A New Class of Nonlinear PID Controllers

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Abstract

This paper introduces a new class of simple nonlinear PID-type controllers comprised of a sector-bounded nonlinear gain in cascade with a linear fixed-gain P, PD, PI, or PID controller. Two simple nonlinear gains are proposed: the sigmoidal function and the hyperbolic function. The systems to be controlled are assumed to be modeled or approximated by second-order transfer-functions, which can represent many robotic applications. The stability of the closed-loop systems incorporating nonlinear P, PD, PI, and PID controllers are investigated using the Popov Stability Criterion. A numerical example is given for illustration.

1 Introduction

Undoubtedly, PID controllers are the most popular and the most commonly used industrial controllers in the past fifty years. The popularity and widespread use of PID or three-term controllers is attributed primarily to their simplicity and performance characteristics, where the I term ensures robust steady-state tracking of step commands while the P and D terms provide stability and desirable transient behavior. PID controllers have been utilized for the control of diverse dynamical systems ranging from industrial processes to aircraft and ship dynamics. In fact, industrial robotic manipulators invariably use PID controllers in their independent joint servo control systems.

While linear fixed-gain PID controllers are often adequate for controlling a nominal physical process, the requirements for high-performance control with changes in operating conditions or environmental parameters are often beyond the capabilities of simple PID controllers. For instance, when a robotic arm is contacting a reaction surface with a known stiffness coefficient, a linear fixed-gain PID controller can be designed to achieve a desirable force response with zero steady-state error, low overshoot, and rapid rise time. However, the same controller typically exhibits a sluggish response in contact with softer surfaces, and becomes unstable when contacting harder surfaces. In other words, because the stiffness coefficients of different reaction surfaces can differ substantially, a fixed-gain PID controller design based on a nominal surface stiffness leads to a non-uniform dynamic performance and often instability. This problem can be alleviated, to a large extent, by employing nonlinear elements in the PID control scheme. These elements can compensate for stiffness variations and yield stable and uniform responses. Even when the reaction surface stiffness is constant and known, a nonlinear PID controller can result in superior command tracking and disturbance rejection performances compared to linear fixed-gain PID controllers.

This paper presents a simple enhancement to the conventional PID controller by incorporating a nonlinear gain in cascade with a linear fixed-gain PID controller. This enhancement enables the controller to *adapt* its response based on the performance of the closed-loop control system. When the error between the commanded and actual values of the controlled variable is large, the gain amplifies the error substantially to generate a large corrective action to drive the system output to its goal rapidly. As the error diminishes, the gain is automatically reduced to prevent large overshoots in the response. Because of this automatic gain adjustment, the nonlinear PID controller enjoys the advantage of high initial gain to obtain a fast response, followed by a low gain to prevent large overshoots.

The paper is structured as follows. The problem is stated in Section 2. Absolute stability of the closed-loop systems incorporating nonlinear P, PD, PI, and PID controllers are investigated in Sections 3–6. A numerical example is given in Section 7 for illustartion. Finally, conclusions drawn from this work are presented in Section 8.

2 Problem Statement

In many robotic applications, the dynamics of the system to be controlled can be adequately modeled by a second-order differential equation. Even when the system dynamics is of higher order, the response of the system is often largely dependent on the location of a pair of *dominant complex poles*, which can be embodied in a second-order model [1]. Examples of such robotic systems are: joint servo dynamics, arm Cartesian dynamics, force control, and compliance/impedance control. In these systems, the second-order transfer-function relating the system output $y(t)$ to the control input $u(t)$ is given by

$$G(s) = \frac{y(s)}{u(s)} = \frac{\omega_n^2 \hat{k}}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{c}{s^2 + as + b} \quad (1)$$

where ξ, ω_n, and $\hat{k}$ denote, respectively, the damping ratio, natural frequency, and open-loop gain of the system, $a = 2\xi\omega_n$, $b = \omega_n^2$, and $c = \omega_n^2 \hat{k}$.

The new class of controllers presented in this paper consists of a nonlinear gain k in cascade with a linear constant-gain PID-type controller $K(s) = k_p + \frac{k_i}{s} + k_d s$, where k_p, k_i, and k_d are the positive or zero proportional, integral, and derivative gains, respectively. The nonlinear gain k acts on the error $e(t) = y_r(t) - y(t)$ between the actual output $y(t)$ and the desired output $y_r(t)$, and produces the "scaled" error $f(t) = k(e).e(t)$, where $k(e)$ denotes a function of e. The scaled error $f(t)$ is then inputted to the PID controller $K(s)$ which generates the control action $u(s) = K(s).f(s)$ to drive the system, as shown in Figure 1. The gain k can represent any nonlinear function which is bounded in the sector $0 < k < k_{max}$. There is a broad range of options available for the nonlinear gain k. Here, we propose two examples of such functions. The first proposed nonlinear gain k as a function of the error e is the smooth *sigmoidal* function

$$k = k_o + k_1 \left\{ \frac{2}{1 + exp(-k_2 e)} - 1 \right\}$$

where k_o, k_1, and k_2 are user-defined positive constants. The gain k is lower-bounded by $k_{min} = k_o - k_1$ when $e = -\infty$, is upper-bounded by $k_{max} = k_o + k_1$ when $e = +\infty$, that is $k_{min} < k < k_{max}$, and furthermore $k = k_o$ when $e = 0$. Thus k_o defines the *central value* of k, k_1 determines the *range of variation* of $k (= k_{max} - k_{min} = 2k_1)$ with $k_1 \leq k_o$ to ensure $k > 0$, while k_2 specifies the *rate of variation* of k. Figure 2a shows a typical variation of k as a function of e when k_o=2, k_1=1, and k_2=0.5, and shows that k has an "S-shaped" curve.

The second proposed choice for the gain k as a function of the error e is the *hyperbolic* function

$$\begin{aligned} k &= k_o + k_1 \left\{ 1 - \frac{2}{exp(k_2 e) + exp(-k_2 e)} \right\} \\ &= k_o + k_1 \{ 1 - sech(k_2 e) \} \end{aligned}$$

where k_o, k_1, and k_2 are user-defined positive constants. The gain k is now upper-bounded by $k_{max} = k_o + k_1$ when $e = \pm\infty$, and lower-bounded by $k_{min} = k_o$ when $e = 0$. Thus k_o defines the ***minimum value***, k_1 denotes the ***range of variation***, and k_2 specifies the ***rate of variation*** of k. Figure 2b shows a typical variation of k versus e when k_o=1, k_1=1, and k_2=0.5. It is seen that k is an "inverted bell-shaped" curve, and is an even function of e, that is $k(-e) = k(e)$. This class of nonlinear gains is applicable when k is required to be a function of the error magnitude $|e|$.

The motivation for using the nonlinear gain k is now discussed qualitatively. When k is a constant, the linear PID controller gains can be chosen such that for a step command input, the closed-loop system exhibits either an oscillatory fast response with overshoot or a monotonic slow response with no overshoot. In other words, the linear PID controller is incapable of accomplishing the two contradictory requirements of a fast response and no overshoot simultaneously. On the other hand, when the gain k is a nonlinear function of the error e, such as the sigmoidal function defined earlier, initially the error e between the command y_r and the output y is large, hence the gain k will be large, producing a fast response. As time proceeds and the error e is diminished, the gain k will be reduced automatically. When the output y overshoots the command y_r, the gain k is reduced even further, thus inhibiting further overshoot. Therefore, the automatic adjustment of the gain k as a function of the error e can produce a fast response with a small overshoot, a behavior that is unattainable by a linear fixed-gain PID controller. This argument can be repeated when the system is subjected to disturbance inputs, whereby the nonlinear gain enables the system to exhibit a fast non-oscillatory response.

Consider now the closed-loop control system shown in Figure 1. Because of the nonlinear nature of k, the stability analysis of the closed-loop system is nontrivial. We shall now present the stability analysis of the closed-loop nonlinear systems with different types of PID controllers.

3 Stability Analysis of Nonlinear P Controllers

In this case, the closed-loop system employs the proportional (P) controller

$$K(s) = k_p \tag{2}$$

in cascade with the nonlinear gain k, where k_p is the constant positive proportional gain.

To investigate the absolute stability of the closed-loop system, we combine the linear components (1) and (2) as

$$W(s) = G(s)K(s) = \frac{ck_p}{s^2 + as + b} \tag{3}$$

which is a second-order transfer-function, and separate out the nonlinear element which is the gain k. We can now apply the Popov Stability Criterion [2,3] to the system by examining the Popov plot of $W(j\omega)$, which is the plot of $\mathcal{R}eW(j\omega)$ versus $\omega\mathcal{I}mW(j\omega)$, with the frequency ω as a parameter and $\mathcal{R}e$ and $\mathcal{I}m$ refer to the real and imaginary parts, respectively. This plot reveals the range of values that the nonlinear gain k can assume while retaining closed-loop stability. The Popov Criterion states that:

"A *sufficient* condition for the closed-loop system to be absolutely stable for all nonlinear gains in the sector $0 < k < k_{max}$ is that the Popov plot of $W(j\omega)$ lies entirely to the right of a straight-line passing through the point $-\frac{1}{k_{max}} + j0$."

In order to apply the Popov Criterion to the system, we need to compute the crossing of the Popov plot of $W(j\omega)$ with the real axis. In this case, from equation (3), we obtain

$$\mathcal{R}eW(j\omega) = \frac{ck_p(b-\omega^2)}{a^2\omega^2 + (b-\omega^2)^2} \tag{4}$$

$$\omega\mathcal{I}mW(j\omega) = \frac{-ack_p\omega^2}{a^2\omega^2 + (b-\omega^2)^2} \tag{5}$$

Thus the Popov plot of $W(j\omega)$ starts at the point $P(\frac{ck_p}{b}, 0)$ for $\omega = 0$ and terminates at the point $Q(0,0)$ for $\omega = \infty$.

It is seen that $\omega\mathcal{I}mW(j\omega)$ is *always* negative for all non-zero ω, that is, the Popov plot of $W(j\omega)$ remains entirely in the third and fourth quadrants and does *not* cross the real axis. This implies that we can construct a straight-line passing through the origin such that the Popov plot is entirely to the right of this line. Therefore, according to the Popov Criterion, the range of the allowable nonlinear gain k is $(0,\infty)$.

4 Stability Analysis of Nonlinear PD Controllesr

In this case, we employ the proportional-derivative (PD) controller

$$K(s) = k_p + k_d s \tag{6}$$

in cascade with the nonlinear gain k, where k_p and k_d are the constant positive proportional and derivative gains, respectively.

To investigate the absolute stability of the closed-loop system, we combine the linear components (1) and (6) as

$$W(s) = G(s)K(s) = \frac{c(k_p + k_d s)}{s^2 + as + b} \tag{7}$$

which is a second-order transfer-function, and separate out the nonlinear element which is the gain k. To find out the range of values that the nonlinear gain k can assume while retaining closed-loop stability, we examine the Popov plot of $W(j\omega)$. In this case, from equation (7), we obtain

$$\mathcal{R}eW(j\omega) = \frac{c[(ak_d - k_p)\omega^2 + bk_p]}{a^2\omega^2 + (b-\omega^2)^2} \tag{8}$$

$$\omega\mathcal{I}mW(j\omega) = \frac{-c\omega^2[k_d\omega^2 + (ak_p - bk_d)]}{a^2\omega^2 + (b-\omega^2)^2} \tag{9}$$

The Popov plot of $W(j\omega)$ starts at the point $P(\frac{ck_p}{b}, 0)$ for $\omega = 0$ and terminates at the point $Q(0, -ck_d)$ for $\omega = \infty$. Two cases are now possible depending on the *relative* values of k_p and k_d.

4.1 Case One: $bk_d \le ak_p$

In this case, from equation (9) it is seen that $\omega\mathcal{I}mW(j\omega)$ is *always* negative for all non-zero ω, that is, the Popov plot of $W(j\omega)$ remains entirely in the third and fourth quadrants and does *not* cross the real axis. Therefore, according to the Popov Criterion, the range of the allowable nonlinear gain k is $(0,\infty)$.

4.2 Case Two: $bk_d > ak_p$

In this case, the Popov plot of $W(j\omega)$ crosses the real axis. The crossover frequency ω_o is found by solving $\omega\mathcal{I}mW(j\omega) = 0$ to yield

$$\omega_o{}^2 = \frac{bk_d - ak_p}{k_d}$$

and the value of $W(j\omega_o)$ is then found to be

$$\mathcal{R}eW(j\omega_o) = \frac{ck_d}{a}$$

which is always positive. Since the Popov plot of $W(j\omega)$ never crosses the *negative* real axis, from the Popov Criterion the range of the allowable nonlinear gain k is $(0, \infty)$.

We conclude that in both cases, the closed-loop system is always stable under PD control with unbounded nonlinear gain k.

5 Stability Analysis of Nonlinear PI Controllers

In this case, the closed-loop system employs the proportional-integral (PI) controller

$$K(s) = k_p + \frac{k_i}{s} \tag{10}$$

in cascade with the nonlinear gain k, where k_p and k_i are the constant positive proportional and integral gains, respectively.

To investigate the absolute stability of the closed-loop system, we group the linear components (1) and (10) as

$$W(s) = G(s)K(s) = \frac{c(k_p s + k_i)}{s(s^2 + as + b)} \tag{11}$$

which is now a third-order transfer-function, and separate out the nonlinear element which is the gain k. To apply the Popov Stability Criterion stated in Section 3, we examine the Popov plot of $W(j\omega)$. This plot reveals the range of values that the nonlinear gain k can assume while retaining closed-loop stability. For this purpose, we need to compute the crossing of the Popov plot of $W(j\omega)$ with the real axis. In this case, from equation (11), we obtain

$$\mathcal{R}eW(j\omega) = \frac{-c[k_p\omega^2 + (ak_i - bk_p)]}{a^2\omega^2 + (b - \omega^2)^2} \tag{12}$$

$$\omega\mathcal{I}mW(j\omega) = \frac{-c[(ak_p - k_i)\omega^2 + bk_i]}{a^2\omega^2 + (b - \omega^2)^2} \tag{13}$$

The Popov plot of $W(j\omega)$ starts at the point $P(\frac{-c(ak_i - bk_p)}{b^2}, \frac{-ck_i}{b})$ for $\omega = 0$ and ends at the point $Q(0, 0)$ for $\omega = \infty$. Two distinct cases are now possible depending on the *relative* values of k_i and k_p.

5.1 Case One: $k_i \leq ak_p$

In this case, $\omega\mathcal{I}mW(j\omega)$ is *always* negative for all ω, that is, the Popov plot of $W(j\omega)$ remains entirely in the third and fourth quadrants and does *not* cross the real axis. This implies that we can construct a straight-line passing through the origin such that the Popov plot is entirely to the right of this line. Therefore, according to the Popov Criterion, the range of the allowable nonlinear gain k is $(0, \infty)$.

5.2 Case Two: $k_i > ak_p$

In this case, the Popov plot of $W(j\omega)$ crosses the real axis. The crossover frequency ω_o is found by solving $\omega\mathcal{I}mW(j\omega) = 0$ to yield

$$\omega_o{}^2 = \frac{bk_i}{k_i - ak_p} \tag{14}$$

The value of $W(j\omega)$ at the crossover is then obtained as

$$\mathcal{R}eW(j\omega_o) = \frac{(ak_p - k_i)c}{ab} \tag{15}$$

Therefore, the *maximum* allowable gain is

$$k_{max} = -\frac{1}{\mathcal{R}eW(j\omega_o)} = \frac{ab}{(k_i - ak_p)c} \tag{16}$$

We can now construct a straight-line passing through the point $-\frac{1}{k_{max}} + j0$ such that the Popov plot of $W(j\omega)$ is entirely to the right of this line. Thus the range of the allowable nonlinear gain k is $(0, k_{max})$.

Observe that the distinction between the above two cases is on the *relative* values of the proportional and integral gains k_p and k_i in the PI controller, and not on their absolute values. Notice that a reasonable estimate of the attenuation factor a of the transfer-function (1) can readily be obtained experimentally from the open-loop response of the output y to the step control input u. Specifically, the step response has the settling time of $t_s = \frac{5}{\xi\omega} = \frac{10}{a}$ to reach within the $\pm1\%$ tolerance band of the final value [1].

6 Stability Analysis of Nonlinear PID Controllers

In this case, we employ the proportional-integral-derivative (PID) controller

$$K(s) = k_p + \frac{k_i}{s} + k_d s \tag{17}$$

in cascade with the nonlinear gain k, where k_p, k_i, and k_d are the constant positive proportional, integral, and derivative gains, respectively.

To investigate the absolute stability of the closed-loop system, we combine the linear components (1) and (17) as

$$W(s) = G(s)K(s) = \frac{c(k_d s^2 + k_p s + k_i)}{s(s^2 + as + b)} \tag{18}$$

which is a third-order transfer-function, and separate out the nonlinear element which is the gain k. In order to assess the stability of the closed-loop system, we examine the Popov plot of $W(j\omega)$. This plot reveals the range of values that the nonlinear gain k can assume while retaining closed-loop stability. In this case, from equation (18), we obtain

$$\mathcal{R}eW(j\omega) = \frac{-c[(k_p - ak_d)\omega^2 + (ak_i - bk_p)]}{a^2\omega^2 + (b-\omega^2)^2} \tag{19}$$

$$\omega\mathcal{I}mW(j\omega) = \frac{-c[k_d\omega^4 + (ak_p - bk_d - k_i)\omega^2 + bk_i]}{a^2\omega^2 + (b-\omega^2)^2} \tag{20}$$

The Popov plot of $W(j\omega)$ starts at the point $P(\frac{-c(ak_i-bk_p)}{b^2}, \frac{ck_i}{b})$ for $\omega = 0$ and ends at the point $Q(0, -ck_d)$ for $\omega = \infty$. To apply the Popov Criterion, we need to compute the crossing of the Popov plot of $W(j\omega)$ with the real axis. From equation (20), it is clear that when $(ak_p - bk_d - k_i) \geq 0$ or

$$(bk_d + k_i) \leq ak_p \tag{21}$$

then $\omega\mathcal{I}mW(j\omega)$ is negative *for all* ω, thus the Popov plot does not cross the real axis. In this case, the range of the nonlinear gain k for stability is $(0, \infty)$. Hence equation (21) gives a *sufficient*, but not a necessary, condition for closed-loop stability for all values of k.

When $(bk_d + k_i) > ak_p$, the closed-loop system may become unstable for some values of k. These values of k correspond to the cases where the Popov plot crosses the real axis, that is, $\omega\mathcal{I}mW(j\omega) = 0$. In the Appendix, the conditions under which this equation has real positive roots are found. Two distinct cases are possible depending on the *relative* values of k_p, k_i, and k_d.

6.1 Case One: $\sqrt{ak_p} > |\sqrt{bk_d} - \sqrt{k_i}|$

In this case, equation (20) cannot have positive real roots for ω. Hence the Popov plot of $W(j\omega)$ does not cross the real axis and stays entirely in the third and fourth quadrants. Therefore, according to the Popov Criterion, the range of the allowable nonlinear gain k is $(0, \infty)$.

6.2 Case Two: $\sqrt{ak_p} \leq |\sqrt{bk_d} - \sqrt{k_i}|$

In this case, the Popov plot of $W(j\omega)$ crosses the real axis. Equation (20) now has two real positive roots ω_1 and ω_2, which are the two crossover frequencies. These frequencies are the roots of the following equation:

$$k_d\omega^4 + (ak_p - bk_d - k_i)\omega^2 + bk_i = 0 \tag{22}$$

The value of $W(j\omega)$ at the crossover is then found from equation (19) as $\mathcal{R}eW(j\omega_i)$ for i=1, 2. When $\mathcal{R}eW(j\omega_i)$ is positive or zero, the range of the allowable nonlinear gain k is $(0, \infty)$. When $\mathcal{R}eW(j\omega_i) < 0$, the range of k is $(0, k_{max})$, where $k_{max} = min[\frac{-1}{\mathcal{R}eW(j\omega_1)}, \frac{-1}{\mathcal{R}eW(j\omega_2)}]$.

Observe that the effect of the derivative gain k_d is to increase the range of the integral gain k_i for stability. Notice that when $k_d = 0$, the results of Section 5 for PI controllers are obtained.

7 Illustrative Example

For the sake of illustration, computer simulations of the Popov plot for a position-controlled arm with a nonlinear PI force controller are presented. Given

$$G(s) = \frac{25}{s^2 + 10s + 25} \quad ; \quad K(s) = k_p + \frac{2}{s}$$

the Popov plots of $W(s) = G(s)K(s)$ for the two values of $k_p = 2$ and $k_p = 0$ are shown in Figures 3a–3b. For $k_p = 2$, it is seen from Figure 3a that the Popov plot of $W(j\omega)$ does not cross the real axis as expected; hence the allowable range of the nonlinear gain k is $(0, \infty)$. In contrast, when k_p is reduced to zero, Figure 3b reveals that the Popov plot of $W(j\omega)$ crosses the real axis at -0.2, hence the allowable range of k is now reduced to $(0, 5)$.

We conclude that reducing k_p has a destabilizing effect and decreases the range of the allowable nonlinear gain k to maintain closed-loop stability.

8 Conclusions

It is widely believed that a "perfect" control system must exhibit a fast response with no overshoot. These two requirements are contradictory when linear controllers are used, and are often impossible to achieve when the system operating conditions undergo gross variations. A fast response requires a large gain which, in turn, gives rise to a large overshoot, manifesting the contradiction of the two requirements. This paper proposes a simple solution to this fundamental problem by enhancing a fixed-gain PID controller with a nonlinear gain k. The nonlinear characteristics of the gain enables the achievement of fast initial response when k is large, followed by a small overshoot when k is small. Thus the nonlinear PID controller does not suffer from the disadvantage of large overshoots which often accompany a fast response. This automatic adjustment of the gain is the main advantage of the nonlinear PID controller over the conventional linear PID controller.

Current research is aimed at the implementation and practical validation of the proposed nonlinear PID control schemes in robotic compliance and force control applications.

9 Acknowledgment

The research described in this paper was carried out at the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration.

10 Appendix

Consider the quadratic equation

$$AX^2 + BX + C = 0 \tag{23}$$

where the coefficients A, B, and C are constants, and A and C are known to be positive. We wish to find the conditions on A, B, and C under which equation (23) will have two real positive roots. Clearly, if the coefficient B is zero or positive, for any positive number X, the expression $(AX^2 + BX + C)$ is positive. Hence, a *necessary*, but not a sufficient, condition for having a positive root is $B < 0$. Now, since the product of roots of equation (23) is positive, equation (23) can only have either two real positive or two real negative roots. Let $\Delta = B^2 - 4AC$ be the discriminant. Then the conditions for existence of two real positive roots are:

$$\Delta \geq 0 \quad ; \quad B < 0 \tag{24}$$

The first condition yields

$$(B - 2\sqrt{AC})(B + 2\sqrt{AC}) \geq 0 \tag{25}$$

Since $(B - 2\sqrt{AC})$ is negative in view of equation (24), the required condition becomes

$$B \leq -2\sqrt{AC} \tag{26}$$

Therefore, when $B > -2\sqrt{AC}$, equation (23) will *not* have two real roots.

To apply this result to the nonlinear PID controller in Section 6, we substitute: $A = k_d$, $B = ak_p - bk_d - k_i$, $C = bk_i$. This yields

$$(ak_p - bk_d - k_i) \leq -2\sqrt{bk_d k_i}$$

Hence, the required condition for the Popov plot to cross the real axis is found to be

$$ak_p \leq (\sqrt{bk_d} - \sqrt{k_i})^2$$

This equation yields the condition for real axis crossing as

$$\sqrt{ak_p} \leq |\sqrt{bk_d} - \sqrt{k_i}| \tag{27}$$

Therefore, when $\sqrt{ak_p} > |\sqrt{bk_d} - \sqrt{k_i}|$, the Popov plot does *not* cross the real axis.

11 References

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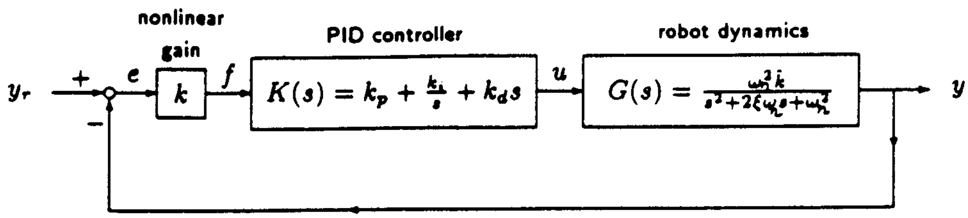
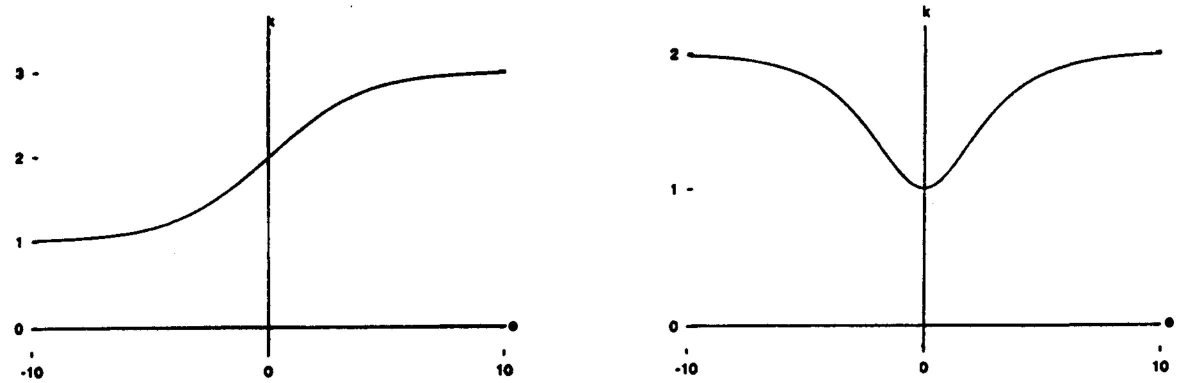


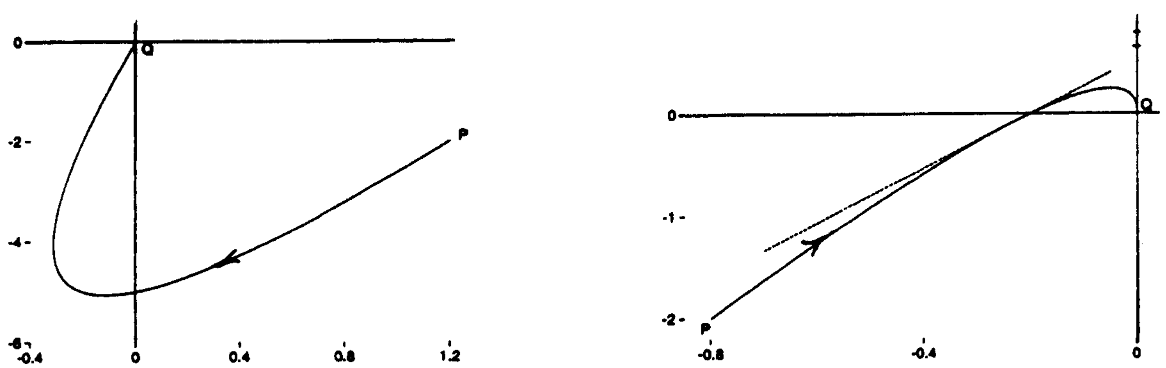
Figure 1: Block diagram of nonlinear PID control system



(a) $k \; = \; 1 + \frac{2}{1 + exp(-0.5 \; e)}$

(b) $k \; = \; 2 - \frac{2}{exp(0.5 \; e) + exp(-0.5 \; e)}$

Figure 2: Two proposed nonlinear gains



(a) $k_p \; = \; 2$

(b) $k_p \; = \; 0$

Figure 3: Popov plots