukumori et al., 1999). Both filters are reduced-state filters defined on coarse horizontal grids.

The barotropic reduced state consists of depth-averaged zonal and meridional velocities and sea level associated with the barotropic circulation (total state dimension 2898). The baroclinic reduced-state is based on vertical empirical orthogonal functions of baroclinic horizontal velocities and combined temperature and salinity modes (total state dimension 1850). As an example, Figure 2 demonstrates the skill of the barotropic filter as defined by changes in model-data differences,

$$\left\langle (d-s)^2 \right\rangle - \left\langle (d-p)^2 \right\rangle$$

where d, s, and p are, respectively, altimetric sea level observations, sea level of the model simulation and that of the filter prediction. (d-p is the innovation sequence.) The brackets denote averages in space and/or time. Positive values indicate the assimilation's skill in improving the model's estimating (predicting) observed variabilities. The skill remains positive throughout the 5-year assimilation and is dominated by improvements at higher latitudes, consistent with theoretical expectations.
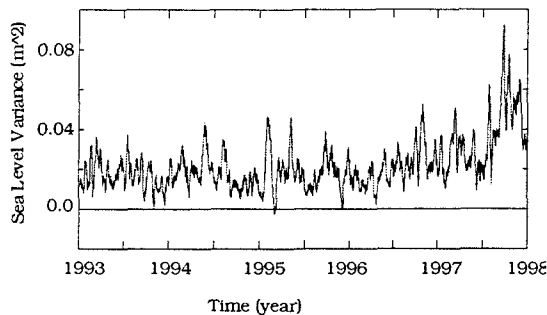

Figure 2: Skill of barotropic Kalman filter as a function of time. Positive values indicate skill of the estimation.

5. ESTIMATION BY THE ADJOINT MODEL

The adjoint code of the forward model is generated by the Tagent linear and Adjoint Model Compiler (Giering and Kaminski, 1999). The adjoint computes the partial derivatives of a cost function with respect to control variables, subject to dynamical constraints of the forward model (model dynamics taken into account in the differential chain rule). The cost function, enforcing the data constraints, is expressed by the following:

$$J = (\mathbf{T}_m - \mathbf{T}_d)^T \mathbf{W}_T (\mathbf{T}_m - \mathbf{T}_d)$$
$$+ (\mathbf{S}_m - \mathbf{S}_d)^T \mathbf{W}_S (\mathbf{S}_m - \mathbf{S}_d) + (\mathbf{h}_m - \mathbf{h}_d)^T \mathbf{W}_h (\mathbf{h}_m - \mathbf{h}_d)$$
$$+ (\tau_m^x - \tau_d^x)^T W_{\tau^x} (\tau_m^x - \tau_d^x) + (\tau_m^y - \tau_d^y)^T W_{\tau^y} (\tau_m^y - \tau_d^y)$$
$$+ (Q_m - Q_d)^T W_Q (Q_m - Q_d) + (E_m - E_d)^T W_E (E_m - E_d)$$

where $\mathbf{T}$, $\mathbf{S}$, $\mathbf{h}$, τ^x, τ^y, Q, and E represent temperature, salinity, sea level anomaly, zonal wind stress, meridional wind stress, surface heat flux, and surface freshwater flux, respectively. W's represent weight matrices, which reflect data errors. The subscript "m" denotes model and "d" indicates data. The first term of J corresponds to the difference between model time-mean temperature and climatological mean temperature data; the second term is similar but for salinity; the third term is the difference of sea level anomaly between the model and that derived from the TOPEX-Poseidon altimeter; the remaining four terms reflect differences of model surface fluxes (averaged over 10-day intervals) from corresponding observations. The "observed" fluxes are obtained from a reanalysis product of the National Center for Environmental Prediction (NCEP).

The control variables are chosen to be initial temperature and salinity and the 10-day averages of surface fluxes. The partial derivatives of J with respect to the control variables computed by the adjoint are used in an optimization procedure (conjugate gradient) to minimize J by optimally adjusting the control variables.

6. MODEL SENSITIVITY ANALYSIS

Parallel to the adjoint-based assimilation and being less computationally intensive is the application of the model adjoint to study sensitivities of ocean processes. An example is presented here in which we examine the sensitivity of the model annual-mean (depth-integrated) transport of the Indonesian throughflow (ITF) V to annual-mean zonal and meridional wind stresses τ^x and τ^y as denoted by $\partial V/\partial \tau^x$ and $\partial V/\partial \tau^y$. (Figure 3). These sensitivities describe the changes from annual-mean ITF transport in response to unit (positive) perturbations to annual-mean zonal and meridional wind stresses at various independent locations. Positive (negative) sensitivity indicates strengthening (weakening) of the ITF due to a unit positive change in τ^x and τ^y.
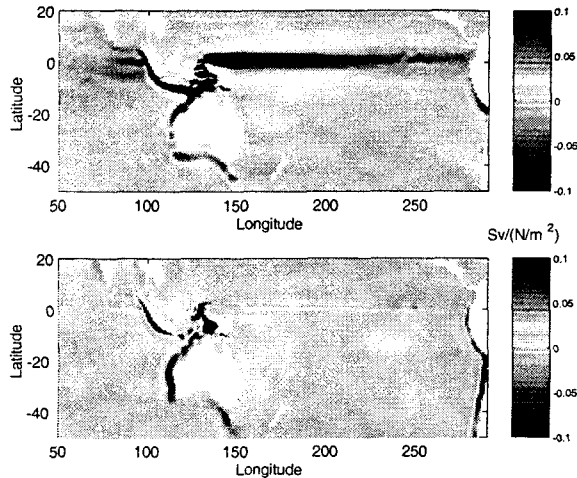
Figure 3: Sensitivity of annual-mean ITF transport to annual zonal (a) and meridional (b) wind stresses.

The sensitivities also serve as a "response function" through which interannual variations of ITF transport can be estimated using interannual anomalies of wind stress, $\tau^{x\prime}$ and $\tau^{y\prime}$. The product of the sensitivity functions with these interannual anomalies of wind stresses gives the contribution of interannual wind anomalies in different locations at different times to interannual changes in ITF transport. This, when integrated over the model domain, yields the total contribution of wind anomalies to interannual ITF transport:

$$V = \int \partial V / \partial \tau^x \times \tau^{x\prime} dxdy + \int \partial V / \partial \tau^y \times \tau^{y\prime} dxdy$$

The estimated interannual ITF transport based on global wind is well correlated with values simulated by the forward model. It is also qualitatively consistent with previous estimates based on XBT data although with a smaller magnitude. Significant correlation is found between the estimated ITF transport anomaly and interannual variation of sea surface temperature in the central to eastern equatorial Pacific (Figure 4): the ITF tends to be weaker during El Niño and stronger during La Niña.
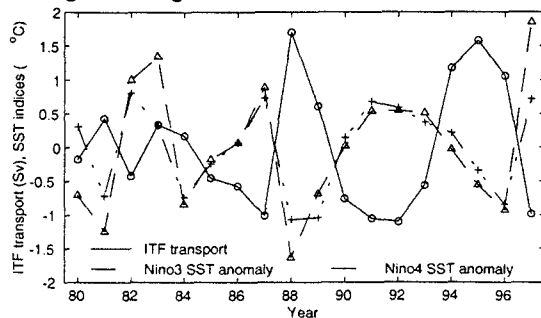

Figure 4: Interannual anomaly of ITF estimated by adjoint sensitivity. Significant correlations are found with sea surface temperature anomalies.

Regional contribution of wind stress to interannual ITF transport is examined by evaluating the integral above over various regions. The largest contribution comes from wind over the equatorial Pacific and coastal region southwest of Java. Secondary contribution is found from wind over the western tropical Indian Ocean, coastal areas off western Australia, New Zealand, and South America. Indian Ocean wind has a smaller contribution to ITF than the Pacific wind does, and tends to counteract the later (Figure 5). This is because zonal wind anomalies over the tropical Pacific and Indian Oceans tend to have opposite signs on interannual time scales, reflecting varying intensity of the Walker circulation cells over the two tropical oceans (having opposite sense of rotation) associated with El Niño - Southern Oscillation. These results can further be compared to the "Island Rule" (Godfrey, 1989) that relates wind stress to transport around isolated land masses.
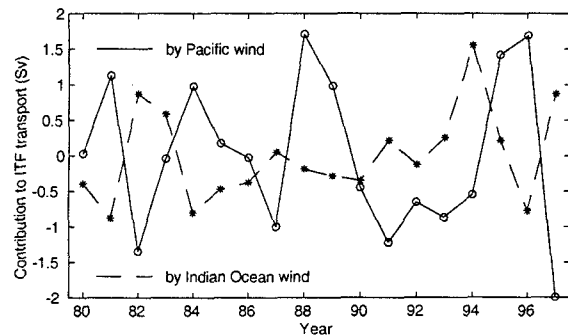

Figure 5: Contributions of interannual anomalies of wind stress over the Pacific and Indian Oceans to anomalies of ITF: The two tend to counteract each other.

7. SUMMARY

A dual ocean data assimilation system is being developed, one based on the Kalman filter and the other based on the adjoint method. The solutions are analyzed with the aim of understanding processes underlying observed seasonal-to-interannual changes of the tropical Pacific Ocean. The solutions of the dual assimilation system are compared to examine the merits of the two assimilation methods.

8. ACKNOWLEDGEMENTS

9. REFERENCES

Fu, L.-L., E. J. Christensen, C. A. Yamarone Jr., M. Lefebvre, Y. Ménard, M. Dorrer, and P. Escudier, 1994: TOPEX/POSEIDON mission overview, *J. Geophys. Res.*, **99**, 24,369-24,381.

Fukumori, I., and P. Malanotte-Rizzoli, 1995. An approximate Kalman filter for ocean data assimilation; An example with an idealized Gulf Stream model, *J. Geophys. Res.*, **100**, 6777-6793.

Fukumori, I., R. Raghunath, L. Fu, and Y. Chao, 1999. Assimilation of TOPEX/POSEIDON data into a global ocean circulation model: How good are the results?, *J. Geophys. Res.*, **104**, 5,647-25,665.

Giering, R., and T. Kaminski, 1998. Recipes for adjoint code construction, *Association for Computing Machinery Transactions on Mathematical Software*, **24**, 437-474.

Gent, P. R., and J. C. McWilliams, 1990. Isopycnal mixing in ocean circulation models, *J. Phys. Oceanogr.*, **20**, 150-155.

Godfrey, J. S., 1989. A Sverdrup model of the depth-integrated flow for the world ocean allowing for island circulation, *Geophys. Astrophys. Fluid Dynamics*, **45**, 89-112.

Large, W. G., J. C. McWilliams, and S. C. Doney, 1994. Oceanic vertical mixing: A review and a model with a nonlocal boundary-layer parameterization, *Rev. Geophys.*, **32**, 363-403.

Marshall, J. C., A. Adcroft, C. Hill, L. Perelman, and C. Heisey, 1997. A finite-volume, incompressible Navier Stokes model for studies of the ocean on parallel computers, *J. Geophys. Res.*, **102**, 5753-5766.

Pham, D. T., J. Verron, and M. C. Roubaud, 1998. A singular evolutive extended Kalman filter for data assimilation in oceanography, *J. Mar. Sys.*, **16**, 323-340.