

A New Model for Retrieving Slant TEC Corrections for Wide Area Differential GPS

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BIOGRAPHY

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ABSTRACT

To provide delay corrections to single frequency users, wide area differential GPS systems depend upon accurate determination of ionospheric total electron content (TEC). This paper describes a new model for retrieving ionospheric calibrations from sets of GPS measurements. A distinctive feature of this approach is the simultaneous retrieval of multiple parameters from the GPS data, representing various integrated quantities in addition to vertical TEC. Instead of converting each slant TEC measurement to a single associated vertical TEC value, the simplest version of this model treats each slant TEC measurement as a linear combination of two-parameters, *i.e.*, integrals of the electron density along raypaths at two or more fixed, fiducial elevation angles. A four-parameter version of the model includes linear terms that correct for horizontal gradients along the raypath of each measurement. Analysis and initial retrieval results indicate that even the two-parameter model produces slant TEC calibrations that are significantly more accurate than those generated using other current models, especially for measurements at low elevation angle, where errors from other approaches tend to be largest.

INTRODUCTION

The Global Positioning System (GPS) can be used to measure the integrated electron density along raypaths between satellites and receivers (Lanyi and Roth, 1988). Maps of the distribution of *vertical* total electron content (TEC) have been generated by analyzing GPS data collected from global and regional networks of receivers (Mannucci *et al.*, 1999). This paper describes a new model, or, more precisely, a new family of models, for obtaining slant TEC corrections for wide area differential GPS systems.

Current slant TEC models typically associate a given slant GPS measurement with the vertical TEC value at the

sublatitude and sublongitude where the raypath intersects a reference ionospheric height (henceforth designated the *ionospheric point*). The slant-to-vertical conversion (*i.e.*, the *mapping function*) is treated as a known function of elevation angle and other model variables. Two types of error restrict the accuracy of such models: (1) error associated with the model mapping function, and (2) error arising from the neglect of horizontal gradients of the electron density along the raypath. A mapping function specifies a predetermined form for the unknown height variation of the electron density profile. For example, the well-known *thin-shell* model relies on the rather crude assumption that the electron density is non-negligible only in the vicinity of the ionospheric reference height. Neglecting horizontal density gradients causes distinct measurements that share a common ionospheric point to produce inconsistent estimates of the same vertical TEC value.

The fundamental postulate of the new model is that, for each measurement, the horizontal variation of the electron density at any altitude can be expressed as a power series expansion centered about the angle formed by three points: the receiver, the center of the earth, and the ionospheric point (typically at an altitude of 450 km). A unique feature of this approach is the simultaneous retrieval of multiple parameters from the GPS data, representing various integrated quantities in addition to vertical TEC. Rather than converting each slant TEC measurement to a single associated vertical TEC value, the simplest version of the model treats each slant TEC measurement as a linear combination of integrals of the electron density along raypaths at two or more fixed, fiducial elevation angles. A four-parameter version includes terms that account for electron density gradients along the raypath.

Analysis and initial retrieval results reveal that even a two-parameter model (*e.g.*, vertical TEC and slant TEC at an elevation angle of 20 degrees) produces more accurate slant TEC calibrations than current models. The improvement is most dramatic for measurements at low elevation angle, where errors from other approaches tend to be largest. However, the two-parameter model doubles the amount of ionospheric correction data that must be transmitted to GPS users. Since the bandwidth for transmitting corrections is limited, it is important to determine the extent to which the magnitude of these additional corrections justifies the increased cost.

The paper is organized in sections as follows: (1) a derivation of the general formalism of the approach; (2) a description of how the dependence of slant TEC on raypath geometry is approximated; (3) application of the slant TEC model (a) in the absence of horizontal gradients

along the raypath and (b) in the presence of linear horizontal gradients along the raypath; (4) a proposal for modeling the azimuthal dependence of observations in parameter estimation; (5) a discussion of the achieved model accuracy; (6) a comparison of initial retrieval results based upon a two-parameter model with similar results obtained using a thin-shell model; and (7) concluding remarks.

GENERAL FORMULATION

An individual slant TEC measurement can be modeled as an integral of the electron density over the raypath:

$$T_{rs} = \int_{\ell_r}^{\ell_s} d\ell n_e(h(\ell), \theta(\ell), \phi(\ell)), \quad (1)$$

where $n_e(h, \theta, \phi)$ is the electron density as a function of h , the height above the earth, θ , the geocentric latitude ($-\pi/2 \leq \theta \leq \pi/2$), and ϕ , the geocentric longitude ($-\pi < \phi \leq \pi$). The limits of integration are ℓ_r and ℓ_s , where the subscripts denote the receiver and the satellite, respectively. This expression may be transformed into an integral over height:

$$T_{rs} = \int_{h_r}^{h_s} dh M(\alpha, h) n_e(h, \theta(\alpha, h), \phi(\alpha, h)), \quad (2)$$

where $M(\alpha, h)$ is $d\ell/dh$ as a function of height h and elevation angle α (see Fig. 1).

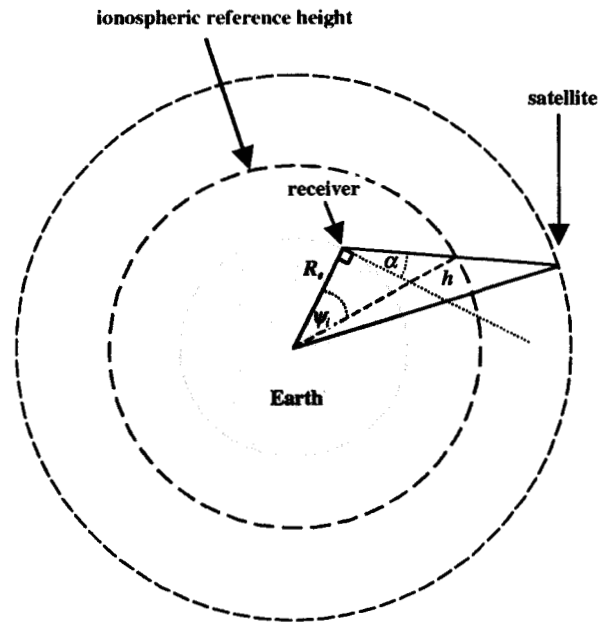


Figure 1. Raypath geometry.

The fundamental assumption of the new slant TEC model is that the horizontal variation of $n_e(h, \theta, \phi)$ at any height along the raypath (*i.e.*, its dependence on θ and ϕ) can be approximated by a power series expansion about the sublatitude and sublongitude of the point where the raypath crosses the mapping reference height h_i (*i.e.*, the ionospheric point). It proves useful to consider the problem in two dimensions before discussing the full three-dimensional problem.

Two-dimensional geometry

Let us first restrict our discussion to the plane of observation, and let $\psi(\alpha, h)$ be the polar angle along the raypath in the plane of observation (see Fig. 1). If refraction is neglected, the raypath and, hence, $\psi(\alpha, h)$ and $M(\alpha, h)$ may be determined from purely geometrical considerations:

$$\psi(\alpha, h) = \frac{\pi}{2} - \alpha - \sin^{-1} \left(\frac{R_e \cos \alpha}{R_e + h} \right), \quad (3)$$

$$M(\alpha, h) = \left[1 - \left(\frac{R_e \cos \alpha}{R_e + h} \right)^2 \right]^{-1/2}. \quad (4)$$

where R_e is the earth radius (evaluated herein as 6370 km).

In the plane of observation, we assume that the spatial dependence of the electron density is of the form:

$$n_e(h, \psi) \approx \sum_{k=0}^{K-1} a_k(h, \psi_i) (\psi - \psi_i)^k, \quad (5)$$

where $\psi_i \equiv \psi(\alpha, h_i)$. In other words, we assume that eq. (5) is valid over the height-dependent domain

$$\psi_i - \delta\psi(\alpha, h) < \psi < \psi_i + \delta\psi(\alpha, h), \quad (6)$$

where $\delta\psi(\alpha, h) \equiv |\psi(\alpha, h) - \psi_i|$. Notice that $\delta\psi(\alpha, h)$ is small in the immediate neighborhood of h_i and larger away from h_i . Thus, h_i should be chosen to be near the height where $n_e(h, \psi)$ varies most rapidly with ψ .

Substituting eq. (5) into eq. (2) gives

$$T_{rs} \approx \sum_{k=0}^{K-1} \int_{h_r}^{h_s} dh a_k(h, \psi_i) G_k(\alpha, h), \quad (7)$$

where each

$$G_k(\alpha, h) \equiv M(\alpha, h) (\psi(\alpha, h) - \psi_i)^k \quad (8)$$

is a known function that depends only upon raypath geometry. Now approximate each $G_k(\alpha, h)$ by $g_k(\alpha, h)$, a summation that separates the dependence on h from the dependence on α :

$$g_k(\alpha, h) \equiv \sum_{m=0}^{M-1} d_{km}(\alpha) q_{km}(h). \quad (9)$$

The purpose of an approximation of this form is to allow us to write eq. (7) in terms of integrals that do not contain an explicit dependence upon the elevation angle:

$$T_{rs} \approx t_{rs} \equiv \sum_{k=0}^{K-1} \sum_{m=0}^{M-1} d_{km}(\alpha) A_{km}(\psi_i), \quad (10)$$

where

$$A_{km}(\psi) \equiv \int_{h_r}^{h_s} dh a_k(h, \psi) q_{km}(h). \quad (11)$$

Note that ψ appears as an independent coordinate in eq. (11): this integral is not over a raypath. Formally, the $A_{km}(\psi)$ are distributions to be retrieved from sets of observations that lie within a single plane of observation. In practice sets of observations rarely reside within a single plane of observation; this necessitates re-examining the problem in three-dimensions.

Three-dimensional geometry

Assuming $K > 1$, the analogue of eq. (5) in spherical geometry is

$$n_e(h, \theta, \phi) \approx n_e(h, \theta_i, \phi_i) + \sum_{k=1}^{K-1} \sum_{j=1}^k a_{kj}(h, \theta_i, \phi_i) (\theta - \theta_i)^j (\phi - \phi_i)^{k-j}, \quad (12)$$

where θ_i and ϕ_i are the geocentric sublatitude and sublongitude of the ionospheric point, respectively, and the spherical coordinates (r, θ, ϕ) are defined with respect to solar-magnetic coordinate axes. [The structure of the ionosphere is relatively stationary in the solar-magnetic reference frame (see Mannucci *et al.*, 1998).] Let us define β_i to be the azimuthal angle of the raypath at the ionospheric point (measured by rotating eastward from due solar-magnetic north). It will prove convenient to rotate the coordinate system, defining new angular coordinates $\theta' \equiv \theta'(\theta_i, \phi_i, \beta_i)$ and $\phi' \equiv \phi'(\theta_i, \phi_i, \beta_i)$, such that the plane of observation coincides with the plane defined by $\phi' = 0$. In this new coordinate system, the

representation of electron density, analogous to eq. (12), is

$$n_e(h, \theta', \phi') \approx n_e(h, \theta_i, \phi_i) + \sum_{k=1}^{K-1} \sum_{j=1}^k a_{kj}'(h, \theta_i, \phi_i, \beta_i) \theta'^j \phi'^{k-j}, \quad (13)$$

where, in general, $a_{kj}'(h, \theta_i, \phi_i, \beta_i) \neq a_{kj}(h, \theta_i, \phi_i)$. Along the raypath, $\phi' = 0$ by design, and, hence, eq. (13) reduces to

$$n_e(h, \theta', 0) \approx n_e(h, \theta_i, \phi_i) + \sum_{k=1}^{K-1} a_{kk}'(h, \theta_i, \phi_i, \beta_i) \theta'^k \quad (14)$$

which is equivalent to eq. (5) with $\psi - \psi_i \equiv \theta'$. Thus, the results from the two-dimensional case immediately become applicable. Equation (10) generalizes to:

$$T_{rs} \approx t_{rs} \equiv \sum_{k=0}^{K-1} \sum_{m=0}^{M-1} d_{km}(\alpha) A_{km}(\theta_i, \phi_i, \beta_i), \quad (15)$$

where

$$A_{km}(\theta, \phi, \beta) \equiv \int_{h_r}^{h_s} dh a_{kk}'(h, \theta, \phi, \beta) q_{km}(h). \quad (16)$$

and $a_{00}'(h, \theta, \phi, \beta) \equiv n_e(h, \theta, \phi)$.

Retrieval of $A_{km}(\theta, \phi, \beta)$

The distributions $A_{km}(\theta, \phi, \beta)$ may be retrieved from sets of slant TEC measurements. Formally, this is accomplished by specifying a set of basis functions such that

$$A_{km}(\theta, \phi, \beta) \equiv \sum_{n=0}^{N-1} c_{kmn} b_n(\theta, \phi, \beta). \quad (17)$$

Substituting this expression into eq. (15) gives

$$t_{rs} = \sum_{k=0}^{K-1} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} d_{km}(\alpha) c_{kmn} b_n(\theta_i, \phi_i, \beta_i). \quad (18)$$

This equation can be used to model *any* observation, not just those in one plane of observation. The coefficients c_{kmn} may be retrieved using, for example, a Kalman filter. In practice, the right-hand side of eq. (18) is augmented by two terms, representing receiver and satellite hardware biases, respectively, that can be retrieved simultaneously with the coefficients c_{kmn} (see Mannucci *et al.*, 1998).

Prior to implementing an algorithm based upon this formal solution, one needs to make two decisions in

addition to defining a set of basis functions: one must decide how many terms to retain in eq. (13), and one must decide how to define the geometric terms $g_k(\alpha, h)$. We will first address the latter of these questions and then consider two cases: (1) neglecting horizontal gradients along the raypath and (2) assuming that these gradients are linear with respect to changes in ψ .

GEOMETRIC APPROXIMATION

Our approach to approximating the geometric terms $G_k(\alpha, h)$ will be to set $g_k(\alpha, h)$ equal to linear combinations of $G_k(\alpha, h)$ evaluated at fiducial elevation angles. The advantage of this approach is that the geometric error associated with modeling a given measurement can be made arbitrarily small by using a sufficient number of such terms, and this error will vanish whenever the measurement elevation angle matches one of the fiducial angles. To be more precise, we set

$$q_{km}(h) \equiv G_k(\alpha_{km}, h) \quad (19)$$

for specific elevation angles α_{km} , where m varies from 0 to $M-1$. The coefficients $d_{km}(\alpha)$ are determined by requiring

$$\sum_{m=0}^{M-1} d_{km}(\alpha) = 1 \quad (20)$$

and

$$g_k(\alpha, h_{km}) = G_k(\alpha, h_{km}) \quad (21)$$

for $M-1$ fiducial values of h . Thus, as an approximation for $G_k(\alpha, h)$, $g_k(\alpha, h)$ will be exact not only at all altitudes for measurements at the fiducial elevation angles, it will also be exact at the fiducial values of h for any arbitrary elevation angle.

When defining a two-term ($M = 2$) approximation for $g_k(\alpha, h)$, we must specify two fiducial angles, α_{k0} and α_{k1} , and one fiducial height h_{k0} . The above prescription then gives

$$g_k(\alpha, h) \equiv d_{k0}(\alpha) G_k(\alpha_{k0}, h) + d_{k1}(\alpha) G_k(\alpha_{k1}, h) \quad (22)$$

where

$$d_{k0}(\alpha) \equiv \frac{G_k(\alpha, h_{k0}) - G_k(\alpha_{k1}, h_{k0})}{G_k(\alpha_{k0}, h_{k0}) - G_k(\alpha_{k1}, h_{k0})} \quad (23)$$

$$d_{k1}(\alpha) \equiv 1 - d_{k0}(\alpha). \quad (24)$$

In general the fiducial height h_{k0} must not equal the ionospheric reference height h_i , since, for $k > 0$, this causes $G_0(\alpha, h_{k0})$ to vanish identically, independent of α . For $k = 0$, however, h_{k0} may be set equal to h_i , since $G_0(\alpha, h)$ is equal to $M(\alpha, h)$ and is thus independent of h_i . The analogous three-term approximation ($M = 3$) is discussed elsewhere (Sparks *et al.*, 2000).

SLANT TEC MODEL NEGLECTING HORIZONTAL GRADIENTS ALONG RAYPATH

The simplest model for slant TEC is to assume that horizontal gradients of the electron density along the raypath may be neglected over the domain in question (as defined, for by eq. (6)). This is equivalent to setting $K = 1$ in eq.(7):

$$T_{rs} = \int_{h_i}^{h_s} dh M(\alpha, h) n_e(h, \theta_i, \phi_i). \quad (25)$$

Note that if $M(\alpha, h)$ is approximated by a constant, namely, $M(\alpha, h_{shell})$ for some representative ionospheric height h_{shell} , then eq. (25) becomes

$$T_{rs} \approx t_{rs} \equiv M(\alpha, h_{shell}) \int_{h_i}^{h_s} dh n_e(h, \theta_i, \phi_i). \quad (26)$$

This is equivalent to assuming $n_e(h, \theta, \phi)$ is non-zero only

in a small neighborhood of h_{shell} and is designated the *thin-shell* model for slant TEC (see Mannucci *et al.*, 1999). $M(\alpha, h_{shell})$ is the thin-shell obliquity factor that relates the slant TEC value to the vertical TEC value. Formally, the thin-shell model may be considered a special case of eq. (10) where $K = M = d_{km}(\alpha) = 1$ and $q_{00}(\alpha) = M(\alpha, h_{shell})$.

By plotting $M(\alpha, h_{shell}) / M(\alpha, h)$ as a function of h , the local geometric error may be represented as a deviation from unity (see Fig. 2). When evaluating the total error in the modeled slant TEC, the geometric error is weighted by the electron density. Since the contributions to the total error from $h > h_{shell}$ and $h < h_{shell}$ are opposite in sign, a judicious choice of shell height can result in a small net error for a given raypath. As will be seen, however, the shell height that achieves this cancellation of error is highly sensitive to changes in the vertical variation of the electron density.

The scheme described in the previous section generally provides a more accurate approach for approximating $M(\alpha, h)$. First consider a two-term approximation $m(\alpha, h) \equiv g_0(\alpha, h)$ as defined in eq. (22) [recall that $G_0(\alpha, h) = M(\alpha, h)$]. Equation (15) now becomes

$$t_{rs} = d_{00}(\alpha) \int_{h_i}^{h_s} dh n_e(h, \theta_i, \phi_i) M(\alpha_{00}, h) + d_{01}(\alpha) \int_{h_i}^{h_s} dh n_e(h, \theta_i, \phi_i) M(\alpha_{01}, h). \quad (27)$$

By selecting $\alpha_{00} = 90^\circ$ (so that, for all h , $M(\alpha_{00}, h) = 1$), the first integral corresponds to the vertical TEC evaluated at (θ_i, ϕ_i) . The problem now is to retrieve two distributions: the distribution of vertical TEC and the distribution of slant TEC evaluated at elevation angle α_{01} . The latter distribution may be defined without reference to measurement azimuthal angles since here we are here assuming that horizontal gradients may be neglected.

Figure 2 also displays the ratio of the two-parameter approximation of $M(\alpha, h)$ to its analytic value as a function of h for various values of α , assuming $h_{00} = 350$ km, $\alpha_{00} = 90^\circ$, and $\alpha_{01} = 20^\circ$. The value chosen for h_{00} sets this ratio to unity in the region where the electron density profile attains its maximum. Note that, for $h < h_{00}$, the ratio is always greater than unity when $\alpha > \alpha_{01}$ and less than unity when $\alpha < \alpha_{01}$; the opposite situation holds when $h > h_{00}$. As α_{01} is increased, the magnitude of the deviation from unity decreases for $\alpha > \alpha_{01}$ and increases for $\alpha < \alpha_{01}$. We may conclude that α_{01} should be chosen no smaller than the smallest elevation angle of the measurements that we wish to include in the retrieval. For example, if we are going to exclude all data with an

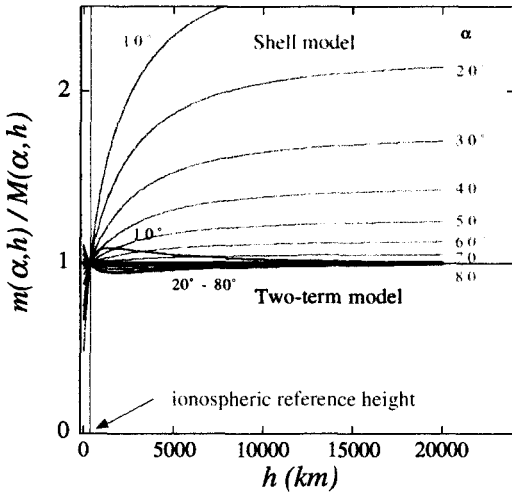


Figure 2. Ratio of the approximation of $M(\alpha, h)$ to its analytic value as a function of height for various elevation angles α , with the ionospheric reference height equal to 350 km: (a) the thin shell model (red lines) and (b) the 2-parameter model (green lines) with fiducial angles $\alpha_{00} = 90^\circ$ and $\alpha_{01} = 20^\circ$.

elevation angle less than 30° , choosing $\alpha_{01} = 30^\circ$ will give, for all h , a maximum error of $< 4\%$.

SLANT TEC MODEL RETAINING LINEAR HORIZONTAL GRADIENTS ALONG RAYPATH

Let us now consider a slant TEC model that incorporates linear horizontal gradients along the raypath. Setting $K = 2$ in eq. (15) we have

$$T_{rs} = \int_h^{h_e} dh M(\alpha, h) n_e(h, \theta_i, \phi_i) + \int_h^{h_e} dh G_1(\alpha, h) a_{11}'(h, \theta_i, \phi_i, \beta_i) \quad (28)$$

The first integral is the zeroth-order term treated in the previous section. The second integral is the first-order correction due to horizontal gradients along the raypath. Assuming a two-term approximation for the geometric term in each integral gives

$$t_{rs} = d_{00}(\alpha) \int_h^{h_e} dh n_e(h, \theta_i, \phi_i) M(\alpha_{00}, h) + d_{01}(\alpha) \int_h^{h_e} dh n_e(h, \theta_i, \phi_i) M(\alpha_{01}, h) + d_{10}(\alpha) \int_h^{h_e} dh a_{11}'(h, \theta_i, \phi_i, \beta_i) G_1(\alpha_{10}, h) + d_{11}(\alpha) \int_h^{h_e} dh a_{11}'(h, \theta_i, \phi_i, \beta_i) G_1(\alpha_{11}, h) \quad (29)$$

Choosing α_{00} and α_{10} both to be 90° causes $M(\alpha_{00}, h)$ to become unity and $G_1(\alpha_{10}, h)$ (and, hence, the third integral) to vanish identically. Thus, t_{rs} reduces to

$$t_{rs} = d_{00}(\alpha) \int_h^{h_e} dh n_e(h, \theta_i, \phi_i) + d_{01}(\alpha) \int_h^{h_e} dh n_e(h, \theta_i, \phi_i) M(\alpha_{01}, h) + d_{11}(\alpha) \int_h^{h_e} dh a_{11}'(h, \theta_i, \phi_i, \beta_i) G_1(\alpha_{11}, h) \quad (30)$$

where the final term represents a first order correction to eq. (27) due to horizontal gradients in the plane of observation.

AZIMUTHAL DEPENDENCE

Parameter estimation requires decomposing the distributions $A_{km}(\theta, \phi, \beta)$ in terms of a set of basis functions. Using eq. (11), .Eq. (30) may be written

$$t_{rs} = d_{00}(\alpha) A_{00}(\theta_i, \phi_i) + d_{01}(\alpha) A_{01}(\theta_i, \phi_i) + d_{11}(\alpha) A_{11}(\theta_i, \phi_i, \beta_i) \quad (31)$$

When the correction due to horizontal gradients (the final term above) is neglected, the decomposition of the $A_{km}(\theta, \phi, \beta)$ represented in eq. (17) may proceed in terms of basis functions $b_n(\theta, \phi)$ that are independent of β . The simplest means of incorporating the correction term into this decomposition is to assume that

$$A_{11}(\theta_i, \phi_i, \beta_i) \approx A_{11}(\theta_i, \phi_i, 0) \cos(\beta_i) + A_{11}\left(\theta_i, \phi_i, \frac{\pi}{2}\right) \sin(\beta_i) \quad (32)$$

This approximation will obviously be exact at $\beta_i = 0$ and $\beta_i = \pi/2$. It will also be exact at $\beta_i = \pi$ and $\beta_i = 3\pi/2$ by virtue of the fact that linearity in the plane of observation ensures that $A_{11}(\theta, \phi, \beta_i) = -A_{11}(\theta, \phi, \beta_i + \pi)$. At intermediate values of β_i , eq. (32) may be considered an interpolation. Substituting eq. (32) into eq. (31) gives

$$t_{rs} = d_{00}(\alpha) A_{00}(\theta_i, \phi_i) + d_{01}(\alpha) A_{01}(\theta_i, \phi_i) + d_{11}(\alpha) \cos(\beta_i) A_{11}(\theta_i, \phi_i, 0) + d_{11}(\alpha) \sin(\beta_i) A_{11}\left(\theta_i, \phi_i, \frac{\pi}{2}\right) \quad (33)$$

In this case, there are four distributions to retrieve: the first two have been discussed previously, namely, the distribution of vertical TEC and the distribution of slant TEC evaluated at elevation angle α_{01} (neglecting horizontal gradients); the latter two represent corrections to slant TEC due, respectively, to north-south and east-west gradients along the raypath.

ACCURACY

To assess the improvement in accuracy over standard models, one can begin by examining how accurately the geometric factor $M(\alpha, h)$ is modeled. Figure 2, for example, reveals that the new model is more accurate, especially at low elevation angle and at altitudes greater than 1000 km. Even when horizontal electron density gradients are ignored, adopting the new model should lead to significant improvement in vertical TEC map accuracy, principally for two reasons: (1) the distribution of slant TEC measurements used to generate a TEC map is typically dominated by measurements at low elevation angle where previous models are least accurate; and (2) the contribution of TEC above 1000 km to the total TEC can be non-negligible. At nighttime, over half the TEC may be contributed from altitudes greater than 1000 km. By overestimating $M(\alpha, h)$ at large altitudes, the thin shell model (and similar models) can be expected to underestimate the vertical TEC. This is a possible explanation for the presence of negative TEC values that

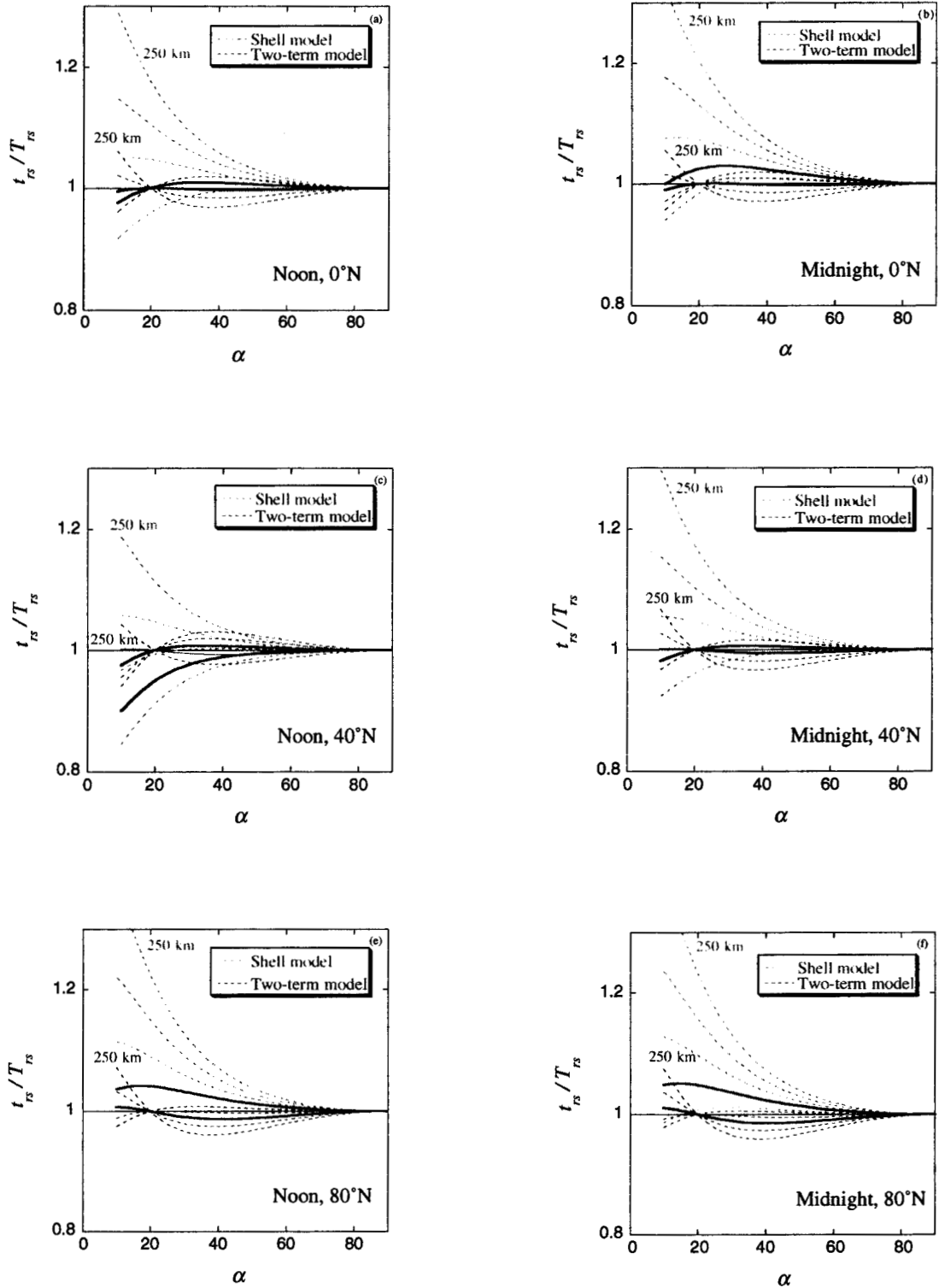


Figure 3. Ratio of approximate slant TEC to analytic slant TEC, using representative profiles defined by eq. (34), for thin shell model (red lines) and two-parameter model (purple lines) with longitude = 0°, the ionospheric reference height = [250, 350, 450, 550, 650] km and fiducial angles = 90° and 20°: (a) latitude = 0°, noon; (b) latitude = 0°, midnight; (c) latitude = 40°N, noon; (d) latitude = 40°N, midnight; (e) latitude = 80°N, noon; (f) latitude = 80°N, midnight.

are sometimes retrieved at nighttime.

As a first step in determining how sensitive the accuracy of the modeled slant TEC is to the geometric error, we have evaluated eqs. (1) and (25) for representative electron density profiles, assuming local spherical symmetry and various approximations for $M(\alpha, h)$. The representative profiles are defined by using the IRI95 (Bilitza, 1993) model for the lower ionosphere and the Gallagher model (Gallagher *et al.*, 1988) for the plasmasphere as follows:

$$n_e(h) \equiv \begin{cases} \text{IRI95}(h) & h < 1000 \\ \text{mixed}(h) & 1000 < h < 2000 \\ \text{Gall}(h) & h > 2000 \end{cases} \quad (34)$$

where $\text{mixed}(h) \equiv (h-1000) \text{Gall}(h) + (2000-h) \text{IRI95}(h)$ and height h is in kilometers. We have evaluated six profiles for June 7, 1999: longitude = 0°, latitude = 0°, 40°N, and 80°N at noon and midnight. Figure 3 displays the ratio of the approximated slant TEC to its analytic value for two models: (1) the thin shell model with a shell height ranging from 250 to 650 km; and (2) the two-term approximation given by eq. (27) with h_{00} also ranging from 250 to 650 km, $\alpha_{00} = 90^\circ$, and $\alpha_{01} = 20^\circ$.

Note that the results for the thin-shell model are much more sensitive to changes in the electron density profile than are the results for the two-term model. A shell height of 550 km tends to be the best choice overall, a result which has been confirmed by prior experience producing global ionospheric maps using different shell heights (the current version of GIM uses an extended slab model that provides an approximation of $M(\alpha, h)$ which closely matches that of the thin-shell model with a shell height of 550 km). Even this optimal choice of shell height, however, can produce errors of up to 10% (see Fig. 3b). In contrast, the error for a two-term model with a shell-height of 450 km never exceeds 3%. Furthermore, since the two-term model is designed so that the error will be small in the neighborhood of the fiducial angles (here 90° and 20°), it can guarantee superior accuracy at low elevation angle in the vicinity of the peak of the distribution of measurements.

To evaluate the improvement in accuracy that the four-term model provides relative to the two-term model (as well as other models), one can conduct a similar study using a fully three-dimensional ionospheric model, *i.e.*, to eliminate the assumption of local spherical symmetry along the raypath. By evaluating eqs. (1) and (28) directly, we can produce maps that display the error as a

function of spatial location, azimuth, and elevation. Such a study is beyond the scope of this paper.

In principle, the formalism of the approach allows as many terms to be added to the slant TEC model as are needed to achieve a desired level of accuracy. Since each additional term requires retrieving an additional parameter, however, it is important to determine when the magnitude of additional corrections justifies the increased computational cost. Furthermore, as the information provided by the measurements is spread among an increasing number of retrieved parameters, the condition of the inverse problem eventually deteriorates, and the accuracy of the solution declines accordingly. The optimal model based upon this formalism remains to be determined.

INITIAL RETRIEVAL RESULTS

In this section, we compare post-fit residuals obtained by processing one-day's worth of GPS data (June 3, 1998) from 98 stations distributed globally. First the thin-shell model was used to fit the data, and then the same data were processed using the two-parameter model described above. Figure 4a shows the distribution of slant TEC measurements at one low-latitude site (Kajelein Atoll in the Marshall Islands, 8.7°N) in TECU units where 1 TECU = 10^{16} electrons / meter². Raypath geometry within the ionosphere ensures that the slant TEC values generally increase as the elevation angle decreases. Also displayed in Fig. 4a are the post-fit residuals for the two-parameter model. Note that the residuals are a small fraction of the slant TEC measurements and that they are largest at low elevation angle.

In Fig. 4b we have superimposed the post-fit residuals of the two-parameter model on top of the corresponding residuals generated using the thin-shell model (with shell height = 550 km). Note that not only are the thin-shell residuals larger, they also tend to be asymmetric about zero. Since even the thin-shell model should work well for measurements at high elevation angle, the fact that the high-elevation angle residuals are offset from zero is evidence that the global least-squares solution generated by the Kalman filter is getting pulled away from the correct answer at high-elevation angle due to mismodeling at low-elevation angle. The residuals from the two-parameter model show no such asymmetry about zero. Results from another low elevation site (Guam, 13.6°N) show similar patterns of behavior.

Figure 4d displays typical post-fit residuals at a mid-latitude site (Krugersdorp, South Africa, -25.9°S). Here the thin-shell model performs better: the residuals are

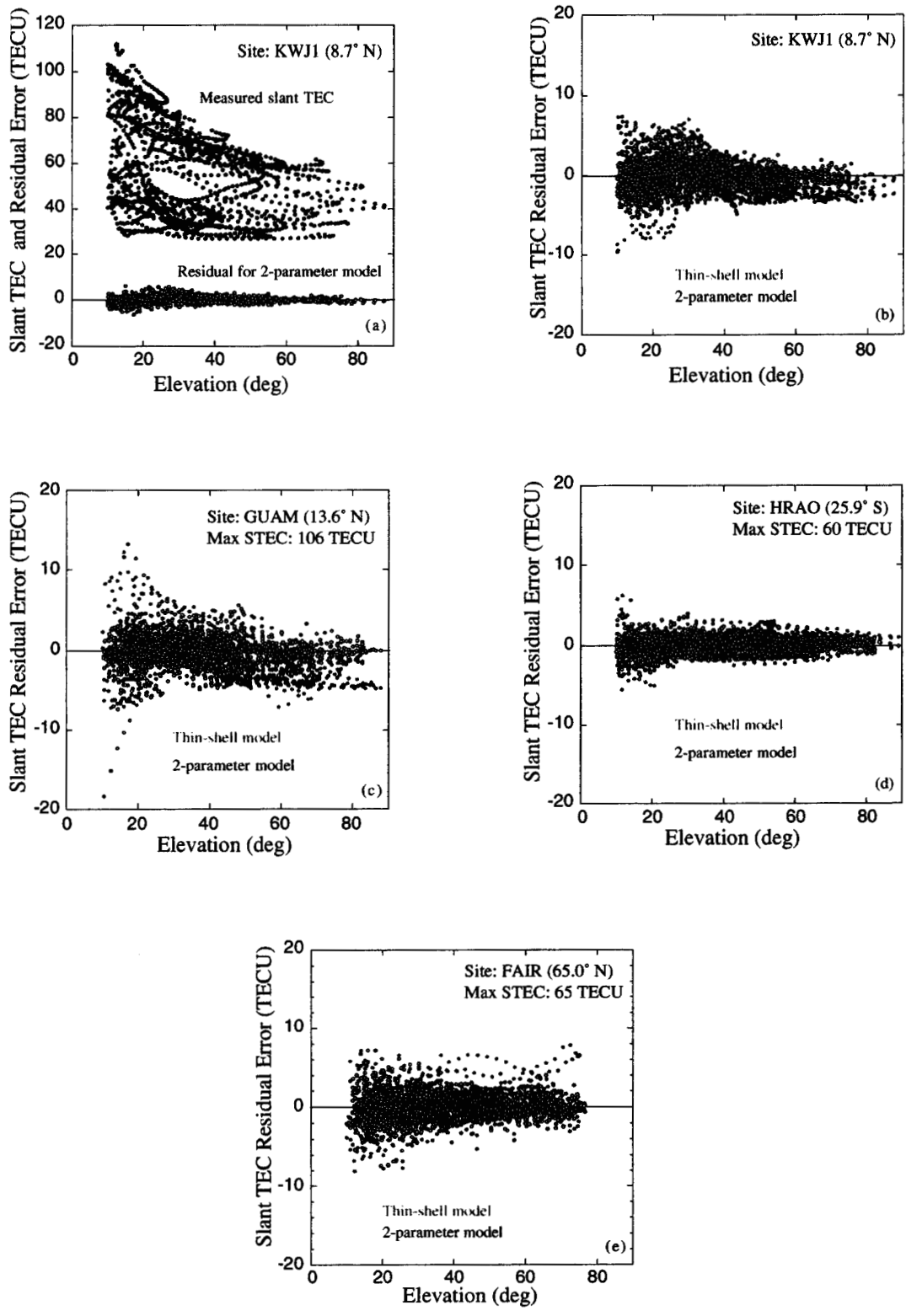


Figure 4. Scatterplots of post-fit residuals obtained by processing GPS data of June 3, 1998, from 98 stations distributed globally, using the thin-shell model (red) and the two-parameter model (blue): (a) measured slant TEC (green) and residuals at Kwajelein Atoll; (b) residuals at Kwajelein Atoll; (c) residuals at Guam; (d) residuals at Krugersdorp; (e) residuals at Fairbanks.

smaller, and no asymmetry is apparent. Nevertheless, the residuals of the two-parameter model are still significantly smaller than those of the thin-shell model. Results for a high-latitude site (Fairbanks, 65.0°N) are displayed in Fig. 4e. Table 1 displays the mean residual and the standard deviation of the residuals for each data set displayed in Fig. 4.

	mean _{TS}	mean _{2P}	σ_{TS}	σ_{2P}
KWJ1	-0.3	-0.002	2.5	1.4
GUAM	-0.6	-0.008	2.7	1.9
HRAO	0.08	0.04	1.2	0.8
FAIR	0.3	-0.04	2.0	1.5

Table 1. The mean residual for the thin-shell model (mean_{TS}) and the two parameter model (mean_{2P}) and the standard deviation of the residuals for the thin-shell model (σ_{TS}) and the two parameter model (σ_{2P}) for receivers at Kwajalein Atoll (KWJ1), Guam (GUAM), Krugersdorp (HRAO), and Fairbanks (FAIR). All values are in TECU units.

CONCLUSION

Current methods for retrieving line-of-sight ionospheric calibrations use models that map slant TEC measurements directly to vertical TEC estimates. Approximations generally include neglecting horizontal gradients along the raypath and assuming a specific vertical electron density profile in the slant-to-vertical conversion. Despite the rather crude nature of these approximations, the ionospheric calibrations retrieved by processing GPS observations have typically been found to be reasonably accurate when compared to independent measurements (Ho *et al.*, 1997).

The model discussed in this paper assumes that, for each measurement, the horizontal variation of the electron density at any altitude can be expressed as a power series expansion centered about the angle formed by three points: the receiver, the center of the earth, and the point where the raypath crosses a reference height. Instead of converting each slant TEC measurement to a single associated vertical TEC value, the simplest version of the model treats each slant TEC measurement as a linear

combination of integrals of the electron density along raypaths at two or more fixed, fiducial elevation angles. More complex versions of this model include terms that correct for electron density gradients along the raypath. Given the level of success already achieved by less accurate models, the improvements represented by this approach suggest that precise ionospheric calibrations for Wide Area Differential GPS can be retrieved whenever data coverage is adequate.

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