RECENT TRENDS IN THE ANALYSIS OF QUASIOPTICAL SYSTEMS

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INTRODUCTION

The recent trend in microwave instruments is the use of multiple millimeter and submillimeter wavelength bands. These systems are typically analyzed by using physical optics, Gaussian beams or ray tracing techniques. Physical optics offers high accuracy at the expense of computation time. This trade-off becomes particularly apparent in the analysis of multiple reflector antennas, such as beam waveguide antennas, where physical optics is used to compute the current on each reflector from the current on the previous reflector. At the other end of the spectrum is ray tracing approaches that ignore diffraction effects entirely. These methods are fast but sacrifice the ability to predict some effects accurately.

An intermediate approach is to use an appropriate set of expansion functions to model the field between the reflectors. If the set is chosen wisely only a few coefficients need to be determined from each reflector current. The field is then computed at the next reflector through the use of the expansion functions and their coefficients rather than by using the previous reflector current. For a beam waveguide system with no enclosing tubes an excellent set of expansion functions is the Gaussian beam mode set. In many cases a preliminary design which includes the effects on diffraction may be obtained by considering only the fundamental mode and a thin lens model for the reflectors. Higher-order modes are included to model the effects of the curved reflector, which include asymmetric distortion of the beam, cross polarization, and beam truncation.

This paper describes a computer code implementing higher-order Gaussian beam scattering by multiple reflector systems. Examples will compare results from the Gaussian beam approach to pure physical optics.

ALGORITHM

A computer program has been written to solve the problem of higher-order Gaussian beam scattering by an arbitrary set of reflectors. The problem geometry is depicted in Fig. 1. The steps involved in the solution are as follows:

(1) Compute the current on the first reflector using physical optics. The incident magnetic field is provided either by a feed model or by an incident set of Gaussian beam modes.

(2) Compute the direction of propagation for the reflected Gaussian beam-set using ray tracing. Using a gut ray in the input direction specified by the feed coordinate system or by the input Gaussian beam set propagation direction and the reflector surface description compute the gut ray direction for the output Gaussian beam set.

(3) Next the waist size and location for the output beam set is found by examining the amplitude and phase distribution of the current on the reflector, as described below. Essentially, the waist and radius of curvature of the output beam set at the reflector are estimated. From these two quantities the beam waist and its location along the output gut ray direction are determined.

(4) Having determined the size of the waist and its location all that remains is to find the amplitudes of the individual modes in the output mode set. This is accomplished through the use of the reciprocity theorem. A calculation of an interaction integral of the mode in question and the reflector current is required.

(5) Steps (1)–(4) are then repeated for each addition reflector in the chain. In each of these cases the input field for the current calculation is provided by the previous Gaussian beam set.

DETERMINATION OF OUTPUT BEAM SET PARAMETERS

One of the key steps in computing scattering of Gaussian beam mode sets by arbitrary reflector is the determination of the best choice for the output beam set's waist and its location. The problem is depicted in Fig. 2, where w_{in} and l_{in} are the input waist and location and w_{out} and l_{out} are the same parameters for the output beam set. Once these parameters are determined computation of the mode amplitudes from the current on the reflector follows easily. The steps followed in the computer code are as follows:

The research described in this paper was carried out by the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration.

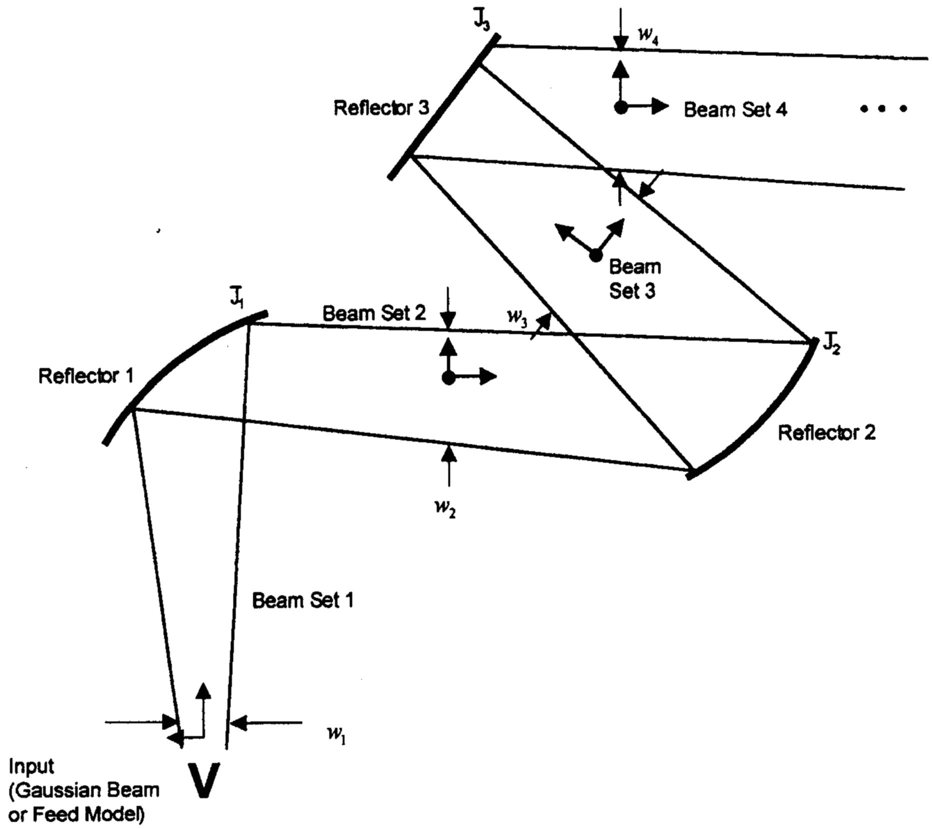

Fig. 1. Problem Geometry

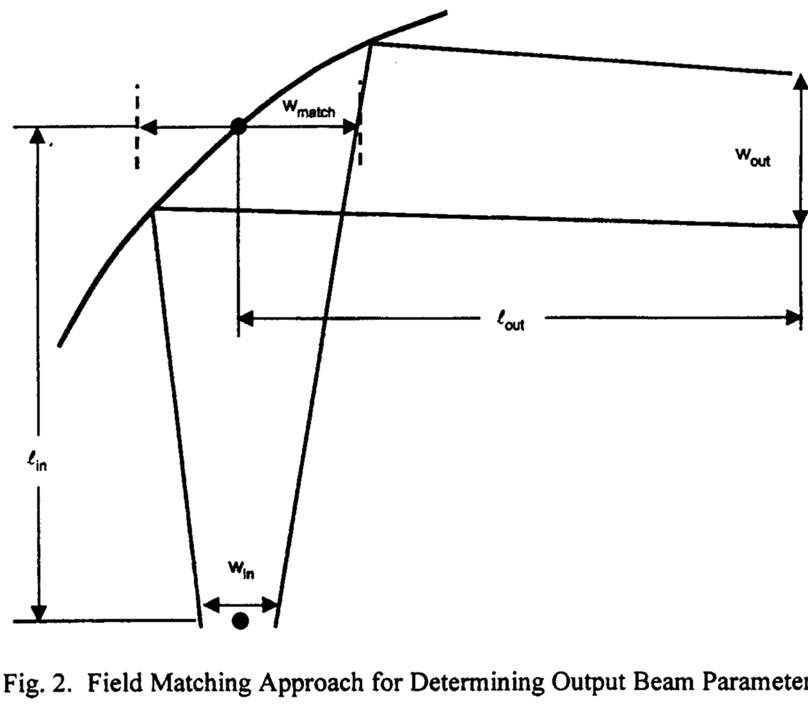

Fig. 2. Field Matching Approach for Determining Output Beam Parameters

(1) Compute a waist at the reflector, denoted as w_{match}. When the input to the reflector is a Gaussian beam mode set the input waist is simply computed at the point of impact on the reflector. When the input to the reflector is a feed an estimate of the waist is determined from the reflector current. The output beam set is required to produce

a waist equal to w_{match} at the point of impact. This provides one of the two equations needed to compute the output beam set parameters.

(2) Next a suitable set of points on the reflector to be used for field matching or path-length matching is derived. These points are generally chosen to be within a waist of the point of impact.

(3) Next one of two approaches is used to determine the best value for l_{out}.

Approach 1: Field Matching

(1) Compute the incident field at each of the match points chosen above. These field values are either determined from the incident beam set or the feed field.

(2) Using these fields and the known direction of the output beam set search l_{out} for an optimum, defined to be where a best-fit match of the reflector fields to the fundamental mode of the output mode set is obtained. In general multiple minima are found and the absolute minimum is chosen. It can be shown that the allowable values for l_{out} are bounded,

$$|\ell_{out}| < \frac{\pi w_{match}^2}{2\lambda}. \qquad (1)$$

Approach 2: Geometrical Optics Determination of Output Radius of Curvature

(1) Compute a phase center location for the input fields. For a Gaussian beam mode set this is determined from the input radius of curvature at the point of impact. For a feed input the phase center is assumed to be at the origin of the feed coordinate system.

(2) Using the set of points on the reflector compute the set of path lengths from the input phase center each point, Fig. 3.

(3) Search for the output phase center (output radius of curvature), sweeping along the direction of the output ray. A minimum in the spread of the total path lengths from input phase center to reflector and then to output phase center is sought. Generally a single minimum is found either in front of or behind the reflector.

(4) Using the output radius of curvature and the required waist at the reflector compute the waist size and location of the output beam set.

COMPUTATION OF MODE AMPLITUDES USING THE RECIPROCITY THEOREM

The approach for determining the amplitudes of the Gaussian beam modes directly from the reflector current is summarized. It should be noted that two approximations are necessary, (1) the reflector current is approximated to be

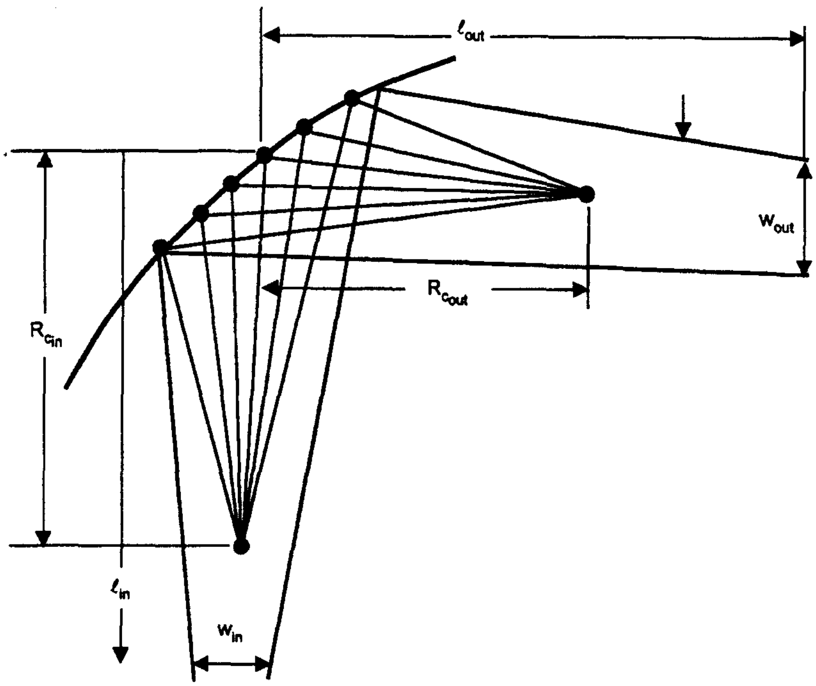

Fig. 3. Ray Optics Approach for Determining Output Beam Parameters

the physical optics current, a good approximation for large reflectors with low edge illumination, and (2) the Gaussian beam modes are solutions to Maxwell's equations in free space, never true but a good approximation if the mode fields are required only in the paraxial region. The Gaussian beam modes used are given in terms of Laguerre polynomials as described by Goldsmith, [1]. Each mode has a polarization, either 'x', or 'y', a radial index, p, and an azimuthal index m.

For two arbitrary fields and their associated sources, denoted by "A", and "B", the reciprocity theorem when applied to an arbitrary volume, V, and its enclosing surface S, may be stated as follows:

$$\oint\!\!\!\oint_S \left(\overline{E}_A \times \overline{H}_B - \overline{E}_B \times \overline{H}_A\right)\cdot d\overline{s} = \iiint_V \left(\overline{E}_B \cdot \overline{J}_A - \overline{B}_B \cdot \overline{M}_A - \overline{E}_A \cdot \overline{J}_B + \overline{B}_A \cdot \overline{M}_B\right) dv. \qquad (2)$$

A half-space completely enclosing the reflector is chosen as V, with the surface S perpendicular to the direction of propagation for the output beam set. For this particular application we choose the output Gaussian mode set, with unknown mode amplitudes, as the "A" field with the reflector current inside the volume being its source. As the "B" field we choose a test field, the conjugate of the ith Gaussian beam mode now propagating toward the reflector. The source for this field are chosen to be outside the volume V.
We have then for the fields,

$$\overline{E}_A = \sum_j a_j \overline{e}_j,\ \overline{H}_A = \sum_j a_j \frac{\hat{z}_{out} \times \overline{e}_j}{\eta_o},\ \overline{E}_B = \overline{e}_i^{\,*},\ \ \overline{H}_B = -\frac{\hat{z}_{out} \times \overline{e}_i^{\,*}}{\eta_o}. \qquad (3)$$

Using the reciprocity theorem and the orthogonality condition for the Gaussian beam modes on the infinite surface S, we can obtain the desired equation for the unknown coefficients,

$$a_i = -\frac{1}{2} \iint_{S_{reflector}} \overline{e}_i^{\,*} \cdot \overline{J}\, ds. \qquad (4)$$

Example Calculation

A comparison of a full Physical Optics calculation and the Gaussian Beam Analysis for the output of a 448 GHz seven mirror radiometer is shown in Fig. 4. The 7-mirror system consisted of a Potter horn on the input with an ellipse, flat mirrors and a parabola to direct and shape the output beam. The computation time for the Physical Optics program was 8 hours on a 200 MHz Pentium and less than 5 minutes for the Gaussian Beam Analysis.

REFERENCES

[1] P.F. Goldsmith, *Quasioptical Systems*, pp. 26–29, IEEE Press, New York, NY, 1998.

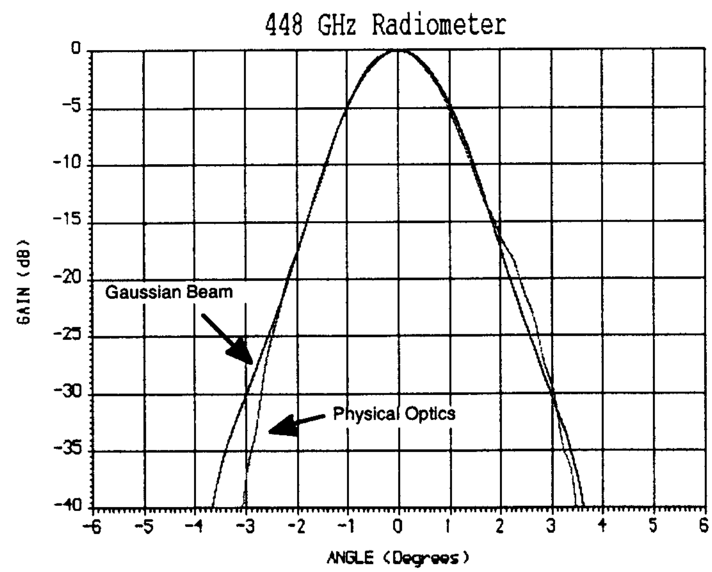

Fig. 4. Comparison of Physical Optics and Gaussian Beam Analysis