AIAA 2002-0596

DTFM MODELING, OPTIMIZATION, AND SENSITIVITY ANALYSIS FOR GOSSAMER STRUCTURES

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Abstract

A new structural modeling and analysis method, the Distributed Transfer Function Method (DTFM), is presented for the applications of space gossamer structures. The method has certain unique features that make it suitable for gossamer space structures. With the DTFM, some problems that are difficult to deal with by other methods can be readily tackled. Examples of this kind of problems include buckling and post-buckling of extremely long booms with geometric and material imperfections, dynamics of long booms with non-uniform cross-sections, dynamics of spinning space structures (gyroscopic effect), and structures with distributed damping. The proposed DTFM is computational efficient, and yields highly accurate results. By the DTFM-based analysis, optimization of a passive vibration control system for an inflatable reflectarray antenna is studied; sensitivity analysis with respect to deviations of bending stiffness with very large length to diameter ratio is investigated. Several examples are provided to show the applications and computational efficiency of the DTFM. In addition, future research directions are discussed.

1. Introduction

Space gossamer systems are generally composed of supporting structures formed by highly flexible, long tubular elements and pre-tensioned thin-film membranes. Shown in Figures 1 and 2 are two examples of gossamer structures—inflatable sunshield and solar sail, which consist of several inflatable booms and single or multiple layers of membrane. Gossamer systems offer order-of-magnitude reductions in mass and launch volume, and will revolutionize the architecture and design of space flight systems that require large in-orbit configurations and apertures. A great interest has been generated in recent years in flying gossamer systems on near-term and future space missions [1].

Figure 1. Inflatable sunshield

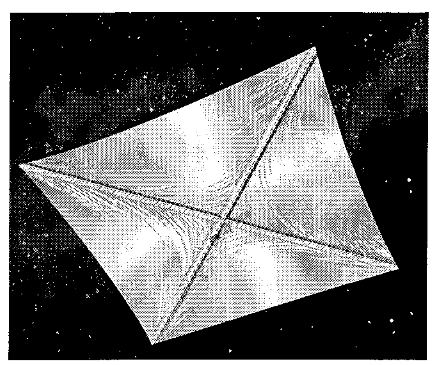


Figure 2. Solar sail

Modeling, analysis, and optimization are essential to the success of development and deployment of gossamer structures. These tasks, however, are unique and extremely complicated, involving a variety of issues such as formation and effects of wrinkles in tensioned membranes, synthesis of tubular and membrane elements into a complete structural system, buckling analysis of inflatable boom components with material and geometric imperfections, and optimization design with non-uniformly distributed boom structures. Because of the above-mentioned difficulties, general-purpose finite-element structural analysis codes are not ready for reliable analysis and optimization of such systems. This has led to an urgent need for the development of new structural modeling and analysis capabilities that are specifically suitable for gossamer structures [2]. The Distributed Transfer Function Method (DTFM) presented herein can potentially meet this need [3, 4].

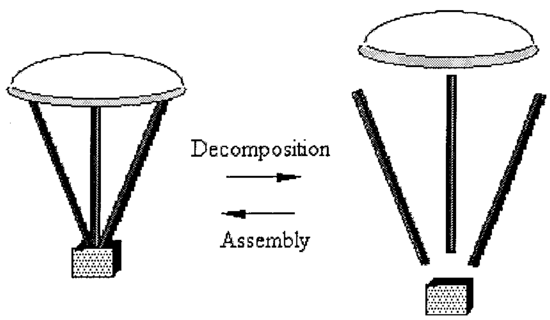


Figure 3. Decomposing and assembling of a gossamer structure

There are three aspects that make the DTFM distinctively suitable for simulation of gossamer structures.

First, the DTFM models a gossamer structure with a minimum number of nodes. This is done by decomposing the structure only at those points where multiple structural components are connected, and by keeping each component as large as possible; see Figure 3 for a demonstrative example. A gossamer structure is usually composed of several very long tubular components as well as several layers of membranes. Those basic building blocks are connected to each other at a small number points. As a result, the DTFM models the gossamer structure with a small number of unknowns, and deals with matrices of low order. Furthermore, the DTFM gives closed form analytical or semi-analytical solutions, which renders the DTFM-based analysis results more reliable.

Second, due to its unique analytical capability of describing local variations of material and geometric properties of structures, the DTFM can be employed to investigate the sensitivities of a structure to the material imperfections or geometrical imperfections. In order for the finite element method (FEM) to investigate the impacts of material or geometrical imperfections, numerous elements have to be employed by a model. Consequently, results might be overwhelmed by numerical inaccuracies induced by numerous elements.

Third, the DTFM is convenient in handling structural systems with passive and active damping, gyroscopic effects, embedded smart material layers as sensing and actuating devices, and feedback controller. On the other hand, commercially finite element codes are not readily applicable to those systems. So, this feature of the DTFM allows convenient re-design and improvement of gossamer systems.

The remainder of the paper is arranged as follows. The general process of the DTFM is first presented. An example of optimization analyses with DTFM follows. Sensitivity analysis procedure with respect to geometrical imperfection of a boom will then be discussed. Several examples of the DTFM-based modeling and analysis will be provided to show the sensitivity analysis process as well as the calculation efficiency of DTFM.

2. Process of Distributed Transfer Function Method

2.1 Decomposition of a Complex Structure

In a DTFM-based modeling and analysis process, a complex structural system is first decomposed into several structural components (substructures). Figure 3 illustrates the idea of how a structure is decomposed into substructures and assembled later on. Unlike the FEM approach that needs to further divide the substructures into small elements, the DTFM approach treats each substructure as a single component. Therefore, the DTFM approach leads to a much smaller number of unknowns to be determined. The governing differential equations of the substructures are then Laplace-transformed (with respect to time). Dynamic stiffness matrices, which include the frequency as a variable, can be established based on Distributed Transfer Function solutions and systematically assembled to form the original structure. Unlike the FEM, the DTFM gives not only displacements, but also higher order derivatives (strains and stresses) with closed-form solutions that precisely describe the behaviors of the structure.

2.2 State Space Form of a One-Dimensional Distributed Parameter System

Without loss of generality, one-dimensional components will be used here to show the DTFM modeling process. (The method has been successfully applied to two- and three-dimensional structural components.)

In the local coordinate system, the displacements $u_i(x,t)$ of a single one-dimensional distributed component are governed by linear partial differential equations,

$$\sum_{j=1}^{n}\sum_{k=0}^{N_j}\left(a_{ijk}+b_{ijk}\frac{\partial}{\partial t}+c_{ijk}\frac{\partial^2}{\partial t^2}\right)\frac{\partial^k u_j(x,t)}{\partial x^k}=f_i(x,t),$$

$$x\in(0,L),\quad t\geq 0,\quad i=1,\cdots,n. \tag{1}$$

Here n is the number of differential equations (which is the same as the number of the unknown displacement functions u_j, $j=1,\cdots,n$), N_j is the highest order of differentiation of u_j with respect to x, $f_i(x,t)$ is the external disturbance, a_{ijk}, b_{ijk}, and c_{ijk} are constants, and L is the length of the one-dimensional component. Coefficients a_{ijk}, b_{ijk}, and c_{ijk} represent inertia, damping, distributed constraint (e.g., elastic foundation), gyroscopic term, axial load, etc. Equations (1) represent various one-dimensional continua such as beam, frame, truss, rotating shaft, and etc.

Equations (1) are Laplace transformed with respect to time (t) and expressed as,

$$\sum_{j=1}^{n}\sum_{k=0}^{N_j}D_{ijk}\frac{d^k\bar{u}_j(x,s)}{dx^k}=\bar{f}_i(x,s),$$

$$x\in(0,L),\quad i=1,\cdots,n, \tag{2}$$

with

$$D_{ijk}=\left(a_{ijk}+b_{ijk}s+c_{ijk}s^2\right). \tag{3}$$

Where the over-bar denotes the Laplace transformation, s is the complex variable of the Laplace domain and zero initial conditions have been assumed.

Equation (3) is cast into a state space form,

$$\frac{d}{dx}\eta(x,s)=F(s)\eta(x,s)+q(x,s),\qquad x\in(0,L). \tag{4}$$

In equation (4), $\eta(x,s)$ is so-called the state space vector.

2.3 Distributed Transfer Functions

Assume the boundary conditions of this component are given by the equation,

$$M\eta(0,s)+N\eta(L,s)=r(s). \tag{5}$$

In equation (5), M and N matrices are named as boundary selection matrices. Entries of these two matrices can be easily changed to assign different boundary conditions.

If the boundary value problem defined by equations (4) and (5) with $q(x,s)=0$ and $r(s)=0$ has only the null solution, then the solution of the state space vector can be given by the expression [3],

$$\eta(x,s)=\int_0^L G(x,\zeta,s)q(\zeta,s)d\zeta+H(x,s)r(s). \tag{6}$$

Where,

$$G(x,\zeta,s)=\begin{cases} e^{F(s)x}(M+Ne^{F(s)L})^{-1}Me^{-F(s)\zeta} & \zeta\leq x\\ -e^{F(s)x}(M+Ne^{F(s)L})^{-1}Ne^{F(s)(L-\zeta)} & \zeta\geq x\end{cases} \tag{7}$$

$$H(x,s)=e^{F(s)x}(M+Ne^{F(s)L})^{-1} \tag{8}$$

are called distributed transfer functions and $e^{F(s)x}$ is the fundamental matrix of the component.

2.4 Dynamic Stiffness Matrix of a Component

The state space vector $\eta(x,s)$ can be divided into two sub-vectors and expressed as,

$$\eta(x,s)=\left[\alpha^T(x,s)\quad \varepsilon^T(x,s)\right]^T. \tag{9}$$

Where,

$$\alpha(x,s)=\left[\alpha_1{}^T(x,s)\quad \alpha_2{}^T(x,s)\quad\cdots\quad \alpha_n{}^T(x,s)\right]^T, \tag{10}$$

is called displacement vector and,

$$\varepsilon(x,s)=\left[\varepsilon_1{}^T(x,s)\quad \varepsilon_2{}^T(x,s)\quad\cdots\quad \varepsilon_n{}^T(x,s)\right]^T, \tag{11}$$

is called strain vector.

Force vector at any point along the component can then be calculated and expressed as,

$$\sigma(x,s)=\bar{E}\varepsilon(x,s). \tag{12}$$

Where, $\bar{E}$ is a given constitutive matrix. Correspondingly, force vectors at two ends of the component can be calculated by equation (12) and expressed as

$$\begin{bmatrix}\sigma(0,s)\\ \sigma(L,s)\end{bmatrix}=\begin{bmatrix}\bar{E}H_{\sigma 0}(0,s) & \bar{E}H_{\sigma L}(0,s)\\ \bar{E}H_{\sigma 0}(L,s) & \bar{E}H_{\sigma L}(L,s)\end{bmatrix}\begin{bmatrix}\alpha(0,s)\\ \alpha(L,s)\end{bmatrix}+\begin{bmatrix}p(0,s)\\ p(L,s)\end{bmatrix}. \tag{13}$$

In equation (13), $\sigma(0,s)$ and $\sigma(L,s)$ are force vectors at two ends of the component, $\alpha(0,s)$ and $\alpha(L,s)$ are displacement vectors at two ends of the component.

$$\begin{bmatrix} \bar{E}H_{\sigma 0}(0,s) & \bar{E}H_{\sigma L}(0,s) \\ \bar{E}H_{\sigma 0}(L,s) & \bar{E}H_{\sigma L}(L,s) \end{bmatrix} \tag{14}$$

is called the dynamic stiffness matrix and all its sub-matrices can be calculated by using equation (8). $p(0,s)$ and $p(L,s)$ are force vectors transformed from distributed external forces.

2.5 Assembly of Components

Equation (13) gives force vectors at two nodes of a component with respect to corresponding displacement vectors. Consequently, dynamic stiffness matrices of all components can be systematically assembled together by using displacement compatibility and force balance at every connecting point to get,

$$\mathbf{K}(s) \times \mathbf{U}(s) = \mathbf{P}(s). \tag{15}$$

In equation (15), matrix $\mathbf{K}(s)$ is the dynamic stiffness matrix of the multi-components structure, $\mathbf{U}(s)$ the nodal displacement vector of the multi-components structure, and $\mathbf{P}(s)$ is the corresponding nodal force vector.

Equation (15) can be used to analyze modal frequencies, mode shapes, frequency responses, time domain responses, stresses, strains, buckling loads, etc.

2.6 Examples

Following is an example that demonstrates the calculation efficiency of DTFM.

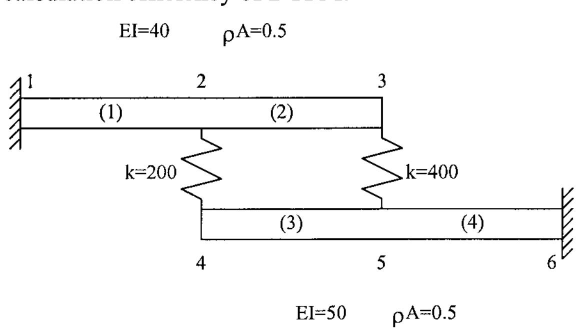


Figure 4. A system with two elastically coupled beams

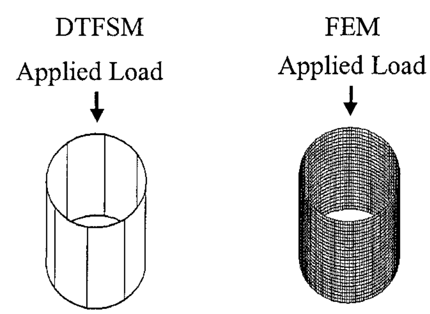
Figure 4 gives a system that is composed of two beams elastically coupled by two springs. Using DTFM, these two beams are decomposed into four components with six nodes as indicated in Figure 4. Due to the six given boundary conditions (displacements at node 1 and 6 are fixed, rotations at node 1 and 6 are fixed, force moments on node 3 and 4 are zero), this system is represented by six unknowns (displacements at nodes 2, 3, 4, 5, and rotations at nodes 2 and 5). As a result, DTFM only deals with a six by six matrix to get the closed form solutions for this system.

Table 1. Resonant frequencies of the two elastically coupled beam system

Mode number	DTFM 6*6 matrix	FEM 18 Elements	FEM 34 Elements	FEM 66 Elements
1	16.3	16.3	16.3	16.3
2	41.0	41.1	41.0	41.0
3	54.6	53.1	54.2	54.5
4	79.2	77.8	78.9	79.1
5	144.7	138.3	143.1	144.3
6	157.0	150.5	155.4	156.6
7	273.9	258.1	269.9	272.9
8	305.2	288.2	289.9	304.1
9	448.7	415.4	440.4	446.6
10	500.5	463.9	491.2	498.1
11	669.1	601.7	653.7	665.3
12	747.5	672.7	730.5	743.3

Table 1 gives first twelve resonant frequencies of this system calculated by both DTFM and FEM. The second column gives frequencies calculated by DTFM. The third, forth, and fifth columns give frequencies calculated by FEM with 18 elements, 34 elements, and 66 elements correspondingly. By comparing results of the twelfth frequency, one can see that at least 66 elements are needed by FEM to get a reasonable result.

Instead of one-dimensional components, DTFM can be further developed to Distributed Transfer Function Strip Method (DTFSM) [6] to address two-dimensional components with semi-exact solutions. An example is given here without going through all the derivations to demonstrate the computational efficiency of DTFSM.



Strip Number	Buckling Force
2	381.28
4	381.28
6	381.28
8	381.28

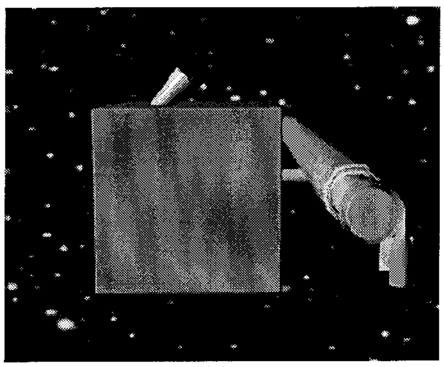
Element Number	Buckling Force
54	1269.6
218	393.3
864	386.8
3456	381.4

Figure 5. DTFSM modeling versus FEM

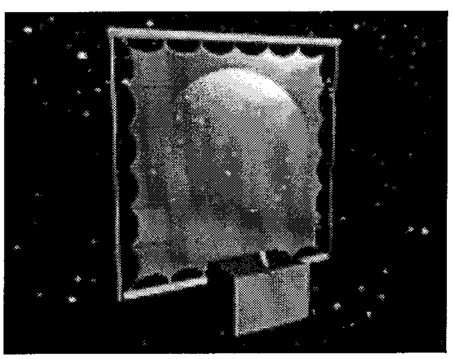
Figure 5 shows the buckling analysis of a thin-walled cylinder using both DTFSM and FEM. From figure 5 one can see that the DTFSM with only two strips yielded very accurate result. However, FEM required at least 864 elements to get a reasonable result and 3456 elements to get an accuracy result.

3. Optimization for Gossamer Structures

Because the DTFM can mathematically describe distributed damping and dampers [7] with high computational efficiency, it can be employed to perform optimization studies in the sense of engineering. This section gives an example.



(a)



(b)

Figure 6. Three-meters inflatable reflectarray antenna

Figure 6 shows a three-meters inflatable reflectarray antenna. Figure 6(a) is the antenna in stowed status and 6(b) is the antenna in deployed status. The supporting structure of the antenna is composed of two inflatable/self-rigidizable booms and two rigid beams. A single layer RF membrane is attached to the supporting structure. Inflatable booms can be flattened and both inflatable booms and the membrane can be rolled-up onto one of the rigid beams [8]. However, the structure of the antenna is relatively large and flimsy. The dynamic response of the antenna to the excitation induced by spacecraft maneuvering is a big challenge.

One of the tasks of this project is to embed damping materials along distributed components and install dampers at several places to reduce the magnitude of the response. In order to minimize the mass penalty of damping material and maximize the damping efficiency, where to place damping material and how much damping material should be used is an optimization problem. Even though it is not impossible, to use a commercial FE software to conduct this study is not an easy job. DTFM is perfectly suitable for this study. Several possible ways of adding damping materials will be proposed. The response reduction of each way as a function of frequency will be analyzed using DTFM. A decision will then be made based on these analysis results as well as weights of damping material.

4. Sensitivity Analyses of Gossamer Structures with DTFM

4.1 The Process of the Sensitivity Analysis

Basic building blocks of most gossamer structures are long booms. The sensitivity of a boom to the material and geometrical imperfections is always a challenging problem to engineers. Figure 7 is the buckling test set up of a five-meter long Spring Tape Reinforced Aluminum Laminate (STRAL) Boom.

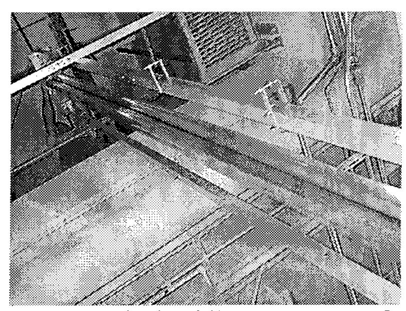


Figure 7. The buckling test scene of an inflatable/rigidizable boom

The STRAL boom is an ultra-light weight and heavy-duty inflatable/self-rigidizable boom [9]. It can withstand 167 lbs of axial buckling load with simply supported boundary conditions on both ends. The weight of the boom is only 2 lbs. The boom is rigidizable, that means pressure is not required to keep its rigidity after it is inflation deployed. The boom is also self-rigidizable, that means the boom does not need any space power or mean to get the rigidity after it is inflated. However, during a series of buckling tests, it is found that the buckling load varied from 118 lbs to 167 lbs. Because the failure mode is Eular buckle, the variation is believed to be caused by eccentricity of the boom centerline and the deviations of bending stiffness along the boom.

In order to investigate the sensitivity of the buckling load to the eccentricity of the boom centerline and the deviation of bending stiffness, a DTFM based analysis process is being developed and presented as following.

Buckling analysis of a boom can be described by the differential equation,

$$\frac{d^2}{dx^2}\left(EI\frac{d^2}{dx^2}w(x)\right)+P\frac{d^2}{dx^2}w(x)=0. \quad (15)$$

Equation (15) belongs to the category defined by equation (1). However, due to the reason that the bending stiffness (EI) is not a constant along the boom, equation (6) cannot be directly employed. There are several ways to handle a non-uniformly distributed component with DTFM. Stepwise uniform is the most efficiency way [4] and is used by this study.

A non-uniform distributed component is first divided into a number of tiny sections and each section is considered to be uniform. Assuming sections S_k and S_{k+1} are interconnected at point x_k, state space vector on section S_k at point x_k is given as,

$$\eta_k(x)=\begin{pmatrix}u_k(x)\\ \varepsilon_k(x)\end{pmatrix}, \quad (16)$$

where $u_k(x)$ is the displacement vector and $\varepsilon_k(x,s)$ is the strain vector. The corresponding force vector can be given as

$$\sigma_k(x)=E_k(x)\varepsilon_k(x). \quad (17)$$

Considering the force balance and displacement compatibility at point x_k, the state space vector on section S_{k+1} at point x_k can be calculated as,

$$\eta_{k+1}(x_k)=T_k(x)\eta_k(x_k). \quad (18)$$

Where,

$$T_k=\begin{bmatrix}I & 0\\ 0 & E_{k+1}^{-1}E_k\end{bmatrix}\in C^{n\times n}. \quad (19)$$

In equation (19), I matrix is an identity matrix.

On the other hand, equations (7) and (8) can be rewritten as,

$$G(x,\xi)=\begin{cases}H(x)M\Phi^{-1}(\xi), & \xi<x\\ -H(x)N\Phi(L)\Phi^{-1}(\xi), & \xi>x\end{cases}, \quad (20)$$

and $$H(x)=\Phi(x)(M+N\Phi(L))^{-1}. \quad (21)$$

In equations (20) and (21), the fundamental matrix $\Phi(x)$ can be approximately expressed as [4],

$$\Phi(x)\approx\hat{\Phi}(x)=e^{F_{k+1}(x-x_k)}T_k e^{F_k(x_k-x_{k-1})}\ldots T_2 e^{F_2(x_2-x_1)}T_1 e^{F_1(x_1)},$$
$$x\in(x_k,x_{k+1}). \quad (22)$$

4.2 Example

The sensitivity analysis of a boom to the deviation of bending stiffness (EI) has been conducted by this example. The length of the inflatable/self-rigidizable STRAL Boom in Figure 7 is 197 inches. It is obtained from previous analysis and test that the original bending stiffness (EI_0) of the boom is 656673 lb*in^2. In order to investigate the impact caused by the deviation of bending stiffness, it is assumed by this example that the bending stiffness of the boom is expressed as,

$$EI=EI_0(1+\varepsilon\times\sin(\frac{x\pi}{L})), \quad (23)$$

where L is the length of the boom. DTFM is able to handle any kind of bending stiffness deviations, even it is a localized deviation.

The state space vector of this example is defined as,

$$\eta(x)=(w(x)\quad w'(x)\quad w''(x)\quad w'''(x))^T. \quad (24)$$

Where w is the deflection of the boom. Based on equation (15), the F matrix of equation (4) can be derived as,

$$F=\begin{bmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\0&0&-\frac{P}{EI(x)}&0\end{bmatrix}. \quad (25)$$

The boundary conditions of the boom are simply supported at both ends. That means on each end, both deflection and bending moment equal to zero,

$$w=0,\ EI\frac{\partial^2 w}{\partial x^2}=0. \quad (26)$$

Which is cast into,

$$M\eta(0)+N\eta(L)=0, \quad (27)$$

with the boundary matrices,

$$M=\begin{bmatrix}1&0&0&0\\0&0&1&0\\0&0&0&0\\0&0&0&0\end{bmatrix},\quad N=\begin{bmatrix}0&0&0&0\\0&0&0&0\\1&0&0&0\\0&0&1&0\end{bmatrix}. \quad (28)$$

The solution of the beam is given by,

$$\eta(x)=\Phi(x;P,\varepsilon)\eta(0). \quad (29)$$

Where $\Phi(x;P,\varepsilon)$ indicates that the fundamental matrix is a function of the axial load P and stiffness variation ε. The $\Phi(x;P,\varepsilon)$ can be obtained by the method given in (22). Substitution of (29) into (27) gives,

$$(M + N\Phi(L;P,\varepsilon))\eta(0) = 0 . \qquad (30)$$

In order for the above homogeneous equation to have a non-trivial solution, we must have,

$$\det(M + N\Phi(L;P,\varepsilon)) = 0 . \qquad (31)$$

The smallest root of the characteristic equation (31) is the critical force P_{cr} (buckling load) of the non-uniform beam.

Table 2. Bucklin force as the function of bending stiffness deviation ε

ε	0%	± 2%	± 4%	± 6%	± 8%	± 10%
Pcr (+ %)	167.0	169.7	172.7	175.4	178.2	181.1
Pcr (- %)	167.0	164.2	161.2	158.5	155.6	152.8

Table 3. The rations of buckling force changing as the function of bending stiffness deviation ε

ε	0%	± 2%	± 4%	± 6%	± 8%	± 10%
Pcr/Pcr_0	1.0000	1.017	1.034	1.051	1.067	1.085
Pcr/Pcr_0	1.0000	0.983	0.966	0.949	0.932	0.915

Table 2 gives the buckling load as a function of ε. Table 3 gives the rations of buckling force changing as the function of bending stiffness deviation ε. From table 3 one can get following conclusions:

- o Buckling force is almost a linear function of the bending stiffness deviation ε.
- o The percentage changing of buckling force is less than the percentage of bending stiffness deviation ε. For example, buckling force changes 8.5% while ε changes 10%.

5. Future Research Directions

The DTFM-based analysis and its applications have been presented. It is shown that this method is suitable for design and development of space gossamer structures. Some problems in modeling and analysis of gossamer structure that are difficult, or even impossible, to be treated by the FEM can be readily treated by the DTFM. Because the DTFM uses analytical solutions for components or substructures, it can obtain accurate results with very high computational efficiency.

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Instead of many advantages of the DTFM, much research and development effort is still needed to make the DTFM a useful tool for gossamer structures. Future research directions for the DTFM include:

- o To conduct buckling force sensitivity analysis of an inflatable boom with respect to the eccentricity of the centerline;
- o To further develop two-dimensional components for investigation of the sensitivity of an inflatable boom to surface imperfections and material imperfections;
- o To develop pre-tensioned two-dimensional membrane components;
- o To develop the thermal analysis capability for gossamer structures;
- o To investigate the dynamics of spinning space structures such as solar sails by the DTFM; and
- o To combine all the capabilities in a user-friendly package for future applications.

6. Acknowledgements

The work described was performed at Jet Propulsion Laboratory, California Institute of Technology under contract with the National Aeronautics and Space Administration of United States.

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