

Gravitational Wave Sensitivity of Alternate LISA Configurations

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We have proposed [1,2,3] LISA data-gathering configurations involving multiple one-way laser links between the test masses; suitable linear combinations of these data and intra-spacecraft calibration signals cancel the leading noise sources (laser phase noise and optical bench motion) while preserving the GW signal. These configurations include, as a subset, the unequal-arm Michelson interferometers currently baselined by the Project.

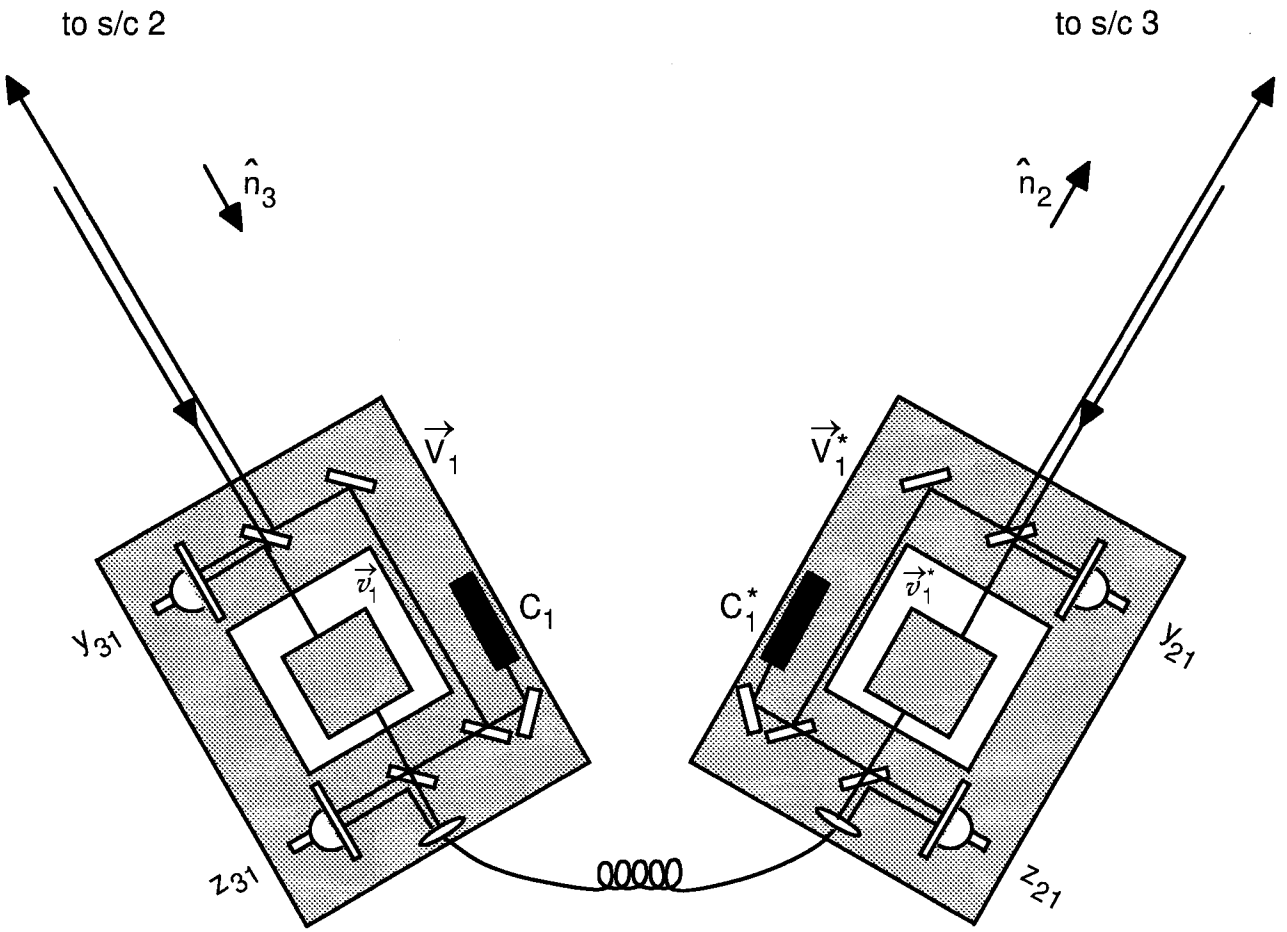
We present here a GW sensitivity-calculation methodology for any noise-cancelling LISA configuration, emphasizing the transfer functions of each of these configurations to the GW signal and to the residual noises (principally: proof mass acceleration noise and optical path noise). Sensitivities will be shown for several alternate configurations, including those where some links are lost due (say) to on-orbit failure of specific subsystems.

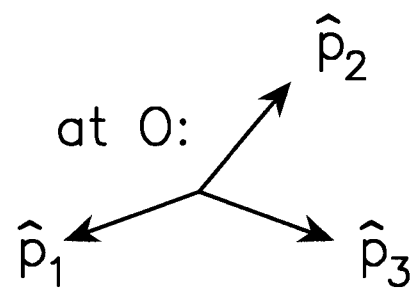
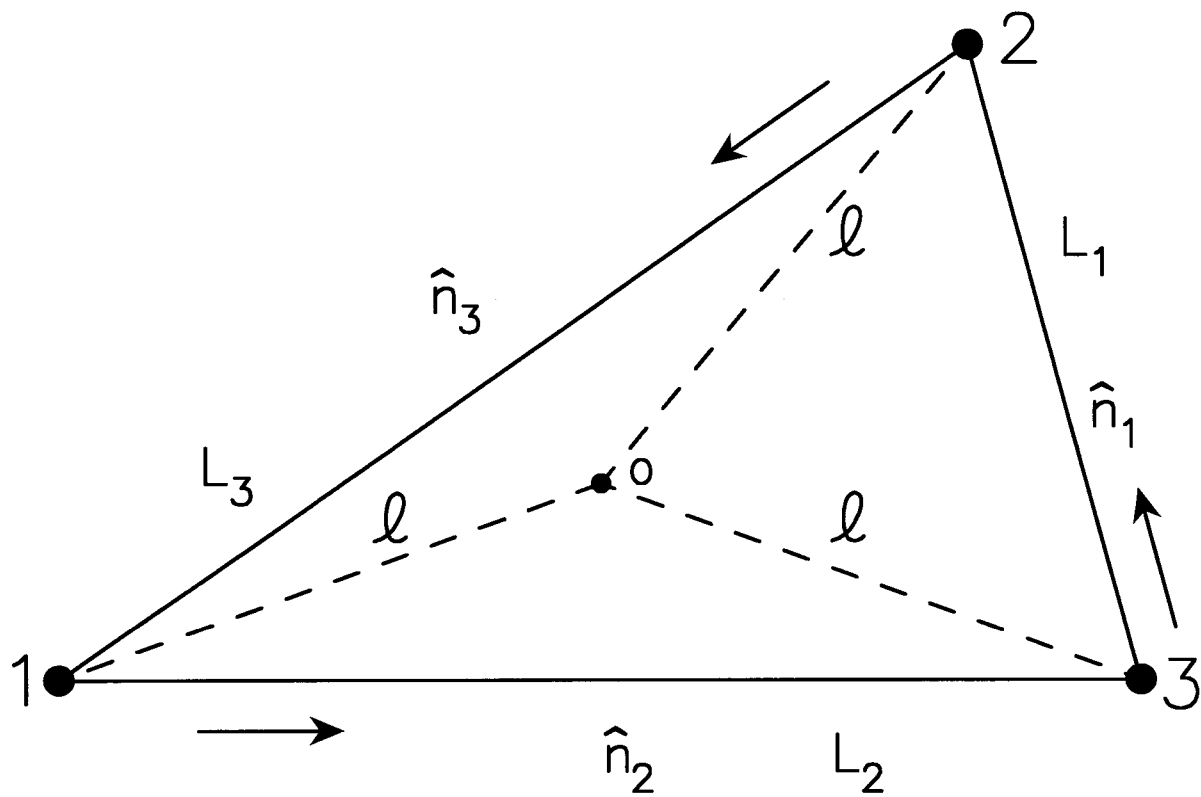
- [1] Tinto & Armstrong Phys. Rev. D, 59, 102003 (1999)
- [2] Armstrong, Estabrook, & Tinto ApJ, 527, 814 (1999)
- [3] Estabrook, Tinto, & Armstrong Phys. Rev. D, in press (2000)

Gravitational Wave Sensitivities of Alternate LISA Configurations

John Armstrong, Frank Estabrook, and Massimo Tinto

- **Why consider alternatives to Michelson?**
- **Formulation and Computation**
 - **One-way link combinations canceling leading noises**
 - **GW signals and their transfer functions to observables**
 - **Noises and their transfer functions to observables**
- **Sensitivities for 6- and 4-link data combinations**
- **Concluding comments**



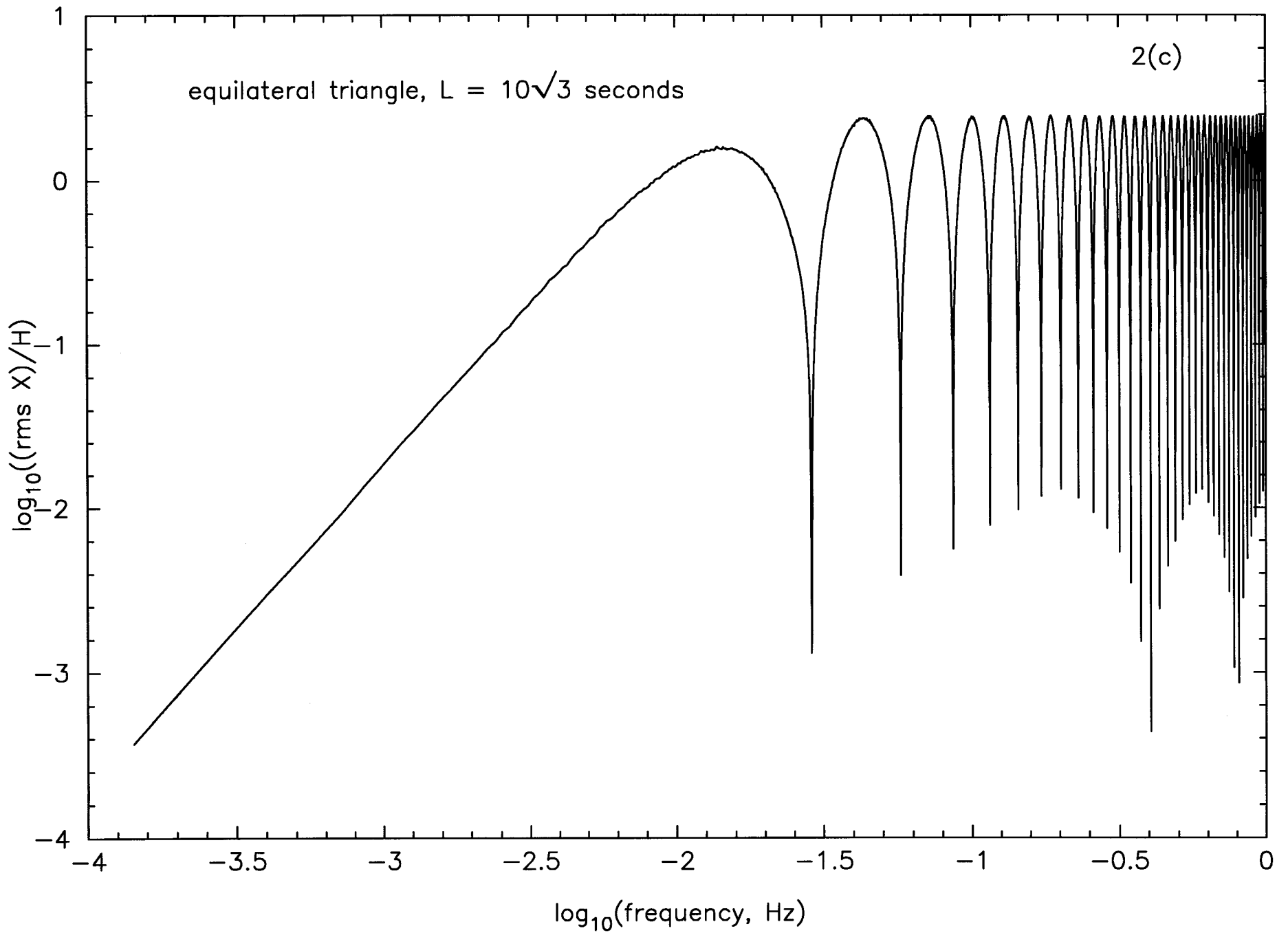


One-Way Links: GW Response

- **Notation and conventions:**
 - **Three LISA spacecraft, equidistant from point "O"**
 - **Unit vectors \mathbf{p}_i locate the spacecraft in the plane**
 - **Unit vectors \mathbf{n}_i connect spacecraft pairs with the indicated orientation**
 - **$y_{21} = \Delta v/v_0$ is the fractional Doppler shift on link originating at s/c 3, measured at s/c 1**
 - **Cyclic permutation of the indices (1 --> 2 --> 3 --> 1)**
- **GW response is "two-pulse" (Wahlquist, GRG, 19, 1101, 1987):**

$$\psi_i(t) = -\frac{1}{2} \frac{\hat{\mathbf{n}}_i \cdot \mathbf{h}(t) \cdot \hat{\mathbf{n}}_i}{(1 - \hat{\mathbf{k}} \cdot \hat{\mathbf{n}}_i)^2}$$

$$y_{21}(t) = [\Psi_2(t - L_3 - \mu_3 \varrho) - \Psi_2(t - \mu_1 \varrho)] [1 - (\varrho/L_2)(\mu_3 - \mu_1)]$$



Noise-Canceling Data Combinations

- **Suitable delayed/added linear combinations of the y_{ij} + internal z_{ij} calibration signals can cancel laser phase noise and optical bench noise**
 - **Tinto & Armstrong (1999)**
 - **Armstrong, Estabrook, & Tinto (1999)**
 - **Peterseim et al. (2000)**
 - **Estabrook, Tinto, & Armstrong (2000)**
- **"Time-delay interferometry"**
- **Example: $\alpha = y_{21} - y_{31} + y_{13,2} - y_{12,3} + y_{32,12} - y_{23,13}$ exactly cancels all laser phase noises**

One-Way Links: Noise Response

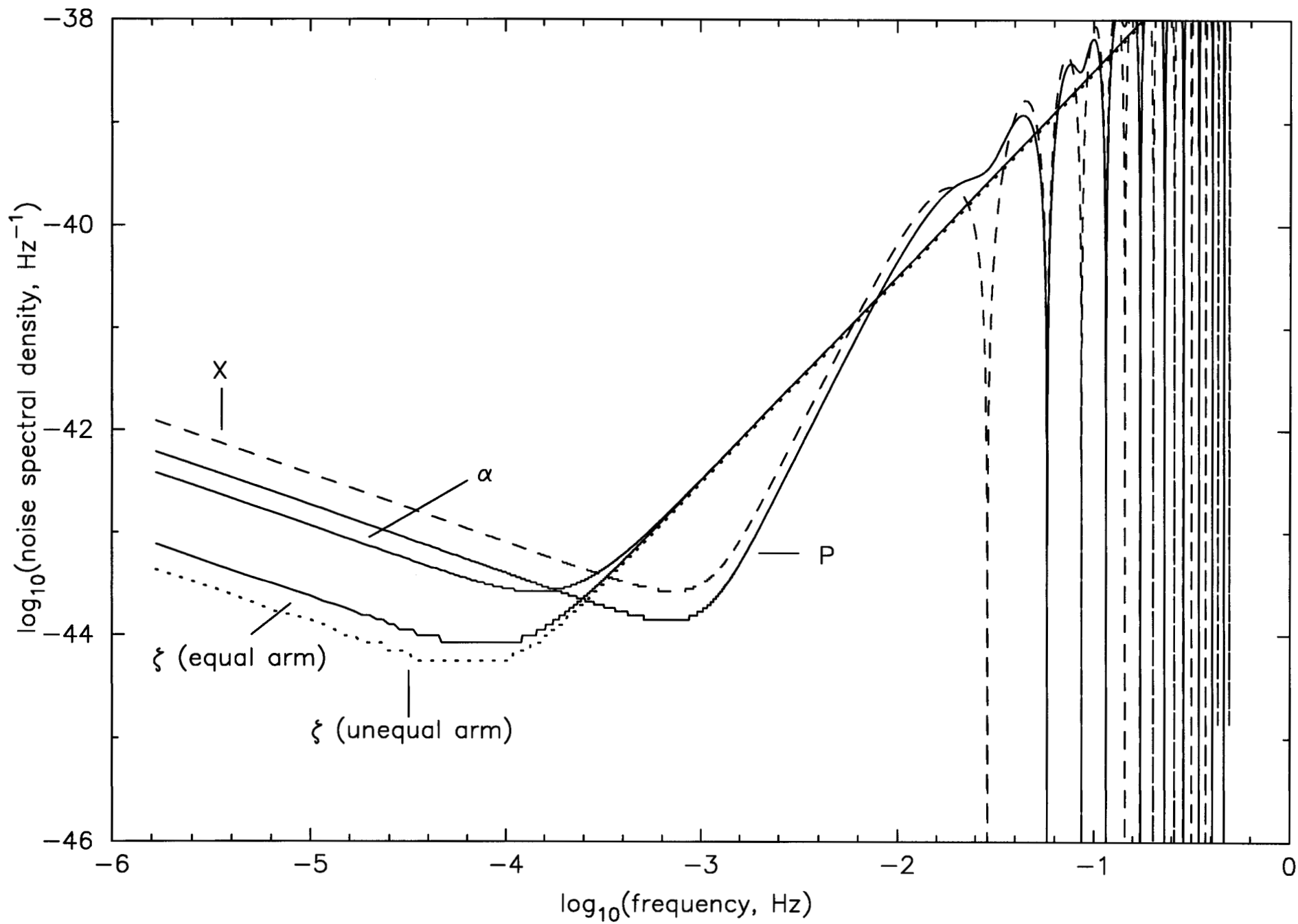
- **Doppler link noises, in simplest case:**

laser phase noise: $y_{31} = C_2(t - L_3) - C_1(t)$

shot noise: $y_{31} = \text{shot}_{31}(t)$

proof mass noise: $y_{31} = \mathbf{n}_3 \cdot \mathbf{v}_2(t - L_3) - \mathbf{n}_3 \cdot \mathbf{v}_1(t)$

- **Introduce comma-notation, e.g. $C_2(t - L_3) = C_{2,3}$**



Procedure: Noises

- **Raw spectra of proof-mass acceleration noise and optical path noise (mostly shot + beam pointing) from the LISA Pre-Phase A report—these are expressed in units of length per square-root-Hz**
- **Square to get power spectra, and convert from units of $\text{length}^2/\text{Hz}$ to fractional Doppler: use derivative theorem for Fourier transforms to express as equivalent velocity spectrum, divide by speed of light squared to convert to spectrum of y . This gives the raw noise spectrum for the fractional Doppler shift of, e.g., a single proof mass noise or the optical path noise of a single laser link.**
- **Write out the data combination under consideration in terms of the difference equation involving the y_{ij} 's defining the noise-canceling combination. Square of z-transform of the difference equation gives the noise's transfer function.**
- **Example: Aggregate noise spectrum for data combination X in equilateral triangle**

Procedure: GW Signal

For a noise-canceling linear combination (e.g. a, X, P, etc) under consideration and for each Fourier frequency in the band:

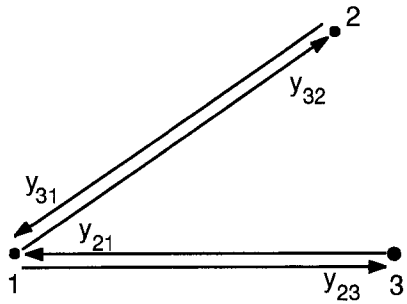
- **Generate N "sources", uniform on the celestial sphere, radiating "waves" with random, arbitrary elliptical polarization state (uniform on the Poincare sphere)—4 numbers: "right ascension", "declination", wave ellipticity, and wave tilt**
- **Generate Ψ_i , which depend on the s/c positions, gravitational wave vector and the polarization state**
- **Compute the y_{ij} , which depend on the Ψ_i , wavevector, arm lengths, and array orientation w.r.t. the source (p_i)**
- **Form linear combination to cancel laser noise (if in long-wavelength limit, check calculation against LWL analytical expansion)**
- **Iterate over Monte Carlo sources/polarizations to compute the mean-square GW signal response for this data combination at this frequency**
- **Example: GW response for Michelson configuration, X**

Sensitivity

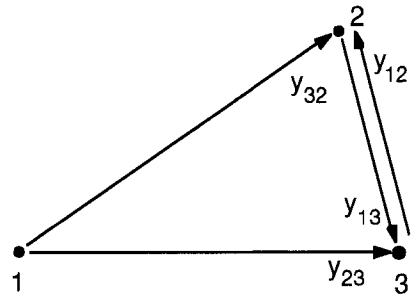
- **Conventional figure-of-merit is RMS sensitivity required for a sinusoidal wave (SNR = 5 in bandwidth $B = 1/(\text{one year})$, averaged over source position and over wave polarization state), as a function of Fourier frequency**
- **All real-world problems (e.g. apparatus rotation) ignored for now**

$$h = \frac{\text{noise}}{\text{signal}} = \frac{5 \sqrt{S_{\text{noise}}(f) B}}{\text{rms GW response}}$$

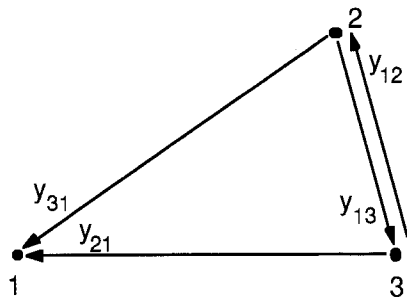
- **Thus need GW signal response for a given data combination and the noise spectra (including transfer functions) for that data combination**



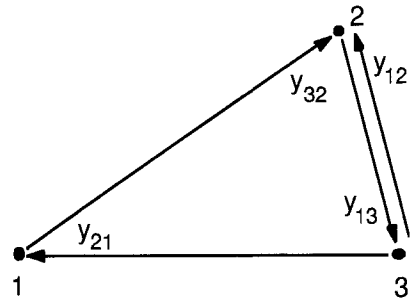
INTERFEROMETER (X,Y,Z)



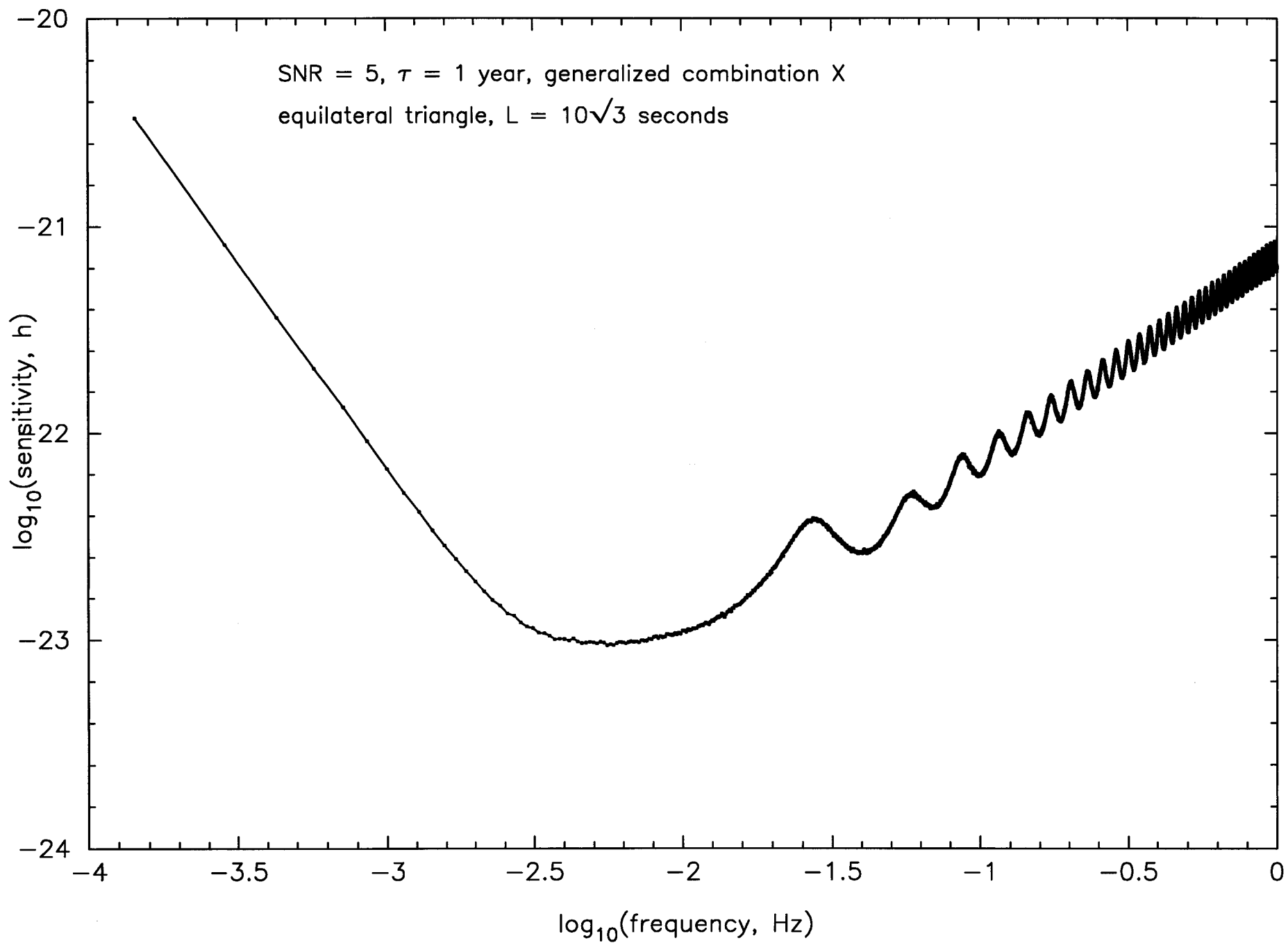
BEACON (P,Q,R)

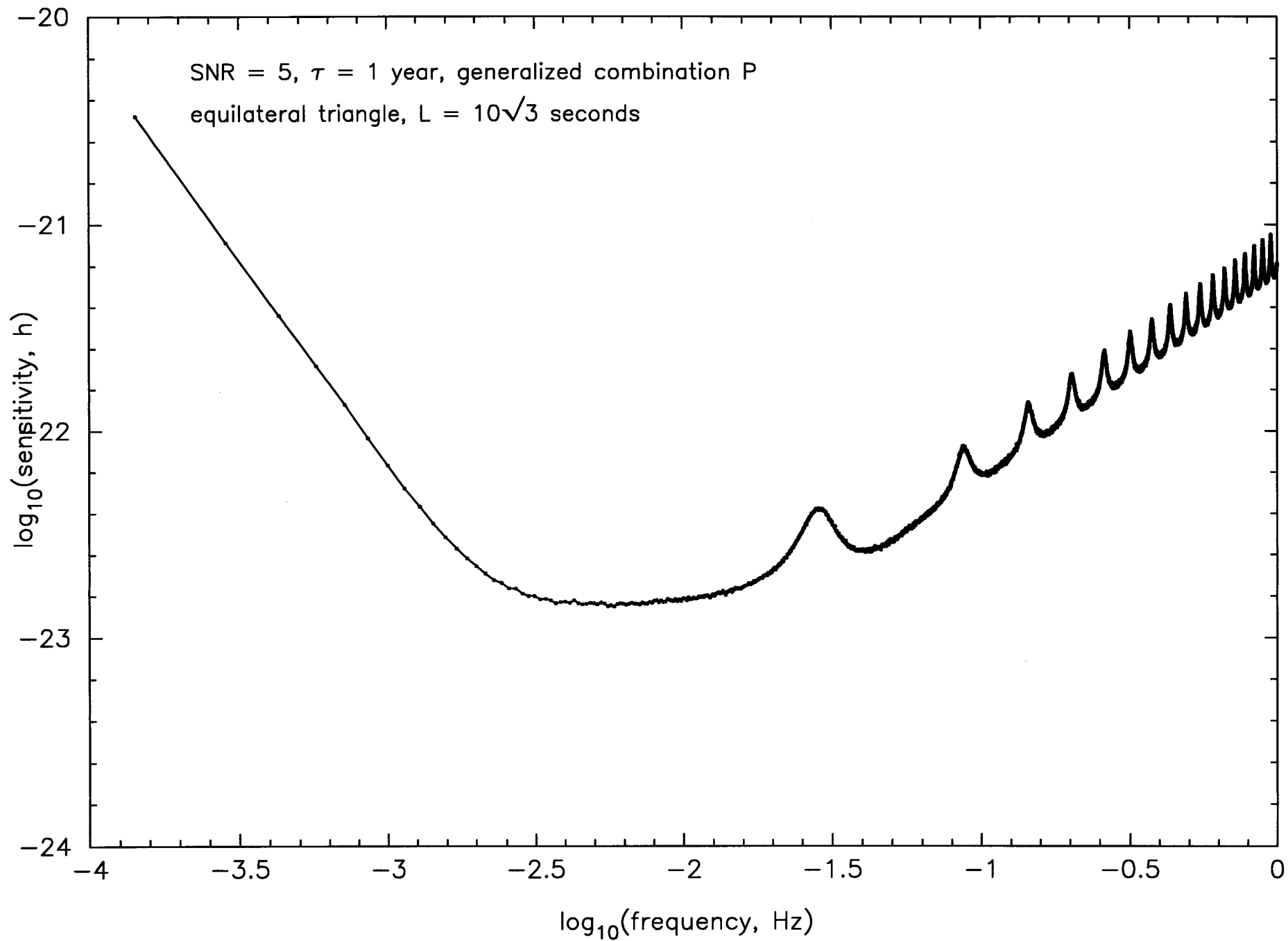


MONITOR (E,F,G)

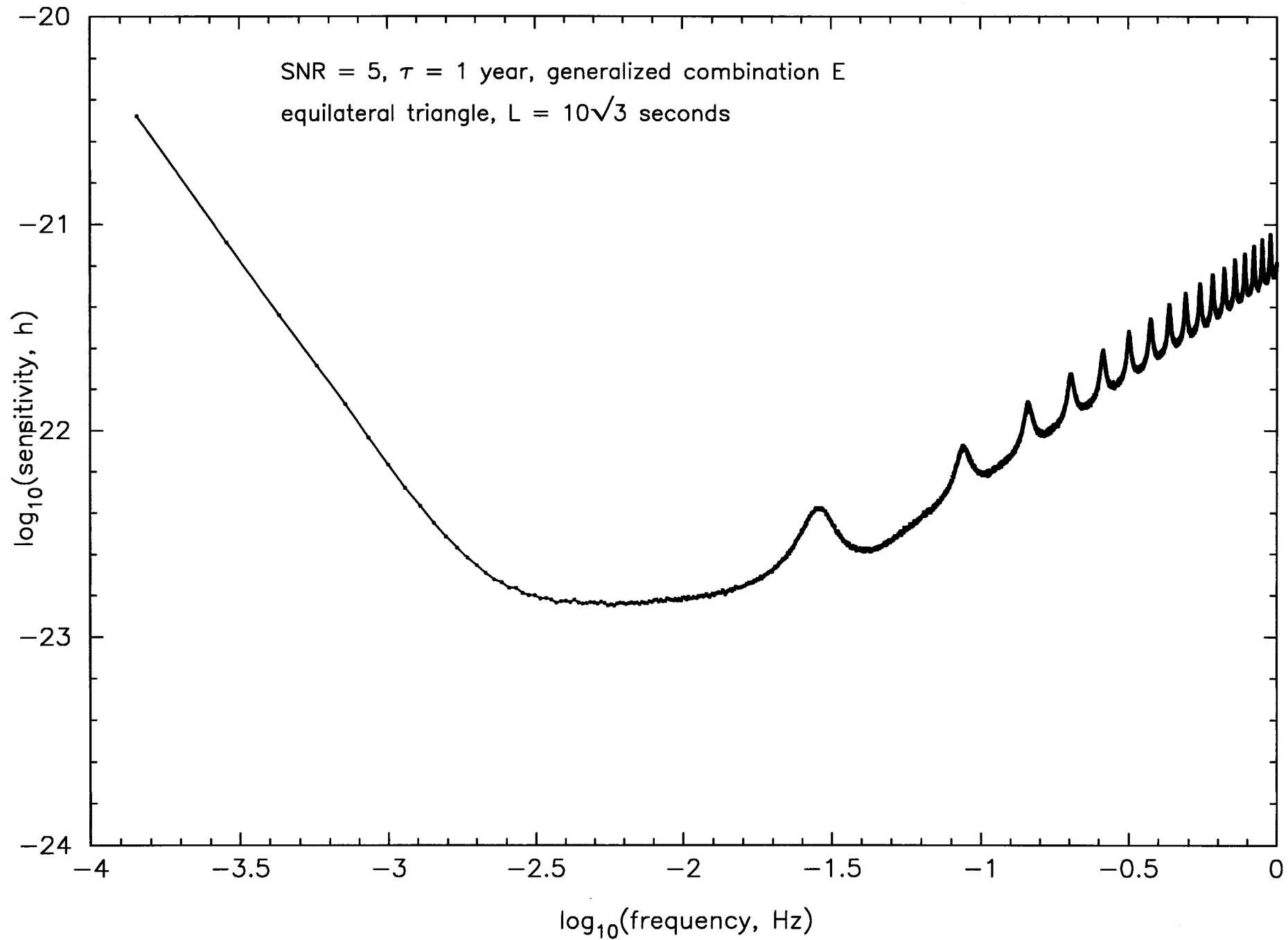


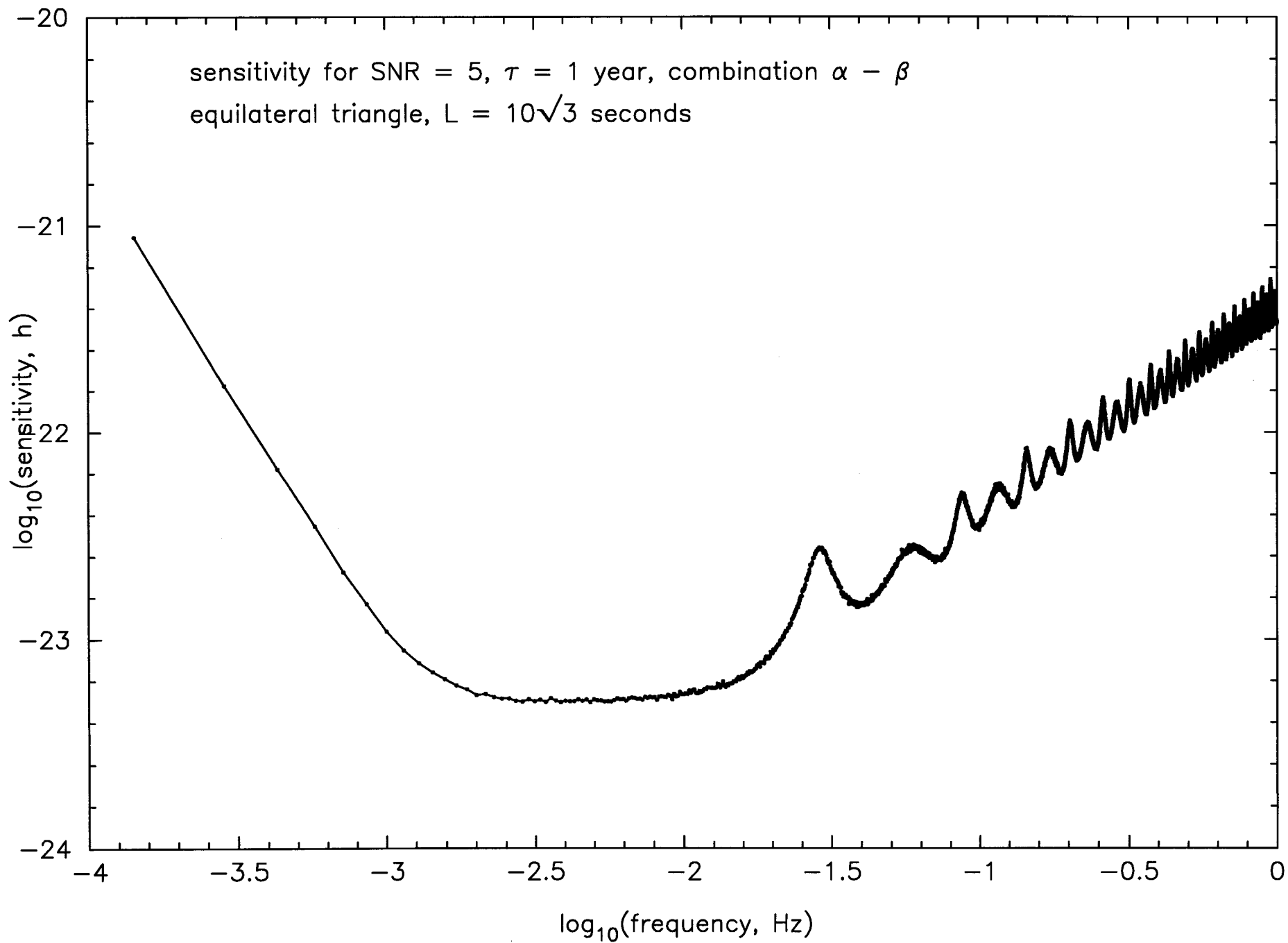
RELAY (U,V,W)





SNR = 5, $\tau = 1$ year, generalized combination E
equilateral triangle, $L = 10\sqrt{3}$ seconds





Concluding Comments

- **LISA can be analyzed in terms of Doppler shifts on 6 one-way laser links connecting the three test masses**
- **Many ways to combine these 6 links, with appropriate time delays ("time-delay interferometry"), to cancel principal noises but retain GW signal**
- **GW sensitivity for each laser-noise-free data combination constructed by ratio of noise to signal at each Fourier frequency:**
 - **Signal determined by computing GW response of the given data combination (averaged over the sky and general elliptical polarization state)—i.e., passing the GWs through the signal transfer function appropriate for that linear combination.**
 - **Noise determined from spectra of the fundamental noises (e.g., proof mass noise, shot noise, etc), modified by the transfer function of laser-noise-canceling combination being considered**
- **Only one measure of sensitivity considered here! (we have a particular source in mind)**
- **Many noise-canceling configurations have comparable best sensitivity and comparable bandwidth**