AUTONOMOUS PATH-PLANNING FOR FORMATION-FLYING APPLICATIONS

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ABSTRACT *- In several formation-flying control applications the inter-spacecraft separations can range from a few meters to several kilometers. It is mission critical to avoid collisions between spacecraft as they move in space. We present a solution to the optimal formation path-planning problem where the formation reconfigurations are required subject to collision avoidance and resource limitation constraints. The problem is formulated as a parameter optimization problem where spacecraft paths are parameterized as splines. An iterative algorithm to solve the problem on-board is proposed. The solution to the problem results in realizable trajectories which avoid collisions between constituent spacecraft*

KEYWORDS: Formation flying, collision-avoidance, path-planning.

1. INTRODUCTION

Several future space science missions, e.g. the Terrestrial Planet Finder (TPF), the Terrestrial Planet Imager (TPI), Starlight missions, involve coordinated flying of multiple spacecraft platforms. The purpose of formation flying interferometry is to form a variable-baseline space interferometer whose baseline may be varied from a few meters to several kilometers. It is mission critical to avoid collisions between spacecraft as they move in space, especially when they are required to be in close proximity of one another. Mission profiles require several formation reconfigurations over the life of the mission. By a formation reconfiguration we mean a change in relative spacecraft *position* vectors. The problem being addressed here is that of autonomous formation reconfiguration planning subject to some optimality criteria and collision avoidance constraints.

The problem of collision avoidance between collaborative systems has been the subject of extensive research in the field of robotics. Several published works [1-5] have addressed the problem of robot path planning in workspace environments. Almost all of these have proposed solutions that make use of artificial potential functions. Application of potential-function based methods is a very effective and powerful technique for handling collision avoidance constraints, which has also been generalized to spacecraft applications [7, 8]. Shan and Koren [6] take an obstacle *accommodation* approach to the problem. Rather than avoid physical contacts between moving objects, their approach controls relative velocities to avoid damage from contact. Some of the formation flying specific research [8-11] considers formation reconfiguration problem but not in the context it is proposed in here. Specifically, the collision avoidance constraints during formation reconfigurations have been ignored in literature published thus far. While these methods are analytically rigorous and are also attractive from an implementation point of view, the collision avoidance for formation flying interferometry applications will need to satisfy additional and more stringent requirements beyond the scope of the work published so far. For example, the collision avoidance constraints must be satisfied *exactly* at all times, the convergence to the desired end-point must not be too slow, which is the case with potential-function based methodologies, and the accelerations required to follow the desired paths must be within the capabilities of the actuation hardware.

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2. PROBLEM FORMULATION

The optimal collision-avoidance problem is formulated as a parameter optimization problem. The translation paths taken by each spacecraft are parameterized as polynomial functions of time. The parameterization is constrained such that the feasible paths satisfy the appropriate boundary conditions. Subsequently we propose an iterative algorithm to solve the resulting parameter optimization problem whose size is proportional to the number of spacecraft in the formation. We show that the solution approach can be made to terminate in a fixed, user-specified number of iterations for the relatively simple case of a two spacecraft formation (e.g. Starlight). The maneuver duration is treated as another optimization parameter. It is computed such that the control required to follow the 'optimal' path does not exceed the linear acceleration capability of any spacecraft in the formation. The optimal path-planning problem does not require a solution in real-time. The proposed algorithm can be programmed to execute a little ahead of time, in anticipation of the impending reconfiguration maneuver.

A spacecraft in the formation is assigned the role of the *reference* spacecraft. This is the spacecraft where formation reconfiguration path planning will actually take place in a real system. Any spacecraft in the formation may serve this role. The positions of all other spacecraft in the formation will be defined with respect to the reference spacecraft. There are a total of N spacecraft in the entire formation and the reference spacecraft will be referred to as the spacecraft N. For the purpose of prescribing collision avoidance constraints, we shall define an exclusion sphere of radius R_k about the spacecraft k. Enforcement of the collision avoidance constraint would require that any two exclusion spheres do not intersect (a point of contact is allowed, however). Let x_k denote the linear position vector of spacecraft k with respect to the reference spacecraft in inertial coordinates (Fig. 1), $k = 1, 2, ..., N-1$; v_k be the time derivative of x_k, $k = 1, 2, .., N-1$; and a_k denote the *absolute* linear acceleration of spacecraft k, $k = 1, 2, .., N$. By absolute acceleration we mean the acceleration with respect to some inertial frame. Note that a_k is not the time derivative of v_k, rather it is the sum of the time derivative of v_k and a_N, the absolute linear acceleration of the reference spacecraft.

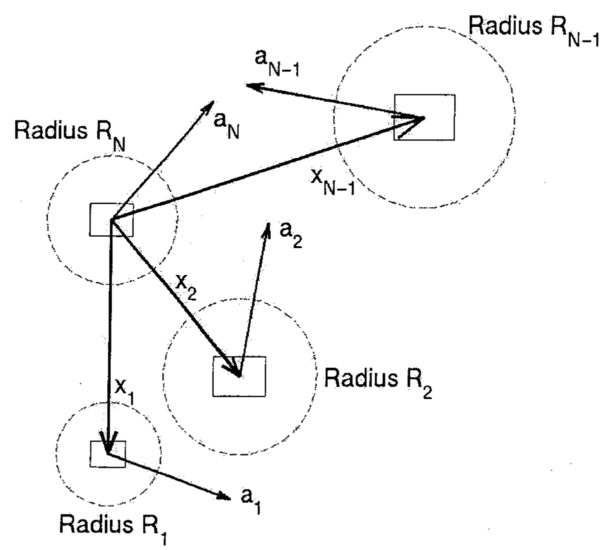

Fig. 1. Formation of N Spacecraft

Furthermore let T denote the reconfiguration maneuver time, a '$\otimes$' the vector cross product operator, and a '•' the vector scalar product operator. The following assumptions are made:

i) The formation reconfiguration maneuvers are of the rest-to-rest variety, i.e. $v_k(t) = 0$, $t = 0, T$, $k = 1, 2, .., N-1$. Almost all formation reconfigurations belong to this class. The one exception is synchronized formation rotation. Avoiding collisions is not a concern in this case since the specific maneuver places additional constraints on the motion, which preclude collisions.

ii) The natural orbital perturbations on system relative equations of motion are small enough to be ignored. This is a realistic assumption for the deep-space formation flying applications. Orbital dynamics induced relative motion accelerations are several orders of magnitude below the path accelerations during formation reconfigurations.

iii) Spacecraft rotational degrees of freedom are ignored. This is not restrictive from a practical application standpoint. It simply requires that either a prescribed fraction of the total acceleration capability be used for collision avoidance path-planning (the balance reserved for attitude planning) or that a momentum exchange device be used for attitude control.

2.1. The Problem Statement

The formation equations of motion may be stated as:

$$\dot{x}_k = v_k, \qquad k = 1, 2, .., N\text{-}1, \tag{1}$$

$$\dot{v}_k = a_k - a_N, \qquad k = 1, 2, .., N\text{-}1, \tag{2}$$

$$x_k(0) = x_k^0,\ x_k(T) = x_k^T, \qquad k = 1, 2, .., N\text{-}1, \tag{3}$$

$$v_k(0) = v_k(T) = 0, \qquad k = 1, 2, .., N\text{-}1. \tag{4}$$

Note that the maneuver duration T is not specified. The problem is to find suitable accelerations $a_k(t),\ t \in [0, T]$, $k = 1, 2, .., N$, such that the sum of total energy, i.e.

$$J = \frac{1}{2} \sum_{k=1}^{N} \int_0^T (a_k \cdot a_k)\, dt, \tag{5}$$

is minimized while satisfying appropriate collision avoidance and control authority constraints, i.e.

$$\{x_k(t) - x_j(t)\} \cdot \{x_k(t) - x_j(t)\} \geq (R_k + R_j)^2, \qquad (k, j) = 1, 2, .., N\text{-}1;\ k \neq j,\ t \in [0, T], \tag{6}$$

$$\{x_k(t)\} \cdot \{x_k(t)\} \geq (R_k + R_N)^2, \qquad k = 1, 2, .., N\text{-}1; \quad t \in [0, T], \tag{7}$$

$$|a_{k_j}(t)| \leq A_{k_j}, \qquad j = x, y, z, \qquad k = 1, 2, .., N; \quad t \in [0, T], \tag{8}$$

The last constraint (eqn.(8)) may take other forms, e.g. a constraint may be imposed on the 2-norm of the acceleration vectors. Although the solution approach can handle a general acceleration bound, we shall, for the sake of problem definition, assume a 'box' type bound noted in (8).

Defining a dimensionless time variable $\xi \triangleq t/T$ and using $(.)'$ to denote a differentiation with respect to ξ, allows us to express (1-5) as follows:

$$x_k' = T v_k, \qquad k = 1, 2, .., N\text{-}1, \tag{9}$$

$$v_k' = T(a_k - a_N), \qquad k = 1, 2, .., N\text{-}1, \tag{10}$$

$$x_k(0) = x_k^0,\ x_k(1) = x_k^T, \qquad k = 1, 2, .., N\text{-}1, \tag{11}$$

$$v_k(0) = v_k(1) = 0, \qquad k = 1, 2, .., N\text{-}1, \tag{12}$$

$$J = \frac{1}{2} T \sum_{k=1}^{N} \int_0^1 (a_k \cdot a_k)\, d\xi. \tag{13}$$

By treating the maneuver duration T as another parameter it becomes possible to enforce (8) *aposteriori*. This results in a significant simplification. We shall initially ignore (8) in computing appropriate collision-avoidance paths $x(\xi)$, $\xi \in [0, 1]$, and, later, choose an appropriate T to enforce (8). In order to do so we shall exploit the following relationship, which follows from (9), (10):

$$a_k(\xi) = a_N(\xi) + x''_k(\xi)/T^2. \tag{14}$$

Optimal accelerations minimize the Hamiltonian:

$$H = T\Big[\frac{1}{2}\sum_{k=1}^{N} \| a_k \|^2 + \sum_{k=1}^{N-1}\{p_k \cdot v_k + q_k \cdot (a_k - a_N)\} + \frac{1}{2}\sum_{k=1}^{N-1}\Big\langle \lambda_k\{\| x_k \|^2 - (R_k+R_N)^2\} + \sum_{\substack{j=1 \\ j \neq k}}^{N-1} \lambda_{kj}\{\| x_k - x_j \|^2 - (R_k+R_j)^2\}\Big\rangle\Big] \tag{15}$$

where $p_k(\xi)$, $q_k(\xi)$ are co-states associated with $x_k(\xi)$ and $v_k(\xi)$, respectively, and λ's are the Lagrange multipliers associated with the collision avoidance constraints. Minimization of the Hamiltonian leads to:

$$a_N(\xi) = \sum_{k=1}^{N-1} q_k(\xi), \tag{16}$$

$$a_k(\xi) = -q_k(\xi), \quad k = 1, 2, .., N\text{-}1 \tag{17}$$

Derivation of Euler-Lagrange equations is straightforward; we obtain for $k = 1, 2, .., N\text{-}1$:

$$p'_k(\xi) = -\lambda_k(\xi)\, x_k(\xi) - \sum_{\substack{j=1 \\ j \neq k}}^{N-1} [\lambda_{kj}(\xi)\{x_k(\xi) - x_j(\xi)\}], \tag{18}$$

$$q'_k(\xi) = -p_k(\xi), \tag{19}$$

$$\lambda_k(\xi) = 0, \text{ when } \| x_k(\xi) \|^2 \geq (R_k+R_N)^2, \tag{20}$$

$$\lambda_{kj}(\xi) = 0, \text{ when } \| x_k(\xi) - x_j(\xi) \|^2 \geq (R_k+R_j)^2. \tag{21}$$

Equations (9-12), (6-7), and (16-21) form the necessary and sufficient conditions for optimality for the sub-problem where there are no control authority limitations. Elimination of the $q_k(\xi)$ between (16) and (17) yields:

$$\sum_{k=1}^{N} a_k(\xi) = 0, \ \xi \in [0, 1], \tag{22}$$

which implies that, for an optimal maneuver, the net formation acceleration must be zero at all times. Using (14) and (22), optimal acceleration required at each spacecraft can be expressed as the following summations:

$$a_k(\xi) = \frac{1}{T^2}\left[\frac{N-1}{N} x''_k(\xi) - \frac{1}{N}\sum_{\substack{j=1 \\ j \neq k}}^{N-1} x''_j(\xi)\right], \ \xi \in [0, 1], \ k = 1, 2, .., N\text{-}1, \tag{23}$$

$$a_N(\xi) = \frac{1}{T^2}\left[-\frac{1}{N}\sum_{k=1}^{N-1} x''_k(\xi)\right], \ \xi \in [0, 1]. \tag{24}$$

2.2. Solution Approach

Without loss of generality, we may express the evolution of $x_k(\xi)$ as:

$$x_k(\xi) \;=\; b_{k0}(\xi)\, x_k^0 \;+\; b_{k1}(\xi)\, x_k^T \;+\; b_{k3}(\xi)\, \{x_k^0 \otimes x_k^T\}, \quad \xi \in [0, 1], \; k = 1, 2, .., N\text{-}1, \tag{25}$$

where $b_{ki}(\xi)$, i = 0, 1, 2, are continuously differentiable scalar functions of ξ. In instances where the cross product direction is not defined, it is still possible to prescribe a similar evolution of relative positions using other basis vectors. Satisfaction of system boundary conditions (11, 12) imposes the following constraints on the boundary values of the functions $b_{ki}(\xi),\ i = 0, 1, 2;\ k = 1, 2, .., N\text{-}1;\ \xi \in [0, 1]$:

$$b_{k0}(0) = 1, \quad b_{k0}(1) = 0, \quad b'_{k0}(0) = 0, \quad b'_{k0}(1) = 0, \tag{26}$$

$$b_{k1}(0) = 1, \quad b_{k1}(1) = 1, \quad b'_{k1}(0) = 0, \quad b'_{k1}(1) = 0, \tag{27}$$

$$b_{k2}(0) = 1, \quad b_{k2}(1) = 0, \quad b'_{k2}(0) = 0, \quad b'_{k2}(1) = 0. \tag{28}$$

Representative graphs of these functions (subsequently referred to as the Path Functions) are shown in Fig. 2.

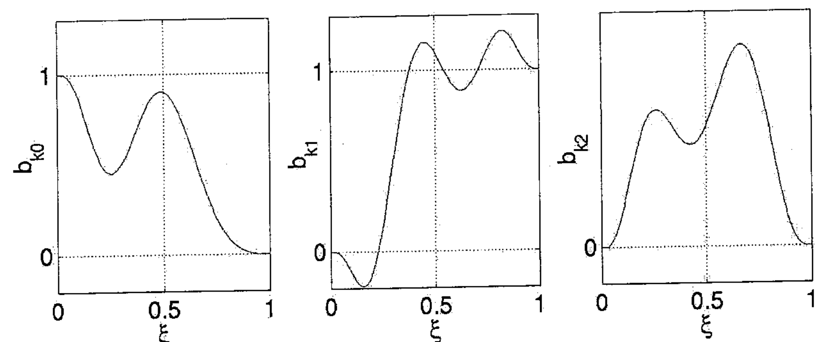

Fig. 2. Representative Graphs of Path Functions

Representation (25-28) defines a feasible path, i.e. a path which satisfies only the boundary conditions. It does not restrict the optimal paths in any way, and is therefore equivalent to the most general representation possible. It is trivial to place additional constraints on optimal paths, e.g. it may be desirable to further restrict optimal paths such that they lie in the plane spanned by the two end points. This is accomplished by setting $b_{k2}(\xi) \equiv 0$.

The optimization problem can now be re-stated as the problem of determining the optimal set $b_k \triangleq \{b_{k0}(\xi), b_{k1}(\xi), b_{k2}(\xi)\},\ k = 1, 2, .., N\text{-}1;\ \xi \in [0, 1]$. Several parameterization of b_k are possible. We make the following choice:

$$b_{ki}(\xi) \;=\; \sum_{j=0}^{n} c_{kij}\, \xi^j, \quad \xi \in [0, 1], \tag{29}$$

where $c_{kij},\ i = 0, 1, 2;\ j = 0, 1, .., n;\ k = 1, 2, .., N\text{-}1,$ are some undetermined coefficients. This specific choice is based primarily on the observation that the optimal solution of the constraint-free optimal path-planning problem belongs to the class of solutions (29) where $c_{kij},\ i = 0, 1, 2;\ k = 1, 2, .., N\text{-}1;\ j \geq 4$. Enforcement of boundary conditions (11, 12) results in the following expressions for the elements of set b_k :

$$b_{k0}(\xi) = 1 - 3\,\xi^2 + 2\,\xi^3 + \sum_{j=4}^{n} [\{(j\text{-}3)\xi^2 - (j\text{-}2)\xi^3 + \xi^j\}\, c_{k0j}],\ k = 1, 2, .., N\text{-}1;\ \ \xi \in [0, 1], \tag{30}$$

$$b_{k1}(\xi) = 3\,\xi^2 - 2\,\xi^3 + \sum_{j=4}^{n} [\{(j-3)\xi^2 - (j-2)\xi^3 + \xi^j\}\, c_{k1j}],\ k = 1, 2, .., N\text{-}1;\ \ \xi \in [0, 1], \qquad (31)$$

$$b_{k2}(\xi) = \sum_{j=4}^{n} [\{(j-3)\xi^2 - (j-2)\xi^3 + \xi^j\}\, c_{k2j}],\ k = 1, 2, .., N\text{-}1;\ \ \xi \in [0, 1], \qquad (32)$$

In general for an n^{th} order expansion (29), there will exist $3(N\text{-}1)(n\text{-}3)$ undetermined coefficients. The number of collision-avoidance constraints to be satisfied are $N(N\text{-}1)/2$. An over-parameterized system would require that n, the order of expansion in (29), be greater than $3+N/6$. For example, a 6^{th} order expansion for Path Functions leads to an over-parameterized system when $N < 18$, i.e. formations of at most 17 spacecraft. The solution methodology proposed here is not a function of n. However, the number of computations required to reach a solution is dependent on n.

As noted earlier, it can be shown that the solution to the (trivial) problem of obtaining minimum energy trajectories where collision avoidance consideration is absent, is given by (30-32) where $c_{kij},\ i = 0, 1, 2; j \geq 4.$ The optimal path in this case is a cubic (spline) function of time:

$$x_k(\xi) = (1 - 3\,\xi^2 + 2\,\xi^3)\, x_k^0 + (3\,\xi^2 - 2\,\xi^3)\, x_k^T\ ,\ k = 1, 2, .., N\text{-}1;\ \ \xi \in [0, 1], \qquad (33)$$

The optimal solution to the constraint-free optimal path-planning problem belongs to the class of solutions being considered here. It is also obvious that consideration of the collision avoidance constraints would require inclusion of at least one more term in the power series (29).

2.3. Numerical Algorithm

Let c denote the set of undetermined coefficients:

$$c = \{c_k\},\ k = 1, 2, .., N\text{-}1, \qquad (34)$$

where $c_k = \{c_{k04}, c_{k14}, c_{k24}, c_{k05}, c_{k15}, c_{k25}, .., c_{k0n}, c_{k1n}, c_{k2n}\}$. The optimal collision avoidance path-planning problem has been reduced to the problem of determination of an appropriate c which minimizes J/T (where J is given by (13)) while satisfying (6) and (7). Rather than minimizing J, we minimize J/T so that the dependency of the initial solution on maneuver duration is completely removed. A suitable value for T will be chosen later on so that (8) is also satisfied. The resulting problem is a non-convex optimization problem (convex cost, non-convex constraints). The solution approach proposed here is numerical. Although it is difficult, if not impossible, to make any claims about the convergence and nature of solutions to such problems, the proposed approach is based on arguments induced by geometry and has not been found to fail yet in our applications. Define the minimum separation between the two spacecraft as follows:

$$d_{kj} = \underset{\xi \in [0,1]}{Minimum} \{\| x_k(\xi) - x_j(\xi) \|_2\},\ \ (k, j) = 1, 2, .., N\text{-}1; j \neq k, \qquad (35)$$

$$d_{kN} = \underset{\xi \in [0,1]}{Minimum} \{\| x_k(\xi) \|_2\},\ \ k = 1, 2, .., N\text{-}1, \qquad (36)$$

and the gradients of the cost J and minimum separations d_{kj} with respect to c:

$$\nabla J = \frac{1}{T}\frac{\partial J}{\partial c}, \qquad (37)$$

$$\nabla d_{kj} = \frac{\partial d_{kj}}{\partial c},\ \ k = 1, 2, .., N\text{-}1; j = 1, 2, .., N; j \neq k. \qquad (38)$$

Details of the numerical algorithm are presented next. We initialize $c = \{0\}$ and proceed as follows:

Step 1: Compute minimum separations d_{kj}

Exit if all d_{kj} are larger than the minimum required, i.e. exit if:

$$d_{kj}^2 \geq (R_k + R_j)^2, \quad (k,j) = 1, 2, .., N\text{-}1; j = 1, 2, .., N; j \neq k, \tag{39}$$

else proceed to *Step 2.*

Step 2: Evaluate gradients

Numerically evaluate gradients ∇J, ∇d_{kj} at c. J and x_k can be expressed in closed forms (functions of c). Evaluation of gradients requires discrete approximations.

Step 3: Determine direction σ in which the vector c must be perturbed

Using geometry-based arguments, a most obvious direction is the one which most nearly lies in the plane orthogonal to ∇J and also yields the maximum possible change in separations. Since there may be several inter-spacecraft separations requiring an improvement, a weighted linear combination of the gradients of all offending d_{kj}'s is formed. The weights used are simply $(R_k + R_j) - d_{kj}$, in other words, the extent of violation. Other weights might also be used here. The resulting gradient is therefore:

$$\nabla d = \sum \{ (R_k + R_j) - d_{kj} \} \nabla d_{kj}, \tag{40}$$

where only the offending ∇d_{kj}'s figure in the sum. The appropriate direction σ is required to satisfy the following conditions:

$$\nabla J \bullet \sigma \leq 0, \tag{41}$$

$$\nabla d \bullet \sigma > 0. \tag{42}$$

A solution to (41, 42) can be expressed as:

$$\sigma = \nabla d - \frac{(\nabla J \bullet \nabla d)}{(\nabla J \bullet \nabla J)} \nabla J. \tag{43}$$

In instances where ∇J and ∇d are nearly collinear, we choose a direction which perturbs c along ∇J.

Step 4: Update c

Update c along σ:

$$c^+ = c^- + \left\{ \frac{\delta d}{\hat{\sigma} \bullet \nabla d} \right\} \hat{\sigma}, \tag{44}$$

where $\hat{\sigma}$ is the unit vector along σ, and δd, the required improvement along ∇d, is a user-specified parameter. The update equation (44) is motivated by the two spacecraft case, where there is only one collision-avoidance constraint. Let d_2 be the minimum inter-spacecraft separation in this case. Therefore, if c is perturbed by $\rho\,\hat{\sigma}$, where ρ is some scalar and we would like to effect a change δd in d_2, then the change in d_2 can be expressed as: $\rho\,\hat{\sigma} \bullet \nabla d \approx \delta d$, which leads to: $\rho = \delta d / (\hat{\sigma} \bullet \nabla d)$, the coefficient of $\hat{\sigma}$ in (44).

Step 5: Return

Set $c = c^+$, and return to *Step 1.*

Once a solution has been obtained, we turn our attention to the satisfaction of (8), the constraints which place limitations on individual spacecraft acceleration components. Numerical solution to the problem also yields the following evaluations:

$$\alpha_{ki} = \underset{\xi \in [0,1]}{Maximum} \left[a_{ki}(\xi) = \frac{1}{T^2}\left[\frac{N-1}{N} x''_{ki}(\xi) - \frac{1}{N} \sum_{\substack{j=1 \\ j \neq k}}^{N-1} x''_{ji}(\xi) \right] \right], \quad i = x, y, z; \; k = 1, 2, .., N-1,$$

$$\alpha_{Ni} = \underset{\xi \in [0,1]}{Maximum} \left[a_{Ni}(\xi) = \frac{1}{T^2}\left[-\frac{1}{N} \sum_{k=1}^{N-1} x''_{ki}(\xi) \right] \right], \quad i = x, y, z .$$

It is trivial to choose an appropriate maneuver duration T such that none of the acceleration components exceed it's prescribed limit, i.e.

$$T = \sqrt{\underset{\substack{(i = x, y, z) \\ (k = 1,2,..,N)}}{Maximum} \left[\frac{\alpha_{ki}}{A_{ki}} \right]}$$

Once the undetermined coefficient set c has been obtained, the 'optimal' solution is trivial to implement. It requires substituting the numerically derived coefficient set c in (29) to obtain the desired Path Functions b_k. Path Functions are then substituted in (25) to obtain the relative position as an explicit function of time, which can be analytically differentiated once to obtain the required relative velocities and twice to obtain relative accelerations.

3. NUMERICAL EXAMPLE

Consider the case of a formation with 5 spacecraft. There are 10 collision avoidance constraints in this case. We shall assume the same avoidance radius for all spacecraft: $R_k = 10$ m, $k = 1, 2, .., 5$. This would require that all spacecraft-spacecraft separations be greater than 20 m. The boundary conditions to be satisfied in this instance are:

$$\begin{aligned}
x_1(0) &= \{ 22.254, \; -36.706, \; 17.052 \} \; m, \\
x_2(0) &= \{ 22.254, \; -14.518, \; 2.261 \} \; m, \\
x_3(0) &= \{ 22.254, \; 7.670, \; -12.532 \} \; m, \\
x_4(0) &= \{ 22.254, \; 29.858, \; -27.324 \} \; m,
\end{aligned}$$

$$\begin{aligned}
x_1(T) &= \{ -38.545, \; -13.285, \; -21.705 \} \; m, \\
x_2(T) &= \{ -25.697, \; 3.953, \; -5.930 \} \; m, \\
x_3(T) &= \{ -12.848, \; 21.192, \; 9.845 \} \; m, \\
x_4(T) &= \{ 0.000, \; 38.431, \; 25.621 \} \; m,
\end{aligned}$$

and the appropriate acceleration limits to be observed in this case are:

$$\begin{aligned}
A_k &= \{ 0.005, \; 0.004, \; 0.003 \} \; m/s^2, \quad k = 1, 2, 3, 4, \\
A_5 &= \{ 0.004, \; 0.003, \; 0.005 \} \; m/s^2.
\end{aligned}$$

All initial position vectors lie in a plane, as do all terminal positions. The plane of initial positions is different from the plane spanned by the terminal positions.

Using a 4^{th} order expansion (i.e. $n = 4$) for paths, we obtain the following solutions for the undetermined coefficients:

$$\{c_{104}, c_{114}, c_{124}\} = \{3.7976, \quad 4.0480, \quad 0.6680\},$$
$$\{c_{204}, c_{214}, c_{224}\} = \{19.448, \quad 19.394, \quad 2.6337\},$$
$$\{c_{304}, c_{314}, c_{324}\} = \{3.0534, \quad 2.0985, \quad -2.3686\},$$
$$\{c_{404}, c_{414}, c_{424}\} = \{1.3019, \quad 0.6078, \quad -0.4503\}.$$

Since the final formation plane is different from the initial plane, out of plane motions are required (the 3^{rd} coefficient in each set above is non-zero). This will not be the case if the final formation lied in the plane defined by the initial formation. The maneuver required *316 seconds* to complete in this case. Maneuver duration is dictated by the z component of a_4, the acceleration of the 4^{th} spacecraft (see Fig. 7, where note that $a_{4z}(0) = 0.003\ m/s^2$, the prescribed limit). In all time histories depicted in Fig. 3-8, unconstrained time histories appear as dashed lines. By unconstrained time histories we mean optimal solutions for the case when there are no collision avoidance constraints (eqn. (33)), i.e. $R_k = 0,\ k = 1, 2, .., 5$. The constrained histories, related to the problem addressed here, appear as solid lines. The spacecraft-to-spacecraft separations and cost functional are plotted vs. non-dimensional time ξ in Fig. 3. There are 10 inter-spacecraft separations and four of them come close to the required 20 m threshold. Note that two of the unconstrained separations clearly violate the 20 m lower limit in the plot on the left in Fig. 3. Satisfaction of all constraints requires 80% more energy.

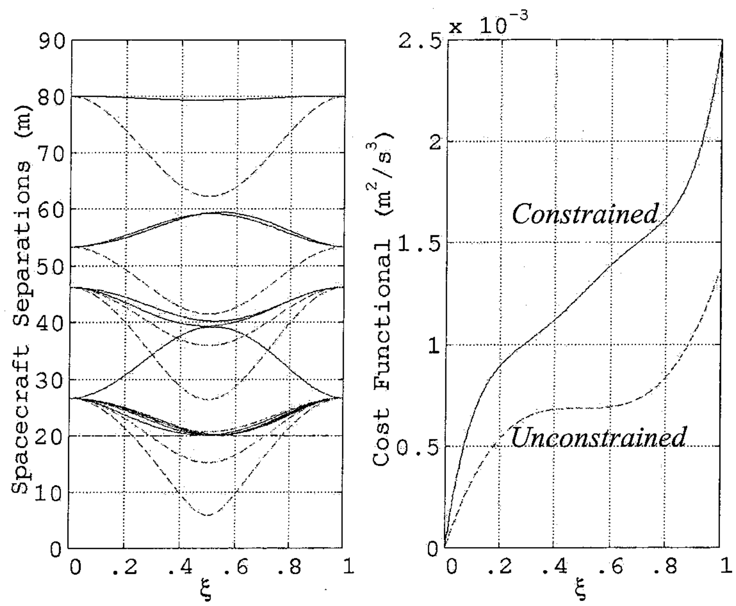

Fig. 3. Variations in inter-spacecraft separations and maneuver cost

Time histories of x_1 and a_1 are depicted in Fig. 4, x_2 and a_2 are shown in Fig. 5, x_3 and a_3 are shown in Fig. 6, x_4 and a_4 are shown in Fig. 7, and a_5, the acceleration required by the reference spacecraft, is shown in Fig. 8.

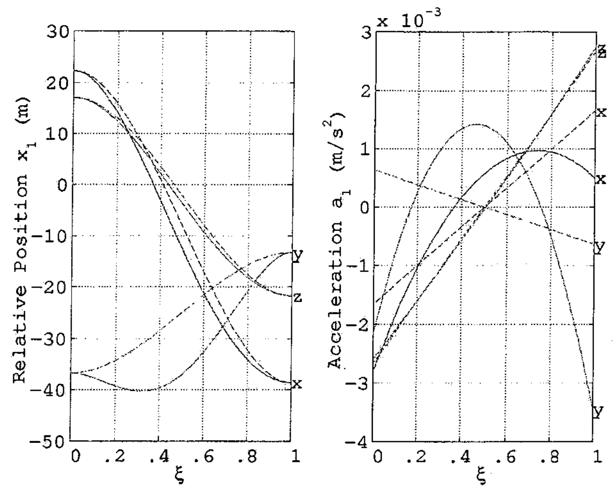

Fig. 4. Variations in x_1 and a_1

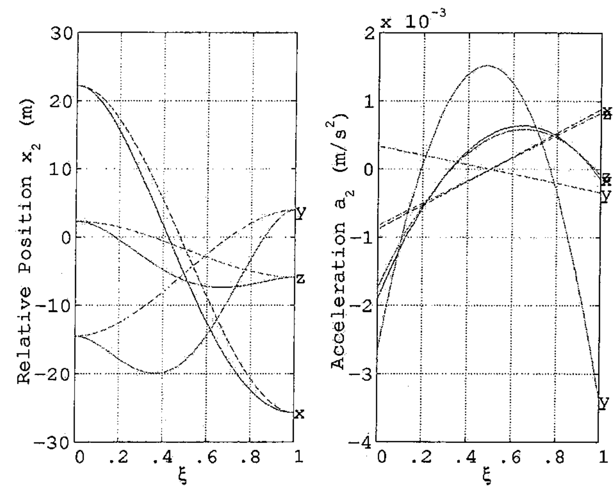

Fig. 5. Variations in x_2 and a_2

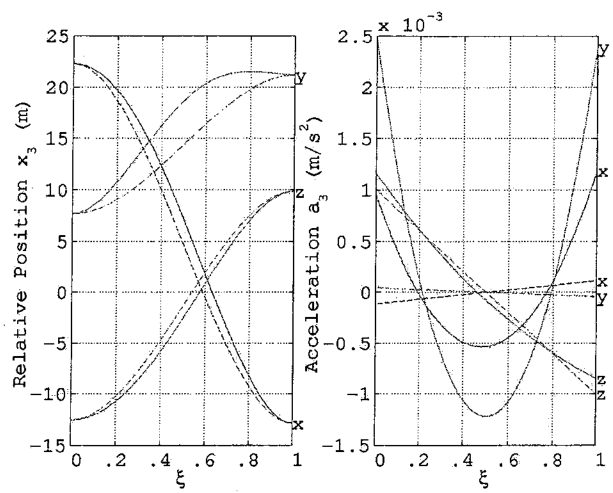

Fig. 6. Variations in x_3 and a_3

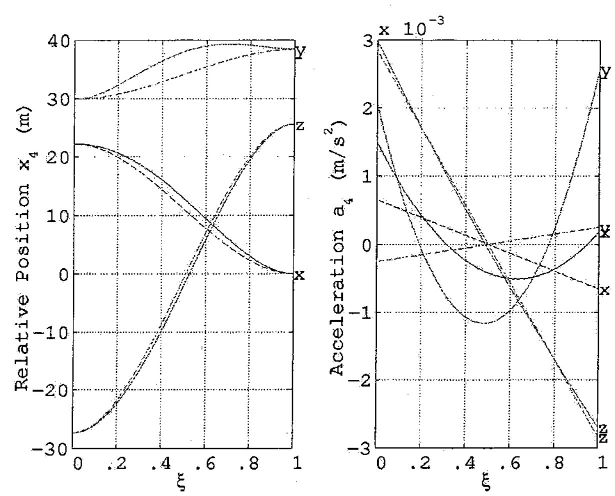

Fig. 7. Variations in x_4 and a_4

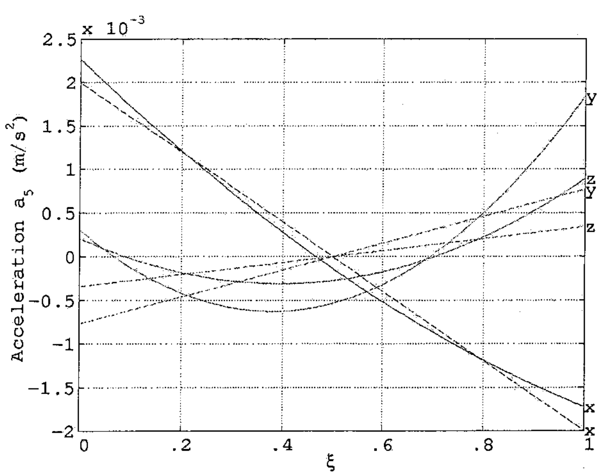

Fig. 8. Variations in a_5

4. CONCLUSIONS

The problem of minimum energy collision avoidance for formation flying applications is considered and a solution in presented. The minimum energy expended is the chosen metric. It is closely related to the total fuel expended. The proposed methodology looks for sub-optimal solutions which are attractive from the standpoint of real-time implementations. The solution is sub-optimal since it tries to locally minimize the appropriate cost-functional within the class of paths under consideration.

Our analysis has shown that, within the class of proposed solutions, consideration of only the first significant term in the time-series approximation yields a solution with the lowest cost. It is also computationally least intensive. It is possible to include additional constraints on relative velocities. Such inequalities can be handled in the manner the acceleration constraint is accommodated here. Also, other metrics can be considered within the same class of solutions. Additional work is also needed in the area of *avoidance-assured* path-planning which guarantees collision-free motions in case of a fault at any time during a maneuver. This places additional constraints on the relative positions and velocities which must be accounted for in the numerical algorithm.

5. ACKNOWLEDGEMENT

The work described in this paper was carried out at the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration.

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