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ON THE FAST EVALUATION METHOD OF TEMPERATURE AND GAS MIXING RATIO WEIGHTING FUNCTIONS FOR REMOTE SENSING OF PLANETARY ATMOSPHERES IN THERMAL *IR* AND MICROWAVE

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Abstract

Evaluation of weighting functions in the atmospheric remote sensing is usually the most computer–intensive part of the inversion algorithms. We present an analytic approach to computations of temperature and mixing ratio weighting functions that is based on our previous results but the resulting expressions use the intermediate variables that are generated in computations of observable radiances themselves. Upwelling radiances at the given level in the atmosphere and atmospheric transmittances from space to the given level are combined with local values of the total absorption coefficient and its components due to absorption of atmospheric constituents under study. This makes it possible to evaluate the temperature and mixing ratio weighting functions in parallel with evaluation of radiances. This substantially decreases the computer time required for evaluation of weighting functions. Implications for the nadir and limb viewing geometries are discussed.

1 Introduction

Ability to evaluate adequate weighting functions is of vital importance to any retrieval algorithm based on solution of corresponding inverse problems. By their physical meaning, the weighting functions in the atmospheric remote sensing provide a quantitative measure of how sensitive are the observed radiances to profiles of atmospheric parameters that are to be retrieved. In the case of non–scattering atmosphere, there exists a simple analytic relation between observed radiances and profiles of atmospheric parameters. Depending on representation chosen for the analysis, this relation can be differentiated in the form of either variational or partial derivatives of radiances with respect to either continuous or gridded atmospheric profiles. In the simplest case of a temperature weighting function for an atmosphere with a blackbody underlying surface at the ambient temperature and nadir viewing geometry, corresponding expressions can be written as:

$$\frac{\delta R_\nu(\mu)}{\delta T(p)} = \left(-\frac{\partial t_\nu}{\partial \ln p}\right) \cdot \left.\frac{\partial B_\nu}{\partial T}\right|_{T(p)} \tag{1}$$

or, in the finite–dimensional (jacobian) representation

$$\frac{\partial R_\nu(\mu)}{\partial T(p)} = \left(-\frac{\partial t_\nu}{\partial \ln p}\right) \cdot \left.\frac{\partial B_\nu}{\partial T}\right|_{T(p)} \cdot \Delta(\ln p) = \frac{\delta R_\nu}{\delta T(p)} \cdot \Delta(\ln p) \tag{2}$$

Here R_ν is a radiance observed at frequency ν; $T(p)$ is the profile of atmospheric temperature as dependent on atmospheric pressure p; $\ln p$ is selected as a vertical coordinate; $t_\nu(p,\mu)$ is the atmospheric transmittance of a slant column between the level p and top of the atmosphere along the line of sight at zenith angle $\cos^{-1}\mu$; $B_\nu(T)$ is Planck function; $\Delta \ln p$ is a (centered) difference of the coordinate $\ln p$ at the level p. In Eq.2 it is also assumed that the atmospheric absorption is not dependent on temperature.

Under the same assumptions on the atmosphere and underlying surface, the volume mixing ratio (VMR) weighting function can be written in the form of analytic expression (Ustinov, 1990, 1991):

$$\frac{\delta R_\nu(\mu)}{\delta \ln f(p)} = -\frac{1}{\mu} H_g(p)\, \kappa_\nu(p) \int_p^{p_0} t_\nu(p',\mu)\, dB_\nu(T(p')) \tag{3}$$

Here $f(p)$ is the VMR profile of the given atmospheric constituent, $H_g(p)$ is a gas scale height, $\kappa_\nu(p)$ is the volume absorption coefficient due to this constituent, and p_0 is the surface pressure. It should be pointed out here that the transmittance weighting function $\partial t\nu/\partial \ln p$ which is directly used in the expression for the temperature weignting function, Eqs.1 has no direct relevance to the VMR weighting function, Eq.3. In the finite–dimensional representation:

$$\frac{\partial R_\nu(\mu)}{\partial \ln f(p)} = \frac{\delta R_\nu}{\delta \ln f(p)} \Delta(\ln p) \tag{4}$$

The above expressions for the temperature and VMR weighting functions can be compared with the expression for the radiances R_ν themselves. Under the same assumptions about the atmosphere and underlying surface, it can be written in the form:

$$R_\nu(\mu) = \int_0^{p_0} B_\nu\Big(T(p)\Big)\Big(-dt_\nu(p,\mu)\Big) + t_\nu(p_0,\mu)B_\nu\Big(T(p_0)\Big) \tag{5}$$

Whereas the expression for the temperature weighting function Eq.1 is quite simple (it is not, in the case of temperature–dependent opacity, as we will see below), the expression for the VMR weighting function involves an integration over the atmospheric column, as does the expression for radiances, Eq.5. The existing literature on the analytic approach to computatations of weighting functions like VMR weighting function, Eq.3 is still scarce and the general approach pursued there, is, like in (Ustinov 1991), based essentially on taking the derivatives of radiances as composite functionals of atmospheric profiles of desired parameters.

In this presentation we consider an alternative way of analytic computations of the weighting functions in the non–scattering atmosphere that involves integration of radiances only. The hint how, can be drawn easily from Eq.3 which, using the integration by parts, can be rewritten in the integral form similar to that of the expression for radiances, Eq.5. In Section 2 we present general considerations for the arbitrary line of sight, based in the Beer–Lambert law. In Section 3 we consider the nadir case with a simple model of underlying surface. The limb case and FOV convolution will be considered in Section 4. Section 5 contains a brief summary and conclusions.

2 General considerations

We start from the Beer–Lambert law. For an element of an arbitrary line of sight in the non–scattering atmosphere it can be written in the differential form as (the subscript ν is dropped for a while):

$$\frac{dI}{d\tau} + I = B \tag{6}$$

where I is radiance in the direction of the line of sight and B is the Planck function. Expressing the element of optical depth $d\tau$ along the line of sight ds through the total absorption coefficient κ

$$d\tau = \kappa\, ds \tag{7}$$

we can rewrite Eq.6 as

$$dI = (B - I)\kappa\, ds \tag{8}$$

Integrating Eq.8 between some points s_0 and s, and adding the subscript $\uparrow$ to the (upwelling) intensity $I(s)$ we have

$$I_\uparrow(s) = \int\limits_{s_0}^{s} \Big(B(s') - I_\uparrow(s')\Big)\kappa(s')\, ds' + I_\uparrow(s_0) \tag{9}$$

Equation 9 lays a basis for the method of computation of the temperature and VMR weighting functions presented in this paper. First, we consider the VMR weighting function, and as an intermediate object, the absorption coefficient (AC) weighting function. To proceed, we also assume that the point s_0 is the lowermost point of the line of sight: the tangent point in the case of limb geometry and the lower boundary of the atmosphere in the case of nadir geometry. The details of this assumption in both cases will be considered in corresponding Sections below. Then, both terms in the right hand side of Eq.9 depend on the values of temperature and VMR in the atmosphere corresponding to the point s of the line of sight. The variational derivative with respect to the value of the absorption coefficient at this point, $\kappa(s)$ can be obtained directly from Eq.9:

$$\frac{\delta I_\uparrow(s)}{\delta\kappa(s)} = \Big(B(s) - I_\uparrow(s)\Big) + \frac{\delta I_\uparrow(s_0)}{\delta\kappa(s)} \tag{10}$$

If the elementary layer in the atmosphere intersecting the line of sight at the point s is the only one where the variation of the absorption coefficient does happen, then the rest of the atmosphere is unchanged. Multiplying Eq.10 by transmittance $t_\uparrow(s)$ along the line of sight from the point s to the observer at infinity we have:

$$\frac{\delta R}{\delta\kappa(s)} = \frac{\delta I_\uparrow(\infty)}{\delta\kappa(s)} = t_\uparrow(s)\,\frac{\delta I_\uparrow(s)}{\delta\kappa(s)} = t_\uparrow(s)\,\big(B(s) - I_\uparrow(s)\big) + t_\uparrow(s)\,\frac{\delta I_\uparrow(s_0)}{\delta\kappa(s)} \tag{11}$$

The physical sense of the first term in Eq.11 is sensitivity due to upwelling radiation only, accumulated between points s_0 and s. Denoting

$$\left(\frac{\delta R}{\delta\kappa(s)}\right)_\uparrow = t_\uparrow(s)\,\big(B(s) - I_\uparrow(s)\big) \tag{12}$$

we can rewrite Eq.11 in the form:

$$\frac{\delta R}{\delta\kappa(s)} = \left(\frac{\delta R}{\delta\kappa(s)}\right)_\uparrow + t_\uparrow(s)\,\frac{\delta I_\uparrow(s_0)}{\delta\kappa(s)} \tag{13}$$

We further observe that due to additivity of absorption coefficients of separate atmospheric constituents, the AC weighting function with respect to the absorption coefficient κ_m of the constituent to be retrieved, and AC weighting function with respect to the total absorption coefficient κ, – do coincide:

$$\frac{\delta R}{\delta\kappa_m(s)} = \frac{\delta R}{\delta\kappa(s)} \tag{14}$$

and for the derivative with respect to $\ln\kappa_m(s)$ we have

$$\frac{\delta R}{\delta\ln\kappa_m(s)} = \kappa_m(s)\frac{\delta R}{\delta\kappa(s)} \tag{15}$$

Noting that $\ln\kappa_m = \ln f + \mathrm{const}(\kappa_m)$ we obtain:

$$\frac{\delta R}{\delta\ln f(s)} = \kappa_m(s)\frac{\delta R}{\delta\kappa(s)} \tag{16}$$

We obtained the expression for the VMR weighting function through the AC weighting function defined by Eqs.13, 12. The intensity $I(s)$ used in Eq.10 can be computed using, e.g., Eq.9 which computationally is a recurrence relation computed along the line of sight, starting from the point s_0 all the way to the observer to produce the observed radiances R. Thus, we need only one integration and the need in the second integration, as in Eq.3 is eliminated.

Now we consider the temperature weighting function $\delta R/\delta T(s)$. Taking the derivative of both sides of the Eq.9 with respect to variation of temperature $T(s)$ at the upper limit of integration s gives:

$$\frac{\delta I_{\uparrow}(s)}{\delta T(s)} = \kappa(s)\frac{dB}{dT}\bigg|_{T(s)} + \Big(B(s) - I_{\uparrow}(s)\Big)\frac{d\kappa(s)}{dT} + \frac{\delta I_{\uparrow}(s_0)}{\delta T(s)} \tag{17}$$

Using the same considerations as above to obtain Eq.11 we can write:

$$\frac{\delta R}{\delta T(s)} = \kappa(s)t_{\uparrow}(s)\,\frac{dB}{dT}\bigg|_{T(s)} + t_{\uparrow}(s)\Big(B(s) - I_{\uparrow}(s)\Big)\frac{d\kappa(s)}{dT} + t_{\uparrow}(s)\frac{\delta I_{\uparrow}(s_0)}{\delta T(s)} \tag{18}$$

Finally, using Eq.12 we obtain:

$$\frac{\delta R}{\delta T(s)} = \kappa(s)t_{\uparrow}(s)\,\frac{dB}{dT}\bigg|_{T(s)} + \left(\frac{\delta R}{\delta \kappa(s)}\right)_{\uparrow} \cdot \frac{d\kappa(s)}{dT} + t_{\uparrow}(s)\frac{\delta I_{\uparrow}(s_0)}{\delta T(s)} \tag{19}$$

Eq.19 can be compared with Eq.1 written for a more simple case. From the definition of the transmittance function $t_{\uparrow}(s)$

$$t_{\uparrow}(s) = exp\Big(-\int_s^{\infty} \kappa(s')\,ds'\Big) \tag{20}$$

follows that

$$\kappa(s)t_{\uparrow}(s) = \frac{\partial t_{\uparrow}(s)}{\partial s} \tag{21}$$

Thus, the right–hand term in Eq.1 corresponds to the first term in Eq.19 (the opposite sign of the derivative in Eq.1 is due to difference in the direction of coordinates s and $\ln p$). Second term in Eq.19 with the the AC weighting function $\delta R_{\uparrow}/\delta T$ is due to the temperature dependence of atmospheric absorption.

In following Sections we apply the general expressions for the VMR and temperature weighting functions obtained here to the nadir and limb viewing geometries. Throughout the rest of this paper, the usual assumption about the lateral homogeneity of the atmosphere will be used.

3 Nadir case

3.1 Specular underlying surface

We first consider the case of specular underlying surface because the results obtained for this case, will be used in the more realistic cases of nadir geometry with diffuse underlying surface, and of the limb geometry which are discussed afterwards. Also, with some stretching of imagination, the water basins can be considered as specular reflecting underlying surfaces. The line of sight can be specified here as consisting of two rays: upwelling, – from the surface to the observer and downwelling, – from space to the surface. Both rays have here the same zenith angles which facilitates the consideration.

Let s be the point on the downwelling ray of the line of sight, corresponding to the same level in the atmosphere as the point s in the previous Section. By analogy with Eq.9 we can write the expression for the radiance at the downwelling ray $I_{\downarrow}(s)$ in the form:

$$I_{\downarrow}(s) = \int_{-\infty}^{s} \Big(B(s') - I_{\downarrow}(s')\Big)\kappa(s')\,ds' \tag{22}$$

The integration here is performed from space at $s' = -\infty$ to the level at s. A realistic assumption is made that the radiance of space $I_{\downarrow}(\infty)$ is negligible and, correspondingly, there is no off–integral term. The upwelling radiation at the surface $I_{\uparrow}(s_0)$ has two components: reflected downwelling radiation due to non–zero albedo A and proper radiation of the surface due to non–zero emissivity ε:

$$I_{\uparrow}(s_0) = A\,I_{\downarrow}(s_0) + \varepsilon B(T_0) \tag{23}$$

Thus, derivative of $I_{\uparrow}(s_0)$ with respect to any atmospheric parameter $X(s)$ can be written in the form:

$$\frac{\delta I_{\uparrow}(s_0)}{\delta X(s)} = A\,\frac{\delta I_{\downarrow}(s_0)}{\delta X(s)} \tag{24}$$

Applying the considerations used in the derivation of Eq.11, to the derivative $\delta I_{\downarrow}(s_0)/\delta\kappa(s)$ we obtain

$$\frac{\delta I_{\downarrow}(s_0)}{\delta\kappa(s)} = t_{\downarrow}(s)\,\Big(B(s) - I_{\downarrow}(s)\Big) \tag{25}$$

Here $t_\downarrow(s)$ is the transimttance along the downwelling ray of the line of sight, from s to the surface at s_0. Equation 13 together with Eq.24 applied to $\kappa(s)$ and Eq.25 define the AC weighting function $\delta R/\delta\kappa(s)$ which, in its turn, is used to evaluate the VMR weighting function $\delta R/\delta \ln f(s)$, Eq.16.

Expression for the derivative of $I_\downarrow(s_0)$ with respect to $T(s)$ used in Eq.19 is obtained by a simple analogy with derivation of Eq.19. We have:

$$\frac{\delta I_\downarrow(s_0)}{\delta T(s)} = \kappa(s)t_\downarrow(s)\left.\frac{dB}{dT}\right|_{T(s)} + \frac{\delta I_\downarrow(s_0)}{\delta\kappa(s)}\cdot\frac{d\kappa(s)}{dT} \tag{26}$$

Equation 19 together with Eq.24 applied to $T(s)$ and Eq.26 define the temperature weighting function $\delta R/\delta T(s)$.

To conclude this Subsection we change the coordinate s to the altitude z:

$$ds = \pm\frac{dz}{\mu} \tag{27}$$

(plus and minus signs correspond respectively to the upwelling and downwelling rays of the line of sight) and summarize the results in the sequence they are used in actual computations.

1. Recursively compute the downwelling radiances starting from the value $I_\downarrow(\infty) = 0$ (using, e.g., Eq.22):

$$I_\downarrow(z) = \int_\infty^z \Big(B(T(z')) - I_\downarrow(z')\Big)\kappa(z')\left(-\frac{dz'}{\mu}\right) \tag{28}$$

2. Evaluate the upwelling radiances at the surface (cf Eq.23)

$$I_\uparrow(z_0) = A\,I_\downarrow(z_0) + \varepsilon B(T_0) \tag{29}$$

3. Recursively compute the upwelling radiances starting from the value $I_\uparrow(z_0)$ (using, e.g., Eq.9):

$$I_\uparrow(z) = \int_{z_0}^z \Big(B(T(z')) - I_\uparrow(z')\Big)\kappa(s')\frac{ds'}{\mu} + I_\uparrow(z_0) \tag{30}$$

4. Compute the upwelling transmittances (using, e.g., Eq.20):

$$t_\uparrow(z) = exp\left(-\int_z^\infty \kappa(z')\frac{dz'}{\mu}\right) \tag{31}$$

5. Compute the downwelling transmittances using a simple relation:

$$t_{\downarrow}(z) = \frac{t_{\uparrow}(z_0)}{t_{\uparrow}(z)} \tag{32}$$

which is based on an identity $t_{\uparrow}(z) \cdot t_{\downarrow}(z) = t_{\uparrow}(z_0)$

6. Evaluate the upwelling, downwelling and AC weighting functions (cf. Eqs.25, 12, 13 and 24)

$$\left(\frac{\delta R}{\delta\kappa(z)}\right)_{\uparrow} = t_{\uparrow}(z)\left(B(T(z)) - I_{\uparrow}(z)\right) \tag{33}$$

$$\frac{\delta I_{\downarrow}(z_0)}{\delta\kappa(z)} = t_{\downarrow}(z)\left(B(T(z)) - I_{\downarrow}(z)\right) \tag{34}$$

$$\frac{\delta R}{\delta\kappa(z)} = \left(\frac{\delta R}{\delta\kappa(z)}\right)_{\uparrow} + t_{\uparrow}(z)\, A\frac{\delta I_{\downarrow}(z_0)}{\delta\kappa(z)} \tag{35}$$

7. Evaluate the VMR weighting function (cf. Eq.16)

$$\frac{\delta R}{\delta \ln f(z)} = \kappa_m(z)\frac{\delta R}{\delta\kappa(z)} \tag{36}$$

8. Evaluate the temperature weighting function (cf Eq.19, 26 and 24)

$$\frac{\delta R}{\delta T(z)} = \kappa(z)t_{eff}(z)\left.\frac{dB}{dT}\right|_{T(z)} + \frac{\delta R}{\delta\kappa(z)} \cdot \frac{d\kappa(z)}{dT} \tag{37}$$

where the effective transmittance $t_{eff}(z)$ is defined as

$$t_{eff}(z) = t_{\uparrow}(z) + At_{\downarrow}(z) \tag{38}$$

It should be noted that most of computational burden is due to computations of the transmittances and radiances. Equations 28–32 used to evaluate them are optional and any other appropriate procedures can be used. Once the transmittances and radiances are computed, evaluation of all weighting functions is done using simple non–recursive relations Eqs.33–37.

3.2 Scattering underlying surface

In this case the upwelling radiation at the surface is dependent on the distribution of the downwelling radiation over the whole hemisphere.Assuming for simplicity that the underlying surface is a Lambertian scatterer we have instead of Eq.29:

$$I_{\uparrow}(z_0) = 2A \int_0^1 I_{\downarrow}(z_0, \mu_{\downarrow}) \mu_{\downarrow} \, d\mu_{\downarrow} + \varepsilon B(T_0) \tag{39}$$

or, using the an affordable Gaussian quadrature

$$I_{\uparrow}(z_0) = 2A \sum_{k=1}^{n} I_{\downarrow k}(z_0,) \mu_{\downarrow k} + \varepsilon B(T_0) \tag{40}$$

(NB! $n = 1$ may be not enough). Correspondingly, all downwelling variables have now to be computed for the selected set of $\mu_{\downarrow k}, k = 1, \ldots n$ and appropriate summation is necessary in the analogs of Eqs.35 and 38. The rest of the algorithm remains unchanged.

4 Limb case

4.1 Computations for a single line of sight

The simple change of coordinate $s \rightarrow z$, Eq.27 is not valid for the limb case since μ varies along the line of sight. Using radial distances r instead of the altitude z we have:

$$ds = \pm \frac{dr}{\sqrt{r - r_0}} \tag{41}$$

where the plus and minus signs correspond to the aft (downwelling) and fore (upwelling) legs of the line of sight and r_0 represents the radial distance of the lowermost point (tangent point) of the line of sight. An optional way of dealing with the well–known singularity in Eq.41 used in this presentation is as follows. The integrand function $f(r)$ in the integral expression

$$F(r, r_0) = \int_{r_0}^{r} f(r') \, \frac{dr'}{\sqrt{r' - r_0}} \tag{42}$$

is represented by its values $\{f_j\}, j = 0, \ldots N-1$ over the appropriate set of gridpoints $\{r_j\}$ (r_0 included) with linear interpolation between them:

$$f(r) = f_j + \frac{r - r_j}{r_{j+1} - r_j}(r - r_j) \tag{43}$$

which is easily rewritten as

$$f(r) = w_j^-(r)\, f_j + w_{j+1}^+(r)\, f_{j+1} \tag{44}$$

where the coefficients $w_j^-(r)$ $w_{j+1}^+(r)$ have nonzero values:

$$w_j^-(r) = \frac{r_{j+1} - r}{r_{j+1} - r_j}, \quad w_{j+1}^+(r) = \frac{r - r_j}{r_{j+1} - r_j}; \quad (r_j < r < r_{j+1}) \tag{45}$$

and are identical zero outside of the interval specified. The "$-$" and "$+$" subscripts simply mean that the corresponding gridpoint is below or above the current value of the argument. These coefficients can be easily integrated numerically along the line of sight:

$$w_j^\pm(r_0) = \int_{r_0}^{r} w_j^\pm(r')\, \frac{dr'}{\sqrt{r' - r_0}} \tag{46}$$

Substituting Eqs.44, 45 into Eq.42, performing integration and combining the terms with the same indices j' gives:

$$F(r_j, r_0) = \sum_{j'=0}^{j} w_{j'}(r_0) f_{j'} \tag{47}$$

where

$$\begin{aligned} w_{j'}(r_0) &= w_{j'}^-(r_0) + w_{j'}^+(r_0), \quad 0 < j' < j; \\ w_0(r_0) &= w_0^-(r_0), \quad w_j(r_0) = w_j^+(r_0) \end{aligned} \tag{48}$$

The singularity that is present in Eq.42 is now contained in the integral coefficients $w_{j'}(r_0)$.

Applying treatment of this Subsection to Eq.9 we obtain:

$$I_{\uparrow j}(r_0) = \sum_{j'=0}^{j} w_{j'}(r_0)\Big(B_{j'} - I_{\uparrow j'}(r_0)\Big)\kappa_{j'} + I_{\uparrow 0}(r_0) \tag{49}$$

and we can obtain an expression for the matrix of partial derivatives with respect to κ_j in the form analogous to the expression for the corresponding variational derivative, Eq.10:

$$\frac{\partial I_{\uparrow j}(r_0)}{\partial \kappa_j} = w_j(r_0)\Big(B_j - I_{\uparrow j}(r_0)\Big) + \frac{\partial I_{\uparrow 0}(r_0)}{\partial \kappa_j} \tag{50}$$

The resulting algorithm for the single line of sight with the tangent point at r_0 written in terms of partial derivatives can be summarized as follows (cf Subsection 3.1):

1. Recursively compute the downwelling radiances (using, e.g., Eq.49) starting from the value $I_{\downarrow,N-1} = 0$:

$$I_{\downarrow j}(r_0) = \sum_{j'=j+1}^{N-1} w_{j'}(r_0)\Big(B(T_{j'}) - I_{\downarrow j'}(r_0)\Big)\kappa_{j'} \tag{51}$$

2. Define the upwelling radiances at the tangent point using an identity that holds there

$$I_{\uparrow 0}(r_0) = I_{\downarrow 0}(r_0) \tag{52}$$

3. Recursively compute the upwelling radiances (using, e.g., Eq.49) starting from the value $I_{\downarrow 0}$:

$$I_{\uparrow j}(r_0) = \sum_{j'=0}^{j} w_{j'}(r_0)\Big(B(T_{j'}) - I_{\uparrow j'}(r_0)\Big)\kappa_{j'} \tag{53}$$

4. Compute the upwelling transmittances:

$$t_{\uparrow j}(r_0) = exp\left(-\sum_{j'=j}^{N-1} w_{j'}(r_0)\kappa_{j'}\right) \tag{54}$$

5. Compute the downwelling transmittances using the relation:

$$t_{\downarrow j}(r_0) = \frac{t_{\uparrow 0}(r_0)}{t_{\uparrow j}(r_0)} \tag{55}$$

6. Evaluate the upwelling, downwelling and AC weighting functions:

$$\left(\frac{\partial R(r_0)}{\partial \kappa_j}\right)_{\uparrow} = w_j(r_0)t_{\uparrow j}(r_0)\,\Big(B(T_j) - I_{\uparrow j}\Big) \tag{56}$$

$$\frac{\partial I_{\downarrow 0}(r_0)}{\delta \kappa_j} = w_j(r_0) t_{\downarrow j}(r_0) \left(B(T_j) - I_{\downarrow j} \right) \tag{57}$$

$$\frac{\partial R(r_0)}{\partial \kappa_j} = \left(\frac{\partial R(r_0)}{\partial \kappa_j} \right)_{\uparrow} + t_{\uparrow j} \frac{\partial I_{\downarrow 0}(r_0)}{\partial \kappa_j} \tag{58}$$

7. Evaluate the VMR weighting function:

$$\frac{\partial R(r_0)}{\partial \ln f_j} = \kappa_{m,j} \frac{\partial R(r_0)}{\partial \kappa_j} \tag{59}$$

8. Evaluate the temperature weighting function:

$$\frac{\partial R(r_0)}{\partial T_j} = \kappa_j t_{eff,j}(r_0) \left. \frac{dB}{dT} \right|_{T_j} + \frac{\partial R(r_0)}{\partial \kappa_j} \cdot \frac{d\kappa_j}{dT} \tag{60}$$

where the effective transmittance $t_{eff,j}(r_0)$ is defined as

$$t_{eff,j}(r_0) = t_{\uparrow,j}(r_0) + t_{\downarrow,j}(r_0) \tag{61}$$

4.2 Integration over the field of view

Although computations of the weighting functions (here, jacobian matrices) using the intermediate radiances substantially reduces the computing time as compared with independent computations, this process, even for a single line of sight, remains to be a time consuming. In the real world we have to integrate over the finite field of view of the instrument, or, if there is any way of scanning across the limb, – over an array of them.

This Subsection provides an optional way of incorporating the radiances and jacobians computed for a *discrete* set of lines of sight into those computed for the field of view with inevitably finite span over the tangent heights above the limb.

Let the set of lines of sight with tangent radial distances $\{\rho_k\}, k = 0, \dots n-1$ provides a detailed enough coverage of limb radiances:

$$R_k = R(\rho_k) \, k = 0, \dots n-1 \tag{62}$$

The radiance $R(\rho)$ for an arbitrary tangent radial distance ρ can be interpolated in a manner analogous to Eqs.43, 44

$$R(\rho) = a_k^-(r) R_k + a_{k+1}^+ R_{k+1} \tag{63}$$

where

$$a_k^-(\rho) = \frac{\rho_{k+1} - \rho}{\rho_{k+1} - \rho_k} \quad \text{if} \quad \rho_k < \rho < \rho_{k+1}) \quad \text{else} \quad a_k^-(\rho) \equiv 0 \tag{64}$$

$$a_{k+1}^+(\rho) = \frac{\rho - \rho_k}{\rho_{k+1} - \rho_k}; \quad \text{if} \quad \rho_k < \rho < \rho_{k+1}) \quad \text{else} \quad a_k^+(\rho) \equiv 0 \tag{65}$$

Combining the coefficients at the same grid point ρ_k

$$a_k(\rho) = a_k^-(\rho) + a_k^+(\rho) \tag{66}$$

we can rewrite Eq.63 in the form of a sum over the radiances R_k:

$$R(\rho) = \sum_{k=0}^{n-1} a_k(\rho) R_k \tag{67}$$

and convolve it with the sensitivity $\phi(\rho)$ across the field of view:

$$\int_{FOV} R(\rho)\, d\rho = \sum_{k=0}^{n-1} \left(\int_{FOV} a_k(\rho)\, d\rho \right) \cdot R_k \tag{68}$$

Thus, the effective radiance $R^{(FOV)}$ observed in the given field of view can be expressed through the grid radiances R_k

$$R^{(FOV)} = \sum_{k=0}^{n-1} a_k^{(FOV)} R_k \tag{69}$$

using the convolved coefficients

$$a_k^{(FOV)} = \int_{FOV} a_k(\rho)\, d\rho \tag{70}$$

Using the same coefficients, the partial derivatives of radiances $R^{(FOV)}$ to any atmospheric parameter's value X_j at a gridpoint r_j can be expressed through the derivatives computed for a given set of lines of sight across the field of view:

$$\frac{\partial R^{(FOV)}}{\partial X_j} = \sum_{k=0}^{n-1} a_k^{(FOV)} \frac{\partial R_k}{\partial X_j} \tag{71}$$

The idea of using the convolution of the interpolation coefficients instead of radiances and jacobians themselves can be extended from the linear interpolation to that using the higher–order polynomial.

5 Conclusion

Analytic approach to computations of the weighting functions for remote sensing, as compared to the finite–difference approach, has substantial advantages in terms of computer time required. Both temperature and VMR weighting functions can be computed using this approach. The closed–form analytic expressions fot the weighting functions still may require expenditures of computer time comparable or exceeding those needed for computations of observed radiances.

The particular analytic approach proposed in this presentation is aimed at further savings of computer time. It is based on the using of the intermediate radiances $I(s)$ and transmitttances $t(s)$ that are byproducts of computations of observed radiances R. They are used in simple closed–form relations to compute the AC weighting functions and, further on, to evaluate the temperature and VMR weighing functions. As has been demonstrated above, this approach is applicable to cases of both nadir and limb geometry of observations.

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