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Altimeter Range Processing Analysis for Spacecraft Navigation about Small Bodies

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The impact of using altimeter range measurements as an observation type for navigation of spacecraft about small bodies is evaluated. The altimeter range measurements can supplement or replace the standard optical landmark and Deep Space Network (DSN) radiometric tracking. Navigation of spacecraft orbiting small bodies, like asteroids, can be challenging since the a priori physical characteristics of the central body can have larger than normal uncertainties. The addition of the altimeter range data into the orbit determination problem can be used to alleviate the increased uncertainty in the dynamics of the spacecraft, due to uncertainties in the gravity model of the body. An advantage of the altimeter data is that the measurements can be taken continuously, without the sunlight restrictions of optical landmark tracking, or the station visibility restrictions of DSN tracking. Simulations of various mission scenarios are used as test cases, to quantify the usefulness of the altimeter range data as a navigation tool.

Introduction

Navigation difficulties arise when spacecraft orbit small bodies, like asteroids and comets. The challenges are mostly a consequence of not knowing the dynamics of the situation accurately before the spacecraft begins to orbit the body. More specifically, the values for the total mass, mass distribution, position, orientation, and shape of the target body are not known very well, making accurate determination of the spacecraft's state difficult. Another unique factor in orbiting small bodies is that many small bodies are distinctly non-spherical, which can result in large perturbations to the two-body orbit. In order to obtain the most accurate dynamic force models and orbits, the orbits are processed with all of the available spacecraft tracking. The purpose of this study is to see if the addition of altimeter range data in the orbit determination procedure can improve not only the orbits, but also the estimates of the central body's physical parameters.

The altimeter provides a measure of the distance from the spacecraft to the surface of the body. An advantage of the altimeter data is that the measurements can be taken continuously, without the sunlight restrictions of optical landmark tracking, or the station visibility restrictions of DSN tracking. On the other hand, the altimeter range measurements are weakened by the fact that they are made relative to the surface, which is an unknown height above the body's center of mass. Therefore, the shape model usually needs to be estimated simultaneously with the spacecraft's orbit, in order to prevent errors in the surface model from being aliased into the orbits. The temporal and geographical density of the observations make it feasible to estimate both the shape model and the orbit at the same time, provided that there is a sufficient amount of a complimentary data type available (e.g., DSN radiometric or optical tracking).

This study will use the Near Earth Asteroid Rendezvous (NEAR) spacecraft orbiting the asteroid Eros as a simulation test case. Farquhar et al. (Ref. 1) provides a complete overview of the NEAR spacecraft mission. Eros is shaped irregularly, with the principal semi-axes measuring from 16.5 to 6.5 km, making it a good example to use for the general case of orbiting a distinctly non-spherical small body. NEAR is scheduled to begin orbiting Eros on February 14, 2000. Initially, it will orbit Eros at relatively high altitudes, ranging from 500 to 100 km (Ref. 2). At these altitudes, the spacecraft is less sensitive to many of the perturbations caused from the poorly known physical parameters of Eros and its orientation. This part of the orbit phase will allow some of the preliminary parameters to be updated. Although, the uncertainties associated with these estimates will still be fairly high. Currently, the mission plan calls for NEAR to move into a polar 50 km circular orbit in May of 2000. This orbit will provide complete coverage of the asteroid at a fairly low altitude, making it a prime opportunity to evaluate the potential of the altimeter range data in the orbit determination process. A six-day span during this time period is used as the first simulation test case. After the 50 km polar orbit segment, the plan is to have NEAR orbit Eros in several different inclination and altitude configurations. The second and third simulation test cases are planned to occur during December of 2000. The second simulated segment is a low inclination retrograde 50 km circular orbit. The final simulation is a retrograde elliptic orbit with an apoapse of 50 km and a periapse of 35 km. After the orbital phase is complete in January of 2001, the spacecraft is scheduled to complete several close flybys of Eros (Ref. 3). Following these flybys, and after NASA Headquarters approval, an attempt may be made to land on the surface of Eros. Although these periods are not analyzed in this study, it is anticipated that the altimeter data might prove to be valuable during this final part of the mission.

Processing the Altimeter Observations

The orbit determination software used for this study is the operational navigation software for the NEAR mission and has been written to process DSN radiometric, optical, and altimeter measurements. The altimeter range measurements can be expressed as the difference in the position of the spacecraft and the location of the illuminated point on the surface of the central body as shown in the following:

$$R_{Alt} = \left\| \bar{R}_{s/c} - \bar{R}_{Sur} \right\| \tag{1}$$

In the equation, $\bar{R}_{s/c}$ is the position of the spacecraft relative to the center of mass, and $\bar{R}_{Sur}$ is the position of the point on the surface of the central body that is illuminated by the altimeter. This formulation is implemented in the orbit determination software in the following fashion. First, an approximation for $\bar{R}_{Sur}$ is made by assuming that it is simply equal to the mean equatorial radius pointed from the center of mass towards the spacecraft. This initial guess for $\bar{R}_{Sur}$ is used in Eq. (1) to come up with a corresponding approximation for the altimeter range. The approximate value for the altimeter range is then used in the following relation to obtain a better estimate for the position of the illuminated surface point.

$$\bar{R}_{Sur} = \bar{R}_{s/c} + R_{Alt} \cdot \hat{A}_{lt} \tag{2}$$

In this equation, $\hat{A}_{lt}$ is the unit vector orientated in the direction that the altimeter is pointed, which is computed from the spacecraft attitude. The computed value of the altimeter range is then improved using the following:

$$R_{Alt_{i+1}} = R_{Alt_i} + h_{diff}\left(dR_{Alt}/dh_{diff}\right)_i \tag{3}$$

In Eq. (3), h_{diff} is the difference between the illuminated surface heights computed from both the a priori shape model and from Eq. (2), as shown below.

$$h_{diff} = r_{shape}(\phi,\lambda) - \left|\bar{R}_{Sur}\right| \tag{4}$$

In Eq. (4), $r_{shape}(\phi,\lambda)$ is the radius of the illuminated surface point as determined from the spherical harmonic shape model of the central body. R_{Alt} and $\bar{R}_{Sur}$ are computed iteratively using Eqs. (2) and (3), until the solutions converge to a specified level.

To process the altimeter data, the partials of the computed observations are needed. The partials of the computed altimeter range observations with respect to the spacecraft position are computed in the following manner. First, the directional derivative of the shape model with respect to the computed altimeter range observation is calculated. This is done by taking the dot product of the gradient of the shape model and the unit vector pointed from the illuminated surface point to the spacecraft:

$$\frac{dr_{shape}}{dR_{Alt}} = \left[T^{x,y,z}_{r,\phi,\lambda}\left(\nabla r_{shape}(\phi,\lambda)\right)\right]\cdot \hat{R}_{Alt} \tag{5}$$

In this equation, $T^{x,y,z}_{r,\phi,\lambda}$ is a transformation matrix from spherical to cartesian coordinates. Next, the inverse of the directional derivative is multiplied by the gradient of the shape model, evaluated at the illuminated surface point. So, the partial derivative of the computed altimeter measurement with respect to the spacecraft position is determined from:

$$\begin{aligned}\frac{\partial R_{Alt}}{\partial \bar{R}_{s/c}} &= \left[T^{x,y,z}_{r,\phi,\lambda}\left(\nabla r_{shape}(\phi,\lambda)\right)\right]\left(\frac{dr_{shape}}{dR_{Alt}}\right)^{-1} \\ &= \left(\frac{\partial r_{shape}}{\partial x}\hat{i} + \frac{\partial r_{shape}}{\partial y}\hat{j} + \frac{\partial r_{shape}}{\partial z}\hat{k}\right)\frac{dR_{Alt}}{dr_{shape}}\end{aligned} \tag{6}$$

The orbit determination software uses the preceding formulation in a Square Root Information Filter to process the altimeter observations along with all other measurement types.

The radius of the body ($r_{shape}(\phi,\lambda)$ in Eq. 4) is determined from the spherical harmonic shape model. Similar to the spherical harmonic representation of the gravity model, the radius is computed using Eq. (7).

$$r = \sum_{l=0}^{\deg}\sum_{m=0}^{l} P_{lm}\left[sin(\phi)\right]\left\{C_{lm}cos(m\lambda) + S_{lm}sin(m\lambda)\right\} \tag{7}$$

In this Equation, P_{lm} are the associated Legendre polynomials and the C_{lm} and S_{lm} terms are the shape model coefficients. Figure 1 shows a plot of the reference Eros shape model used in this study (Ref. 5), with a 50 km orbit. The orbit not only gives scale to the figure, but also indicates the relative altitude of the spacecraft above the surface of Eros.

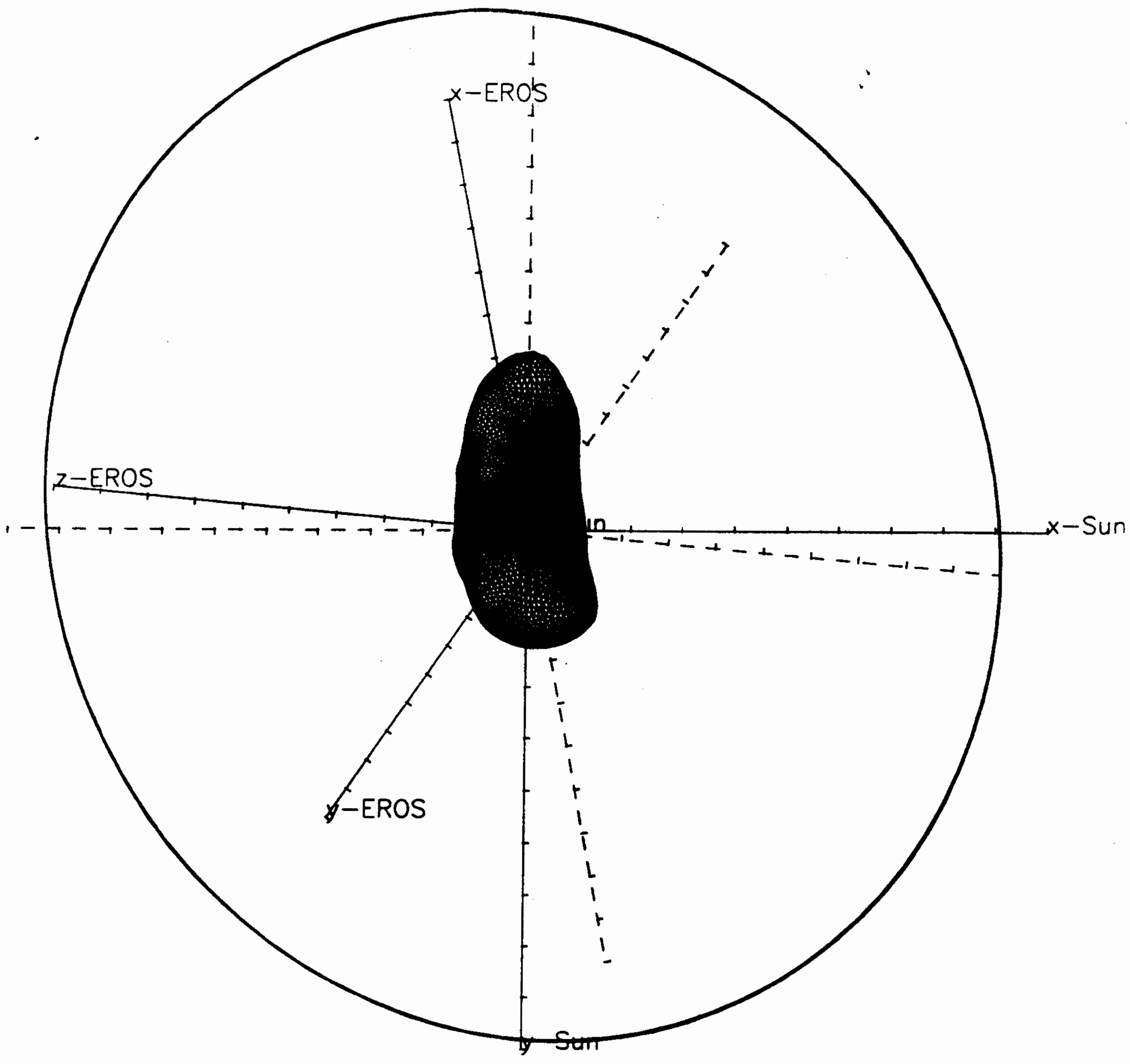

Figure 1 Eros Shape Model with 50 km Orbit

The Simulation Procedure

The simulations done in this study assume that the dynamic and measurement models are known completely, and are 100% accurate. This approach allows the simulated observations to be generated from a perfect reference orbit. The current best estimates of some of the physical parameters for Eros are shown in Table 1, and are taken from the paper by Miller et al. (Ref. 5). As described in that paper, the harmonic shape model is derived from the triangular plate shape model created at Cornell University. The gravity field model, which is complete through degree and order 16, is computed by assuming a uniform density and integrating over the shape model. Figure 2 shows the radial acceleration due to this gravity field at an altitude of 50 km.

Table 1

EROS PHYSICAL PARAMETERS

Variable	Value (Uncertainty)
X-axis radius	16.5 km
Y-axis radius	8.0 km
Z-axis radius	6.5 km
GM	4.8×10^{-4} ($\pm 1.2 \times 10^{-4}$) km^3/s^2
Mean equatorial radius	16.0 km
Rotation rate	5.270371 ($\pm 0.5 \times 10^{-4}$) hr/rev
Pole orientation:	
Right ascension	15.6 (±3.7) deg
Declination	16.4 (±1.8) deg
Prime Meridian	324.1 deg

Radiometric DSN doppler and range measurements along with the altimeter range measurements are generated with an appropriate level of noise applied. For the DSN data, the 1-σ noise levels are set to .012 Hz for the doppler integrated over 60 seconds and 500 m for the range data. For the altimeter range data, a 1-σ noise level of 50 m is applied to the observations. Additionally, it is assumed that tracking is provided from the DSN stations whenever the spacecraft is above 15 degrees elevation, while the altimeter measurements are generated at a rate of once every 2 minutes.

After the observations are generated, reasonably sized errors are added to the a priori initial conditions and force models. These errors are applied to the initial position and velocity of the spacecraft, the position and orientation of Eros, the effective thermal emissivity of the spacecraft, the Eros gravity field model, and the Eros shape model.

The next step is to converge the orbits using the simulated observations, once with and once without the altimeter data. The weight given to each measurement type corresponds to the amount of noise applied to that data type. To evaluate the performance of the estimation process, the estimated parameters can be compared to the correct values, which were used to generate the observations. Additionally, the difference between the estimated spacecraft trajectory and the true trajectory gives a measure of the error in the estimated spacecraft trajectory. The trajectories are usually compared in the body-fixed coordinate frame. Comparisons in the inertial coordinate frame will not only measure the error in the spacecraft state, but also will reflect errors in the estimated position of the central body. A body-fixed comparison is of more interest, since it will only measure the errors in the spacecraft position relative to the central body. These are the errors that are most critical for the measurements made by the scientific instruments onboard the spacecraft, as well as for navigation purposes.

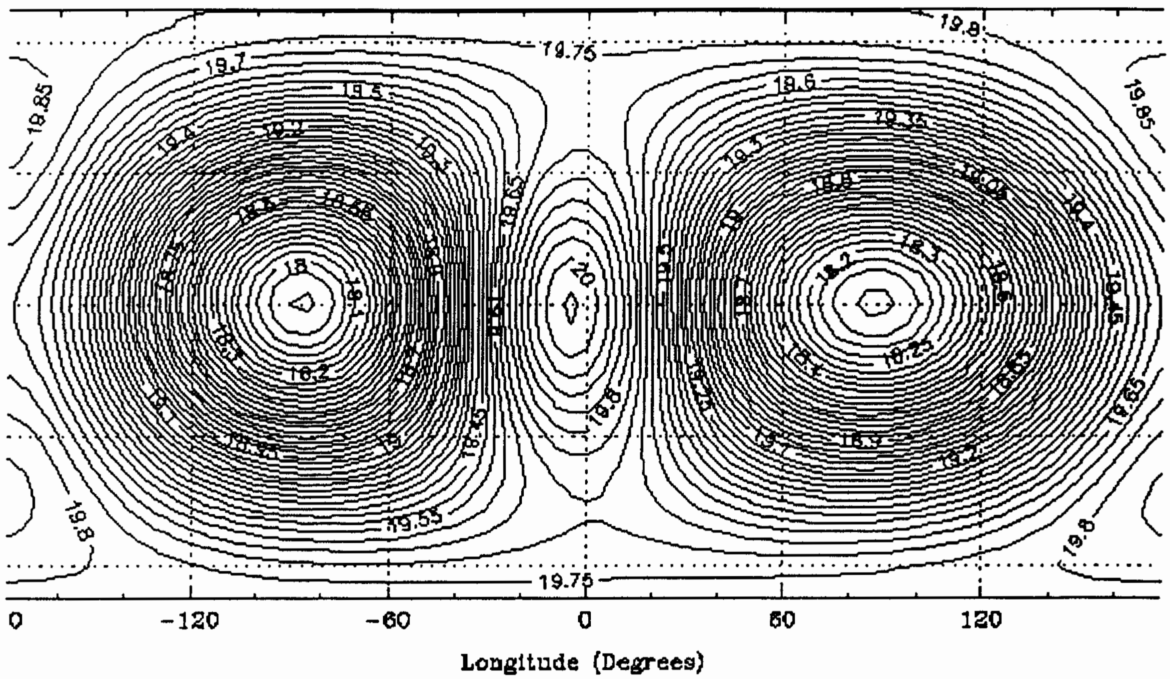

Figure 2 Radial Gravitational Acceleration Around Eros at 50 km Radius (mgals)

In processing the altimetry, it was found that there was no advantage in using the altimeter data during the first few iterations. In fact, if the a priori shape model has large errors, the altimetry can actually cause the solution to diverge. Therefore, the orbits are computed by first converging the solution using the radiometric tracking data only, and then adding the altimetry into the process. This approach also cuts down on the required amount of computer time, since the large number of altimeter observations significantly increases the run time for each iteration.

Circular Polar Orbits

The first example to evaluate the usefulness of the altimeter data is during the planned polar 50 km circular orbit segment. This segment is scheduled to take place after NEAR has been orbiting Eros for several months at higher altitudes. The simulation starts on May 5, 2000 and continues for six days. This time span is long enough to provide good coverage of Eros, and is short enough to make the run time for each case fairly quick. The orbital period of the 50 km orbits is about 28 hours, so during the simulation the spacecraft orbits Eros less than 6 times. However, the amount of coverage is aided by the quick rotation rate of Eros

(5.27 hr/rev). This is illustrated in Figure 3, which shows the spacecraft groundtrack during the 6 day simulation.

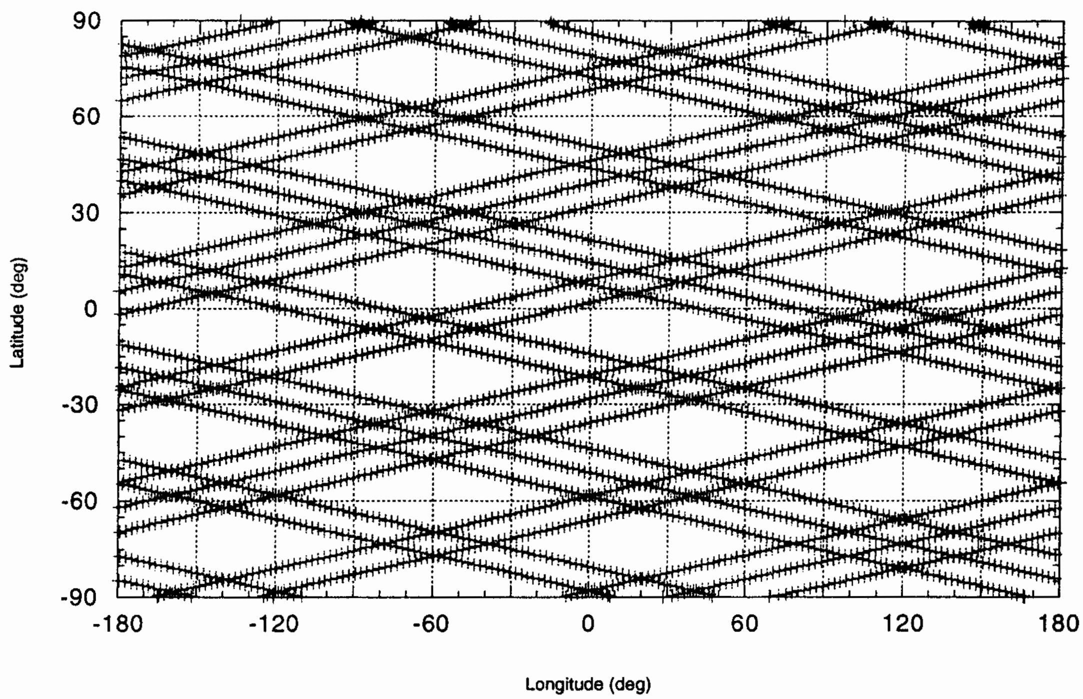

Figure 3 Spacecraft ground track during 5 day simulation

The size of the errors applied to the initial state and force models are chosen to reflect the fact that the spacecraft has been in orbit for a while and there should be at least some level of accuracy in the previous estimates. Likewise, the a priori constraints are chosen to reflect an appropriate level of confidence in the initial conditions. The parameters and the associated a priori uncertainties listed in Table 2 are estimated for all of the 50 km polar orbit cases, unless specified otherwise.

Before performing the simulations, the sensitivity of the 50 km orbits to the gravity coefficients needs to be determined. By integrating the orbits using a truncated gravity field and comparing it to the true trajectory, it can be determined through what degree and order the gravity field coefficients should be estimated. In this case, the RMS of the differences between the true trajectory and a trajectory computed using the gravity field truncated to degree and order six, eight, and ten are 3.8 m, 1.4 m, and 0.1 m, respectively. From this comparison, it appears that recovering coefficients beyond degree and order eight would be difficult at this altitude, since the orbits are expected to have an accuracy on the order of 10 m during this phase. Therefore, the harmonic gravity coefficients are estimated through degree and order eight for all cases of this circular polar orbit simulation.

To start with, a very simple case is considered. For this case, the only error applied to the a priori initial conditions and force models is a 200 m offset in the initial position of the spacecraft, which is much greater than the expected error in the orbits during this phase. All the values in Table 2 are estimated, even though they are assigned the correct a priori values. It's recognized that this test case is not very realistic, but it provides an opportunity to see if the altimeter measurements can provide any improvement to orbits that should already be well determined using only the DSN tracking data. The difference between the true trajectory and the two estimated orbits, converged by either using just the DSN data or by using the combination of DSN and altimeter data, are shown in the top of Table 3. In all the cases displayed in this table, the RMS of the differences between the trajectories is computed by differencing the trajectories every two minutes. The results presented in this table are interpreted as the RMS of the orbit errors, since each case is compared to the truth. As shown, including the altimetry in the solution improves the accuracy of the orbit from 20 m to 11 m, in an RMS sense.

Table 2

A PRIORI UNCERTAINTIES FOR ESTIMATED PARAMETERS

Parameter	Uncertainty
Spacecraft position	100 m
Spacecraft velocity	100 mm/s
Asteroid position	1 km
Asteroid velocity	100 mm/s
Asteroid orientation – prime meridian	10 deg
Spin axis angles	5 deg
Asteroid orientation rate	0.01 deg/s
Effective thermal emissivity of spacecraft	100%
Stochastic Accelerations	0.1 nm/s^2
Normalized Gravity Harmonics (8x8)	0.001
Shape Model Harmonic Coefficients (12x12)	0.1

To determine the best contribution we could expect from the altimeter data, we assume perfect knowledge of the shape model. Case 2 uses this assumption, meaning that the shape model coefficients are no longer estimated. This should strengthen the altimeter data, since there is no longer any uncertainty associated with the height of the central body in each of the altimeter range measurements. As shown in Table 3, this further improves the accuracy of the orbits to the 9 m level. To see in what specific directions the orbits are being improved, the orbit differences can be broken into the radial, transverse, and normal (RTN) directions. For case 2, the RMS of the differences between the true orbit and the one estimated using just the DSN data are 0.7 m in the radial direction, 15.4 m in the transverse direction, and 13.8 m in the normal direction. When the altimeter data is used, the RMS of the orbit errors is 0.5 m in the radial direction, 5.9 m in the transverse direction, and 6.3 m in the normal direction. This comparison suggests that the altimeter data is most significantly contributing to the orbits in both the transverse and normal directions. This may seem surprising, since the majority of the information in the altimeter observations is in the radial direction. This is partly explained by the fact that the altimeter observations provide a strong tie to the orientation of the central body. This is confirmed by inspection of the estimated orientation parameters, which shows that these angles are slightly closer to the truth when the altimeter data is used in the estimation process.

Table 3

ORBIT ERRORS (RMS)

Case #: Errors applied to initial conditions and models	RMS (m)
Case 1: 200 m offset in initial spacecraft position	
a. DSN tracking only	21.0
b. DSN tracking and altimetry	11.0
Case 2: Same as Case 1, without estimating shape coefficients	
a. DSN tracking only	21.0
b. DSN tracking and altimetry	8.7
Case 3: Errors in spacecraft & asteroid states, gravity & shape harmonics, and spacecraft thermal emissivity	
a. DSN tracking only (average of 5 runs)	23.2
b. DSN tracking and altimetry (average of 5 runs)	14.4

Case 3 represents a more realistic situation, with errors included in the spacecraft and asteroid initial conditions, the asteroid orientation, the gravity and shape model harmonic coefficients, and the effective thermal emissivity factors. The errors applied to the position and velocity of the spacecraft state have magnitudes of 100 m and 2 mm/s, respectively. The initial orientation of the asteroid is given an error of 10 degrees in the prime meridian, and two degrees in the orientation of the spin angle. To simulate gravity errors, the harmonic gravity coefficients beyond degree and order four are truncated from the reference gravity model. Additionally, each of the remaining coefficients is given a random error with a 1-σ magnitude equal to 10% of the magnitude of the correct value for that particular coefficient. In a similar manner, random 20% 1-σ errors in the shape model coefficients are applied to the full 20 by 20 shape model. The effective thermal emissivity factors are given 5% errors in each direction.

One thing to keep in mind is that the errors applied to the gravity models and the shape models will not be fully recovered during the estimation process. This is a result of the errors being applied to the full reference models, which were used to generate the observations, while only sub-sets of these models are estimated. The reference gravity model is complete through degree and order 16, while the gravity harmonics are only estimated through degree and order 8. However, as explained earlier, the effects of the higher order terms are not very significant at this altitude. Likewise for the shape model, the errors are applied to all the coefficients (through degree and order 20), while the shape harmonics are only estimated through degree and order 12.

In order to give a little statistical significance to the case 3 results, the orbits are computed 5 different times. For each run, the initial errors are changed, although the magnitude of each of the error sources remains the same. The accuracy of the estimated orbits shown in case 3 of Table 3 represents the average orbit accuracy of the 5 different runs. The scatter of the orbit errors between each of the five runs is approximately 1.5 m RMS, making it likely that these results are representative of other simulations with similar errors in the initial conditions. This case shows that even with significant errors in the a priori shape model, the altimeter data can improve the accuracy of the orbits from the 23 m level to the 14 m level, in an RMS sense.

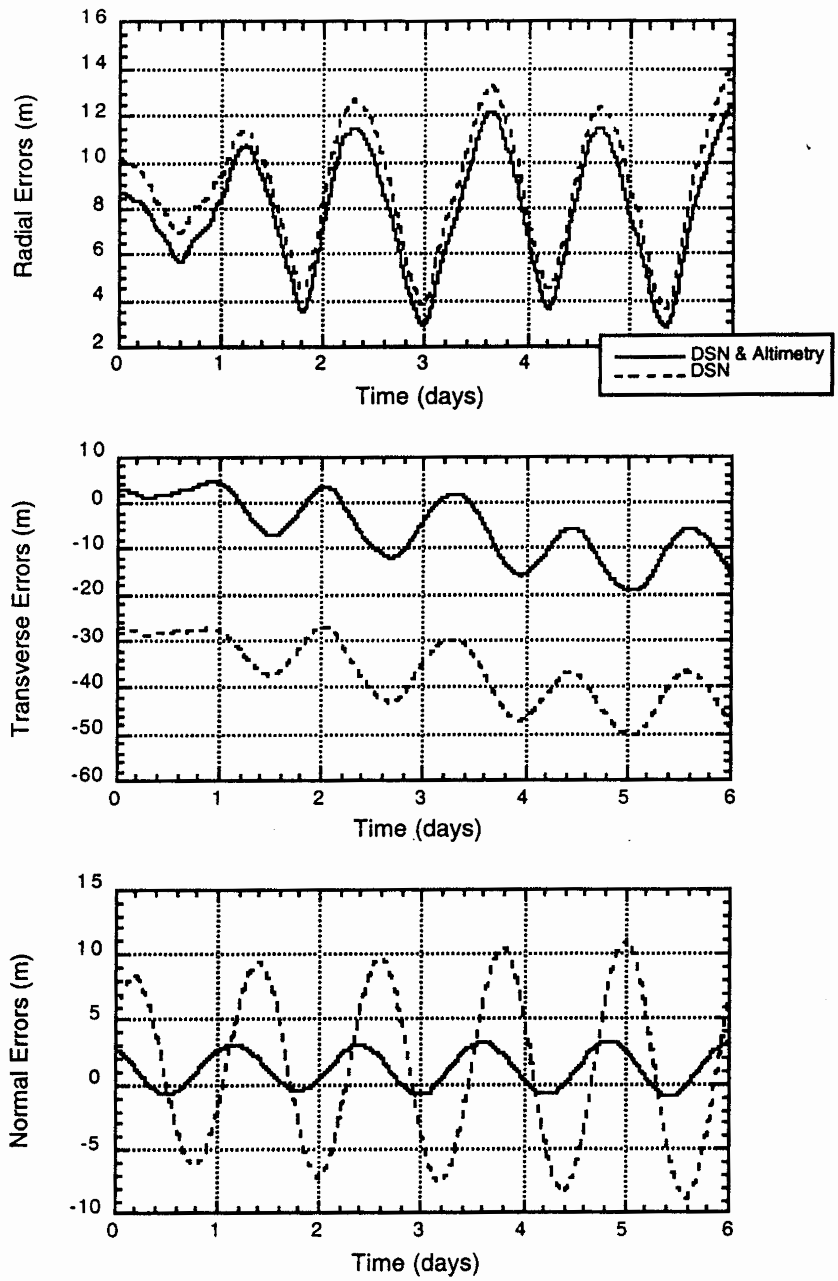

Figure 4 The Impact of Altimetry on Orbit Errors During one of the Case 3 Simulations

As in case 2, the orbits from one of the case 3 runs are compared in RTN coordinates. Figure 4 shows the errors in the orbits computed with and without the altimeter data for one of the five case 3 runs. The top plot shows that the orbits only improve marginally in the radial direction, while the second and third plots show that the addition of the altimeter data significantly improves the accuracy of the orbits in both the normal and transverse directions. This again illustrates that the strength of the altimeter data is in providing a strong tie to the orientation of the central body.

Near Equatorial Circular Orbits

The next simulated test segment is a 50 km orbit at an inclination of 151 degrees. NEAR is currently scheduled to enter this near equatorial retrograde orbit on December 13, 2000. As in the last case, a 6-day orbit arc length is used for the simulation. Obviously, the coverage provided by this orbit configuration will only be between ±30 degrees latitude. Therefore, it is expected that the accuracy of the estimated gravity or shape models will be less than the estimates from the last simulation. Since the spacecraft is at the same altitude, the same parameters will be estimated as those estimated in the last section. The a priori uncertainties used for the 50 km polar orbits are used here, as well (see Table 2)

Table 4

ORBIT ERRORS (RMS)

Case #: Errors applied to initial conditions and models	RMS (m)
Case 1: 200 m offset in initial spacecraft position	
a. DSN tracking only	23.7
b. DSN tracking and altimetry	17.0
Case 2: Same as Case 1, without estimating shape coefficients	
a. DSN tracking only	23.7
b. DSN tracking and altimetry	13.2
Case 3: Errors in spacecraft & asteroid states, gravity & shape harmonics, and spacecraft thermal emissivity	
a. DSN tracking only (average of 5 runs)	49.4
b. DSN tracking and altimetry (average of 5 runs)	22.4
Case 4: Same as Case 3, except a priori shape and gravity models are the estimates from Case 3 in Table 3	
a. DSN tracking only (average of 5 runs)	26.8
b. DSN tracking and altimetry (average of 5 runs)	18.4

As in the last section, the only error applied in case 1 is a 200 m offset in the spacecraft initial position. The resulting levels of orbit accuracy are shown in the top part of Table 4. The improvement in orbit accuracy from the 24 m to 17 m level with the addition of the altimeter data is not quite as good as what was seen in the polar orbit simulation, where the accuracy dropped from 21 to 11 m. It is not clear exactly why the altimeter data seems to contribute less in this case. However, the reason for the reduction in accuracy across the board may be that the limited coverage of the asteroid enlarges the correlations between the estimated parameters. When the shape model is completely known a priori (case 2 of Table 4), the accuracy of the orbits computed with the altimeter data improves to about 13 m, in an

RMS sense. Again, this is not as accurate as the 9 m obtained when using the polar orbits, although in light of the previous results is not surprising.

Once again, case 3 is an attempt at a more realistic simulation. The same magnitude errors are applied to the initial conditions and models as were in case 3 of the last section. Also similar to the last section, the values displayed in Table 4 are the averages of 5 different runs. In each run, the initial errors are applied differently, although the magnitude of the errors remains the same. The surprising result is that the accuracy of the orbits computed without the altimeter data seems to suffer significantly, in comparison to the results from the polar orbit simulation. In this simulation, the average RMS of the orbit error is about 49 m when only the DSN tracking is used. Meanwhile, the average RMS of the orbit error is about 22 m when the altimeter data is included.

To see what is responsible for the large orbit errors in case 3, when the altimetry is not used, each of the error sources are individually removed. In this manner it was found that the primary cause of the large 49 m orbit errors is the initial errors applied to the gravity models. It seems that this, in combination with the limited coverage of the asteroid and having only the DSN tracking makes separating the gravity coefficients from the other estimated parameters difficult. Apparently, the addition of the altimeter data gives enough information to help separate these parameters a bit more, and therefore reduces the errors in the estimated orbits.

To show the impact of smaller initial errors in the a priori gravity models, case 3 is repeated as case 4. This time, the updated shape and gravity models from case 3 of the polar orbit simulation are used as the a priori models. This approach makes sense, since this orbit segment is planned to begin after completion of the polar orbit segment of the last section, and realistically the gravity models should be more accurate by the time this orbit segment begins. As shown in Table 4, this reduces the average orbit errors in the DSN-only orbits from 49 m to 27 m, which is more in line with the magnitude of the orbit errors observed in the polar orbit simulation. Likewise, using the previously adjusted gravity fields improves the accuracy of the orbits computed with the altimeter data (in addition to the DSN tracking) from 22 m to 18 m. Therefore, even when the a priori gravity field model is fairly accurate, using the altimeter data improves the orbits significantly. Also of note, when the a priori gravity model is not very accurate, the addition of altimeter data results in a significant improvement in orbit accuracy for cases where the orbit geometry does not provide broad coverage of the asteroid.

Low Altitude Elliptic Equatorial Orbits

The last simulation to be examined is the orbit segment planned to begin December 31, 2000. This orbit is a 50 by 35 km retrograde orbit at an inclination of 179 degrees. Because of the lower altitude the orbit is slightly more sensitive to higher degree and order gravity harmonics than what were estimated in the previous two examples. However, when the true gravity field is truncated to degree and order 8, the RMS of the resulting orbit error is 8.1 m. In light of this and the previous results, it seems that continuing to estimate the gravity field coefficients through degree and order 8 is sufficient.

The same analysis is done for this simulation as in the last two, with the results presented in Table 5. For case 3, the a priori gravity and shape models are the models estimated during the polar orbit segment (similar to case 4 in Table 4). If the original gravity models with all of the errors applied are used, the orbits will not converge because these errors will result in larger perturbations at this lower altitude. Since this orbit segment occurs after the polar orbit segment, this approach is justified. Additionally, this low of an orbit would not be attempted without adequate knowledge of the gravity field of the central body.

Table 5

ORBIT ERRORS (RMS)

Case #: Errors applied to a priori initial conditions and models	RMS (m)
Case 1: 200 m offset in initial spacecraft position	
a. DSN tracking only	12.3
b. DSN tracking and altimetry	10.3
Case 2: Same as Case 1, without estimating shape coefficients	
a. DSN tracking only	12.3
b. DSN tracking and altimetry	7.2
Case 3: Errors in spacecraft & asteroid states, gravity & shape harmonics, and spacecraft thermal emissivity	
a. DSN tracking only (average of 5 runs)	27.0
b. DSN tracking and altimetry (average of 5 runs)	20.1

The results again show that using the altimeter data improves the orbits. Although in cases 1 and 2, the accuracy of the orbits computed with just the DSN tracking is better than what was achieved in the other two simulations. A reason for this may be that the lower altitude helps separate the estimated parameters, and helps prevent errors in those estimates from being absorbed into the estimates of the spacecraft state. The end result is that the addition of the altimeter data in cases 1 and 2 does not improve the accuracy of the orbits as much as in the first two simulations. In the more realistic case 3, the accuracy drops from approximately 27 m to the 20 m level with the addition of the altimetry. This level of orbit improvement is in line with what has been observed in the previous two simulations (case 3 in Table 3 and case 4 in Table 4). It appears safe to conclude that the addition of altimeter observations can increase the accuracy of the estimated orbits of spacecraft about small bodies.

Reducing the Amount of DSN Tracking

Besides improving the accuracy of the estimated orbits, the addition of altimetry in the orbit determination process could allow for reduced requirements on DSN tracking. In an attempt to show this, the final five runs from the last two simulations (case 4 from Table 4 and case 3 from Table 5) will be rerun, with half of the DSN tracking passes eliminated. For each of the runs, the a priori gravity and shape models used are those estimated from the corresponding five polar orbit runs (case 3 from Table 3). This means that the a priori models are not too far off from the true gravity field and shape models.

For the near equatorial 50 km circular orbit simulations, removing half of the DSN tracking data and using the altimetry results in orbits with similar accuracy to those computed using all of the DSN tracking without altimetry. Previously, it was found that the average accuracy of the orbits computed with just the DSN tracking is about 27 m (from case 4a in Table 4). When half of the DSN tracking passes are removed and the altimeter data is included, the average accuracy of the five sets of computed orbits is 23.8 m. Meanwhile, the accuracy of the orbits computed using only half of the DSN tracking (without any altimetry) is 57.4 m. To make the comparison easier, the averages of the orbit errors for each of the five runs are displayed in the top half of Table 6. It is clear from these results that the presence of the altimeter observations allows the amount of DSN tracking to be reduced without adversely affecting the estimated orbit accuracy.

For the elliptic orbit simulation, the altimeter data is not able to maintain the accuracy of the orbits to the level obtained when the DSN tracking is nearly continuous. However, as displayed in the bottom portion of Table 6, the altimetry does significantly improve the orbit accuracy, when half of the DSN tracking passes are removed. From the Table 5, the average accuracy of the orbits computed with just DSN tracking is 27 m. When half of the DSN passes are removed and the altimetry data is used, the average orbit accuracy is 38 m. Although an accuracy of 38 m is significantly less than 27 m, it is also much better than the average accuracy attained without the altimeter data of 59 m.

Table 6

ORBIT ERROR COMPARISON USING LIMITED DSN TRACKING

Low inclination 50 km circular orbits	RMS (m)
Full DSN tracking	26.8
Altimetry and full DSN tracking	18.4
Half DSN tracking	57.4
Altimetry and half DSN tracking	23.8
Elliptic retrograde orbits (50 km by 35 km)	
Full DSN tracking	27.0
Altimetry and full DSN tracking	20.1
Half DSN tracking	59.1
Altimetry and half DSN tracking	38.3

Summary

This study has attempted to quantify the impact of adding altimeter range observations into the orbit determination problem for spacecraft orbiting small bodies. To this end, simulations of the NEAR spacecraft orbiting the asteroid Eros are performed. Simulations are done for three different orbit configurations, all of which are segments that are currently planned to occur in the year 2000. The simulations have all been for spacecraft orbiting Eros, but similar results are expected for the more generic case of orbiting any small body. The tests have shown that the addition of altimeter data into the orbit determination problem increases the accuracy of the estimated orbits. Even when large errors were applied to the a priori shape models, the orbit accuracy always improved when the altimeter data was added, although the magnitude of the improvements is not always overwhelming. Typically, when just the DSN tracking was used, the RMS of the orbit errors is in the range of 24 to 27 m. When the altimeter data is added the RMS of the orbit accuracy is usually between 14 and

20 m, depending on the orbital characteristics and the errors in the initial conditions. Additionally, the altimeter data proved valuable in its ability to lessen the requirement on the amount of DSN tracking needed. By using the altimeter data, up to half of the DSN tracking passes can be eliminated without adversely effecting the accuracy of the estimated orbits.

On the other hand, it was found that the addition of the altimeter data did not quicken the time required to converge the solutions. The only exception to this is when the a priori shape model does not contain any errors, which is not a very realistic scenario. When errors are present in the a priori shape model, it is usually best to converge the solution first with just the DSN tracking, and add in the altimeter data later.

Future studies may show that adding an algorithm to process altimeter crossovers in the orbit determination software may prove beneficial. This type of measurement would eliminate errors in the shape model from being aliased into the estimated orbits. This may give more strength to the altimeter data, and make it a more valuable tool, in terms of orbit determination.

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