

# IN-FLIGHT ESTIMATION OF THE CASSINI SPACECRAFT'S INERTIA TENSOR

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## Extended Abstract

The Cassini spacecraft was launched on 15 October 1997 by a Titan 4B launch vehicle. After an interplanetary cruise of almost seven years, it will arrive at Saturn in July 2004. To save propellant, Cassini will make several gravity-assist flybys: two at Venus and one each at Earth and Jupiter.

Unlike Voyagers 1 and 2, which only flew by Saturn, Cassini will orbit the planet for at least four years. Major science objectives of the Cassini mission include investigations of the configuration and dynamics of Saturn's magnetosphere, the structure and composition of the rings, the characterizations of several of Saturn's icy moons, and others. The Huygens probe, developed by the European Space Agency, will be released in November 2004 and will study the atmosphere of Titan, the only moon in the solar system with a substantial atmosphere.

Several Attitude and Articulation Control Subsystem (AACS) flight software algorithms onboard the spacecraft use the Cassini spacecraft 3-by-3 inertia matrix. For example, it is used by the AACS Fault Protection algorithm, by the Attitude Estimator in the "gyroless" attitude determination mode, and by the Thruster Vector Control algorithms used during all propulsive maneuvers performed by the 445-Newton main engine. As such, a highly accurate estimate of this inertia matrix is important to the spacecraft operations.

Pre-launch, the inertia matrix for the Cassini spacecraft was estimated by adding together the moments of inertia of the individual components of the spacecraft with respect to the estimated overall spacecraft center of mass. After launch, the onboard spacecraft inertia matrix is being continuously updated using estimates of how much propellant (e.g. hydrazine, fuel, and oxidizer) has been used to date. The inertia matrix of the spacecraft on March 15, 2000, using the “sum-of-all-components” method, is estimated to be:

$$\bar{I}_{SC} = \begin{bmatrix} 8810.8 & -136.8 & 115.3 \\ -136.8 & 8157.3 & 156.4 \\ 115.3 & 156.4 & 4721.8 \end{bmatrix} kg \cdot m^2 \quad (1)$$

This estimate had not been validated in-flight using any alternate approach until this study. The primary goal of this study was to validate the current ground-based estimate of the Cassini inertia matrix with an estimate made using flight data and via an independent mean.

When the spacecraft is slewed about its X, Y, or Z-axis using the three Reaction Wheels Assemblies (RWAs) there is negligible external torque acting on the spacecraft. Hence, the total angular momentum vector of the spacecraft is conserved in the inertial coordinate frame during such a maneuver. In this study, we exploit this fact to estimate the inertia matrix using telemetry data from RWA-based spacecraft per-axis slews.

The total angular momentum of the spacecraft has two components: one from the spacecraft itself and the other from the RWAs. By definition, the total angular momentum vector of the spacecraft is given by the following equation:

$$\vec{H}_{Total} \text{ (in inertial coordinates)} = [P]^{-1} [I_{SC}] \vec{\Omega} + [P]^{-1} [T] [I_{RWA}] \vec{\omega}_{RWA} \quad (2)$$

In Equation (2),  $P$  is the direction cosine matrix from the inertial to the body coordinate frame that could be computed using the estimated spacecraft quaternion.  $T$  is the transformation matrix from the three RWAs to the body frame, and  $I_{RWA}$  is the inertia matrix for the RWAs, both measured pre-launch. The vectors  $\bar{\Omega}$  and  $\bar{\omega}_{RWA}$  are the spacecraft and RWA angular rate vectors, respectively, both of which are available from the spacecraft telemetry data. The spacecraft inertia matrix,  $I_{SC}$ , is the only unknown in Equation 2.

Since the spacecraft angular rates are practically zero prior to the slew, the component of the angular momentum due to the spacecraft rates is cancelled out, and the initial angular momentum vector ( $\bar{H}_{Total}(0)$ ) is dependent only on the RWA rates. The conservation of angular momentum, due to negligible external torque acting on the spacecraft, allows the total angular momentum vector evaluated at the initial time (prior to the beginning of the slew) to be set equal to the total angular momentum vector evaluated throughout the slew.

$$[P(t)]^{-1}[I_{SC}]\bar{\Omega}(t) + [P(t)]^{-1}[T][I_{RWA}]\bar{\omega}_{RWA}(t) = [P(0)]^{-1}[T][I_{RWA}]\bar{\omega}_{RWA}(0)^\dagger \quad (3)$$

The only unknown in this equation is  $I_{SC}$ . Telemetry data for  $\bar{\Omega}$ ,  $\bar{\omega}_{RWA}$ , and  $[P]$  are available at a rate of once every four seconds. Over a slew that lasts about one hour, many data points are available. Hence, a least squares fit among the solutions to Equation (3) for all time steps will provide the best estimate of  $I_{SC}$  using in-flight data. This approach is completely independent of “sum-of-all-components” method, and serves as an accurate validation and comparison to current estimates.

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† Where variable(0) is defined as the value of the variable at time t=0.

Cassini flight data from a set of maneuvers made using the RWAs on March 15, 2000 was analyzed using the method described above. As shown in Figure 1, this set of maneuvers included one slew about the X and Z axes, and three slews about the Y-axis.

The resulting best estimate for the inertia matrix of the spacecraft is:

$$I_{SC} = \begin{bmatrix} 8655.2 & -144 & 132.1 \\ -144 & 7922.7 & 192.1 \\ 132.1 & 192.1 & 4586.2 \end{bmatrix} \text{kg} \cdot \text{m}^2 \quad (4)$$

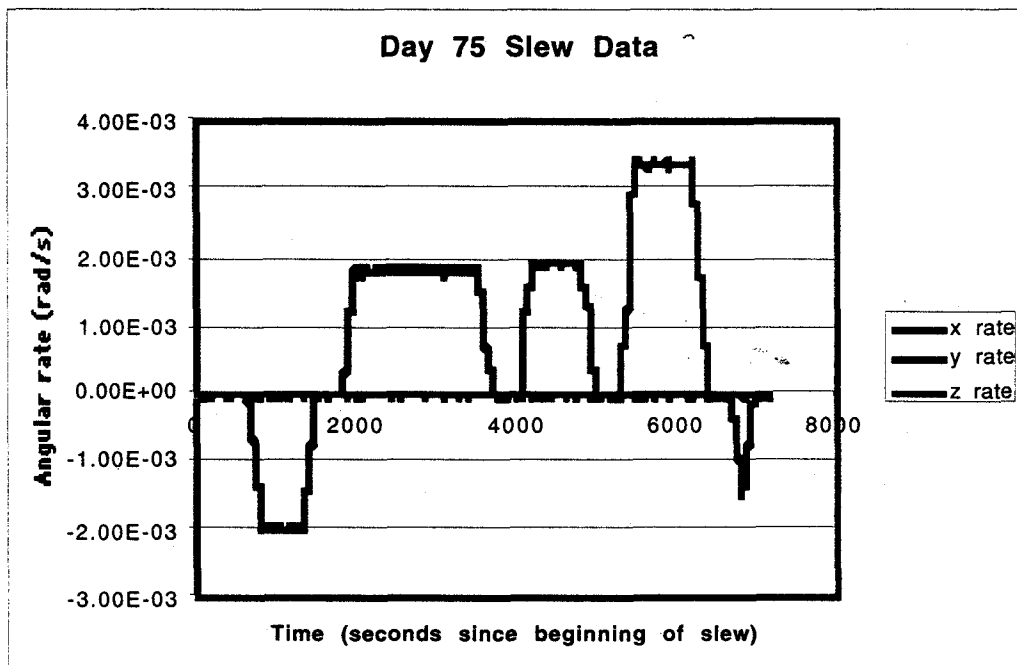


Figure 1 : Time histories of the spacecraft's per-axis rates

The Moments of Inertia ( $I_{xx}$ ,  $I_{yy}$ , and  $I_{zz}$ ) resulting from the least squares method described above are consistently approximately 3% lower than those determined using the current method. This could point to a bias in the estimate of the inertia matrix made prior to launch. In addition, the Products of Inertia ( $I_{xy}$ ,  $I_{xz}$ , and  $I_{yz}$ ) resulting from the least squares method described above are within 50 kg-m<sup>2</sup> of those determined using the

current method. The discrepancy between the methods falls within the knowledge requirement for both the moments and products of inertia.

The method of estimating the moments and products of inertia that is described in this study applies whenever telemetry data associated with slewing the spacecraft by reaction wheels is available. Such a method, proven by its successful application to the Cassini spacecraft, can be utilized by any spacecraft under reaction wheel control.