Comparison of Full-Aperture Interferometry to Sub-Aperture Stitched Interferometry for a Large Diameter Fast Mirror

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ABSTRACT

The Herschel Space Observatory (formerly known as FIRST) consists of a 3.5 m space telescope. Stitching sub aperture interferograms may offer considerable cost savings during testing of the flight telescope as compared to other techniques. A comparative demonstration is presented of interferogram stitching techniques that enable a composite map of a 3-D surface to be assembled from a sequence of sub-aperture measurements. This paper describes the fundamental procedures for stitching together component data sets and demonstrates such techniques with real data sets. A set of 14 sub-aperture measurements was made of a 2 m diameter all-composite mirror developed as part of the Herschel Space Observatory program and two different stitching software packages were employed to stitch together the sub-aperture surface maps. The software packages differ fundamentally in the way the sub-aperture maps are three-dimensionally stitched, one employing a local technique and the other using a global technique. The processed results from both algorithms are compared with each other and with a full-aperture reference measurement made of the same test optic. A summary of the results is presented and potential modifications and enhancements to the stitching techniques are discussed.

Keywords: Optical Metrology, Interferogram Stitching, Sub-Aperture Scanning, Space Telescope, Carbon Fiber Composite, Cryogenic Optics, High Slope Errors

1. INTRODUCTION

The Herschel Space Observatory (formerly known as FIRST), is a European Space Agency (ESA) cornerstone mission, that will be used for photometry, imaging and spectroscopy in the 80 to 670 \textmu m range (see Figure 1). The key science goals that this observatory will achieve concern how galaxies formed in the early universe, and how stars form, and have been forming, throughout the history of the universe. NASA, through the Jet Propulsion Laboratory (JPL), has contributed to the telescope and its design. [1] This paper will discuss the work done by JPL, Composite Optics, Incorporated (COI), and Light Works Optics, Incorporated (LWO) to characterize a large aperture (2 m) demonstration mirror at 70 K. Requirements traceable to the Herschel Space Observatory are wavefront error (6 \textmu m RMS), mass (280 kg), primary mirror diameter (3.5 m), F-number (F/0.5), and the operational temperature (70 – 90 K).

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In response to the telescope requirements a low mass, low coefficient of thermal expansion (CTE) telescope [1] has been designed using carbon fiber reinforced polymer (CFRP) and a 2 m diameter demonstration mirror was fabricated [2] for the purpose of cryogenic performance characterization (see Figure 2). The 2 m diameter demonstration mirror is spherical in shape with a radius of curvature of 4.2 m. It is coated with a protected gold and has an areal density of 10.1 kg/m². The mirror has a unique, segmented facesheet. The core that supports the facesheet is contiguous, but the facesheet is separated into six (6) petals. A specific goal for the demonstration mirror was the optical characterization of surface errors at 70 K.

Several features of the carbon fiber 2 m mirror differentiate it from traditional mirrors. [2] The material, CFRP, is an extremely low expansion material even at cryogenic temperatures. The manufacturing process involved curing the facesheets on a mold in order to generate a smooth near net shape surface and an assembly technique that created the final figure of the mirror on an assembly mold. Although the manufacturing mold was polished, the mirror itself was never polished. The stiffness of the mirror met all of the launch requirements traceable to the Herschel Space Observatory. However, it was sufficiently lightweighted that gravity did affect the surface figure. Generally the surface figure was specular (surface
roughness: $< 0.6 \, \mu m$ RMS from profilometry data). Material uniformity was closely controlled and bond lines were minimized to ensure minimal quilting upon exposure to cryogenic temperatures. However, some quilting was anticipated.

Several methods exist for the characterization of the 2 m demonstration mirror. After a survey of methods and preliminary testing, an infrared phase shifting interferometer (IR PSI) and an infrared Shack-Hartmann (IR SH) wavefront sensor were chosen to collect cryogenic characterization data. [3] Additional data were collected during this test in order to explore the applicability of sub aperture stitching of interferograms. Although this data was not used as a final evaluation of the 2 m demonstration mirror, it serves as a novel case for comparing full aperture interferometry with sub aperture stitched interferometry.

Presented is a motivation for exploring stitching and comparing it to full aperture interferometric measurements. Following this is a discussion of the specific layout of the sub apertures used in the stitching experiment as well as the experimental methods used to collect the data. This is followed by a discussion of the algorithms used to stitch together the sub apertures and a comparison of the stitched surfaces with the full aperture measurement.

2. MOTIVATION FOR COMPARING STITCHING/FULL APERTURE

The comparison of stitched interferometry and full aperture interferometry was inspired by two factors: testing of the 2 m demonstration mirror and traceability to flight telescope testing.

![Interferograms](image)

$\lambda = 10.6 \, \mu m$  
$\lambda = 80 \, \mu m$

Figure 3: Comparison of Interferometric Measurements at $\lambda = 10.6 \, \mu m$ with Simulated Interferogram at $\lambda = 80 \, \mu m$

Although the telescope was designed for use at wavelengths between 80 $\mu m$ and 670 $\mu m$, the shortest wavelength available for optical testing the 2 m demonstration mirror was 10.6 $\mu m$. The 2 m demonstration mirror was anticipated to be diffraction limited at 80 $\mu m$ even at cryogenic (70 K) temperature. However, when testing at 10.6 $\mu m$, the interferogram would be significantly denser (8X). Figure 3 illustrates this phenomenon by showing an interferogram taken using $\lambda = 10.6 \, \mu m$ and an artificial interferogram with the same data calculated for $\lambda = 80 \, \mu m$.

In addition to the challenges of the wavelength of the test source, the effects of cryogenic temperature (70 K) on the 2 m demonstration mirror were significant. Previous testing [2] indicated that the effects of quilting would create very high slope errors; slopes errors high enough that perhaps a full aperture interferogram could not be analyzed. This prompted an effort to improve instrumentation and explore alternate methods of collecting surface data at cryogenic temperatures. [3] One technique to capture high slope errors is to collect data on a sub aperture of the 2 m demonstration mirror. The maximum slope error that can be captured in an interferogram is proportional to the magnification of the interferogram. Therefore, by magnifying the interferogram and reducing the area viewed, very high slope errors can be recorded. These sub aperture interferograms can later be combined through a stitching process to generate a full aperture surface map.
Figure 4: Sub-Aperture Test Concept

In addition to the challenges faced with measuring the 2 m demonstration mirror, sub aperture testing was also inspired by consideration to the optical testing of the flight telescope. The test plan for the Herschel Space Observatory calls for an end-item optical test at or near operating temperature. A full aperture test would require a reference flat equal to the diameter of the primary: 3.5 m. This flat would conceivably be required to be mounted inside the cryogenic chamber, although it is possible that it may be at a warmer temperature than the telescope. One technique that avoids fabricating, mounting, and thermally managing a 3.5 m high quality (nominally < 0.25 μm RMS) reference mirror is to design a sub aperture test with a smaller (approx. 1 m), moving reference flat (see Figure 4). With this in mind, an early understanding of sub aperture measurement and stitching was a means of exploring this technique for use in the flight telescope test.

Stitching of surface data has been explored by others. [4, 5] However, the practice is not routine in optical metrology and commercial software for this technique is in a nascent stage. In addition, there are no references for stitching surface measurements for a mirror as fast (F/1) as the 2 m demonstration mirror or for cryogenic mirrors. Therefore, experimental data was critical to understanding the viability of this technique for the Herschel Space Observatory.

3. LAYOUT OF SUB-APERTURE TEST

Due to the cost of the cryogenic facility and its distance from the mirror fabrication site (test facility – Tennessee/fabrication site - California), preparation was made prior to shipment of the mirror for testing. The pattern of sub aperture measurement was well defined prior to test and the locations of fiducials, critical to stitching, were carefully measured.

The sub aperture patterns were arranged with the following conflicting constraints: minimize the number of measurements, maximize data collected for each sub aperture, and maximize overlap.

Minimizing the number of measurements decreases the time required to take the measurements. For cryogenic tests, time is particularly important due to changes in thermal conditions and the cost of operating the test. Reducing the number of measurements also reduces the opportunities for human error. Minimizing the number of measurements either increases the sub aperture size or decreases the overlap.

Reducing the size of each of the sub apertures increases the slope errors that can be captured by interferometry; fringes with high density are magnified to a lower density. Cryogenic quilting of the mirror can cause large slopes that, without properly selecting the sub aperture diameter, may result in high density fringes that cannot be analyzed, resulting in data loss. Maximizing the data collected from each sub aperture drives the size of the sub aperture smaller.

Increasing the overlap presumably improves the accuracy. An increase in overlap allows better matching of the areas common to each sub aperture; reducing the errors calculating and matching the tip, tilt, and piston terms required for stitching the sub apertures together. Increasing the overlap requires either an increase in the sub aperture size or an increase in the number of sub apertures.
Guided by experimental data from previous cryogenic measurements, [2] a sub aperture approximately 0.7 m in diameter was determined to be adequate for resolving any high slope errors. As a consequence the layout shown in Figure 5 was created for the sub apertures. The overlap between two adjacent sub apertures varied between 18% and 31%.

Fiducial marks were placed on the mirror to allow for registration of the sub apertures during post processing of the optical measurement data. The fiducials were 10 mm diameter with adhesive targets. The targets were measured by theodolite to an accuracy of ± 50 μm. The targets were large enough that they were visible in the interferometric data because they were non-reflective and did not produce fringes. A static image of the mirror shows the fiducials as small black spots (see Figure 6). As such, the software identified them easily as data dropouts.
4. EXPERIMENTAL DATA

Experimental data was collected to compare a stitched surface with a full aperture measurement. The experimental data required an adequate test facility, instrumentation for full aperture and sub aperture measurement, and strict protocols to ensure that the measurements would be consistent.

![Diagram of optical test setup](image1)

**Figure 7: Schematic of Optical Test**

The optical test was conducted at the Arnold Engineering Development Center (AEDC), a United States Air Force test facility. A schematic of the layout of the equipment, chamber, and test article is shown in Figure 7. All optical instrumentation, with the exception of the test article was outside of the chamber. A zinc selenide window was used by the instruments to view the 2 m demonstration mirror.

![Image of optical setup with labels](image2)

**Figure 8: Metrology Instrumentation**
The metrology instrumentation germane to interferometry consisted of a modified 10.6 μm PSI, originally built by Breault Research Organization, Incorporated (BRO), with a custom manufactured scanning objective mounted on a metrology platform with a reference mirror (see Figure 8). The BRO interferometer was modified in the following ways (in order of importance):

1. Pyroelectric vidicon detector was replaced with a microbolometer array,
2. Scanning F/2 objective was designed and fabricated,
3. Adjustments were made to the magnification telescope to remove astigmatism,
4. Upgraded to Intelliwave™ (Engineering Synthesis Design) data acquisition/analysis software.

Figure 9: Custom F/2 Scanning Lens Objective

The full impact of these changes is covered in detail elsewhere. [3] A key element to sub aperture data collection was a custom scanning objective lens. Nominally, to capture high resolution interferograms of small portions of the wavefront from the 2 m demonstration mirror, the interferometer would need to be repositioned. This repositioning is a rotation about the center of curvature of the mirror. In order to reposition about the center of curvature, the interferometer would have to be elevated, translated, and rotated. This amount of movement would have jeopardized the integrity of the full aperture interferometric data that were being collected during the same test. Furthermore, the BRO IR PSI is large (1.75 m x 0.3 m x 0.75 m) and heavy (> 50 kg) and therefore difficult to rotate and translate. Instead, an F/2 objective was designed with a 30 mm thick rotatable germanium window. The F/2 objective was fast enough to capture the entire 2 m demonstration mirror. When sub aperture data was collected, the interferometer magnification was changed internally. The window was then rotated and tilted to scan the view of the interferometer over the 2 m demonstration mirror. This technique minimized any changes in the reference point for each of the sub aperture interferograms. It also inspired the radially symmetric layout of the sub apertures.

The metrology platform included a spherical reference mirror. It was desirable to maintain a constant reference point for all the sub aperture measurements. This would minimize computation errors later during the stitching process. If the reference were accurate enough, the power term would not need to be removed from each sub aperture interferogram. Given that the power term in the actual mirror may vary over the aperture of the mirror due to low order aberrations, this was a potential source of error. Before each sub aperture was measured, the interferometer was nulled with the reference mirror to ensure the measurements were taken from the same point. A trial interferogram was taken at room temperature without implementing this nulling process. The results of the stitched surface had a very poor comparison to the full aperture surface. The nulling process was adopted for all the measurements reported here.
The process for taking measurements are described:

1. The germanium window was tilted and rotated to center the appropriate fiducials in the field of view of the interferometer,
2. The interferometer was nulled with respect to the spherical reference mirror,
3. The surface was measured five (5) times using phase shifting interferometry.

A five-bucket algorithm was used for phase shifting. Each surface was the result of calculations based on five (5) interferograms each separated by 90° of phase (0°, 90°, 180°, 270°, 360°). The entire process of collecting five (5) 90° shifted interferograms was repeated five (5) times to reduce errors through averaging. Therefore a total of 25 interferograms were recorded for each sub aperture. The entire data collection process occurred in less than 45 minutes for the entire set of 14 sub apertures.

Measurements were taken at room temperature (before and after cryogenic test conditions), at 70K, and at several intermediate temperatures. The data from room temperature and 70K was used for the comparison. Figure 10 shows representative surface measurements from sub apertures at 70K.

![Sub Aperture A](image1)
![Sub Aperture B](image2)
![Sub Aperture C](image3)
![Sub Aperture D](image4)

*Figure 10: Sample of Sub Aperture Surface Data Taken at 70 K*
5. STITCHING PROCESS

The process of stitching sub aperture measurements can be broken into two major steps: registration and surface matching.

The first step in the stitching process is the lateral registration of each of the sub apertures. Visually matching regions using local texture or features can provide an intuitive method of registration, but it is usually limited in the degrees of freedom that can be exercised. There are several ways of registering the sub apertures computationally:

1. Linear translation, rotation, magnification, and skew,
2. Affine transformation [6] of registration points,
3. Least squares fitting of registration points.

\[
\begin{bmatrix}
  x \\
  y
\end{bmatrix} =
\begin{bmatrix}
  a & b \\
  c & d
\end{bmatrix}
\begin{bmatrix}
  u \\
  v
\end{bmatrix} +
\begin{bmatrix}
  e \\
  f
\end{bmatrix}
\]

where
\[
\begin{bmatrix}
  a & b \\
  c & d
\end{bmatrix} =
\begin{bmatrix}
  \cos \theta & -\sin \theta \\
  \sin \theta & \cos \theta
\end{bmatrix}
\begin{bmatrix}
  1 & S_x \\
  0 & 1
\end{bmatrix}
\begin{bmatrix}
  M_x & 0 \\
  0 & M_y
\end{bmatrix}
\]

**Figure 11: Affine Transformation of Registration Coordinates**

The linear translation, rotation, magnification, and skew can be accomplished with a simple matrix multiplication and vector addition. An affine transformation is a single matrix multiplication that achieves the same result as rotation, skew, and scaling (see Figure 11). A least squares fitting of registration points is implemented calculating a pseudo inverse matrix from the expected and measured positions of the registrations points.

The registration methods can use global reference data (e.g. the theodolite data taken of the fiducials on the 2 m mirror) or local reference data (e.g. fiducials from adjacent overlapping sub apertures).

**Match Overlap Region in 3-D Space**

\[
\begin{bmatrix}
  \text{Sub-Aperture Surface 1} \\
  \text{Sub-Aperture Surface 2}
\end{bmatrix}
\]

**Figure 12: Overlapping Regions Describe Identical Surfaces. Therefore Adjacent Sub Apertures Must Have Matching Tip/Tilt and Piston**
The second step in the stitching process is matching the surfaces in the overlapping regions (see Figure 12). The overlapping regions of sub apertures should describe identical surfaces. Quantities such as tip, tilt, and focus can be calculated for these regions and matched, providing a basis for continuity across the entire full aperture surface. There are at least two methods of selecting surfaces for matching: sequentially or global least squares fitting. Sequential matching can often lead to cumulative errors that distort the full aperture stitched surface. Least squares fitting provides a more global optimization of minimizing differences in the overlapping regions. The figures of merit for matching overlapping regions are typically the minimization of tip, tilt, and focus error in overlapping sections. Calculating tip, tilt, and focus can be done either by Zernike decomposition or by least squares fitting the overlapping region to the best-fit plane/parabola.

6. STITCHING RESULTS

The two software packages used for this comparison, Phase MOSAIC 2.0b (Fringlesoft) and WaRPP 2.00 (REOSC), use slightly different approaches to each of the two fundamental stitching steps: registration and surface matching. Both software techniques were applied to the same sub aperture data and each was compared to direct measurements of the full aperture data.

Phase MOSAIC featured:

- Least squares fitting of global registration points,
- Sequential surface matching,
- Removal of tip, tilt, and focus by Zernike coefficients.

![Figure 13: Sequential Order of Sub Aperture Surface Matching for Phase MOSAIC Computation](image)

Exercising the Phase MOSAIC software revealed sensitivity to the sequential order of sub aperture surfaces. To minimize this effect, surfaces were matched in groups and then groups of surfaces were matched. The order that the surfaces were matched is shown in figure 13. This order was chosen to attempt to distribute the cumulative errors encountered when sequentially matching sub apertures. Focus was not removed from each sub aperture during the surface matching operation.

WaRPP featured:

- Affine transformation of global registration points,
- Surface matching by least squares fitting,
- Removal of tip, tilt, and focus.
During processing with the WaRPP software, sub aperture resolution was reduced (3X) to decrease the size of the full aperture surface map file.

Locating the fiducials on the interferograms was much less accurate than the theodolite measurements. However, each software package reported a relatively small error for registration mismatch (±7.2 mm for Phase MOSAIC and ±7.3 mm for WaRPP).

Figure 14: Stitched Surfaces from 70 K Surface Data

Figure 14 shows the results from stitching the 70 K sub aperture measurements with Phase MOSAIC and WaRPP. Bear in mind that these surfaces are raw surface measurements. Results reported in other references on this 2 m demonstration mirror [3, 7] were difference measurements between two temperatures. Since these measurements are raw surface measurements, features such as window aberrations and gravity effects are not removed.

A comparison was made between the stitched surfaces with the full aperture surface by analyzing the low order Zernike coefficients of the surfaces. A point-by-point difference of the full aperture measurement and the stitched surface is susceptible to high frequency noise in the data. Instead, Zernike coefficients (1 – 36) were calculated for each of the surfaces. The registration accuracy was far better (0.36% = 7.2 mm/2000 mm) than necessary for this type of data analysis. Zernike polynomials of the 36th order have typically 5 – 6 cycles across the clear aperture. This would require a registration accuracy of between 2% – 5%. The Zernike coefficients were used to generate artificial surfaces. These artificial surfaces were differenced as a comparison of low order errors for the techniques.

The following surfaces were compared:

1. Full Aperture/Stitched Using Phase MOSAIC – 70 K (see Figure 15),
2. Full Aperture/Stitched Using WaRPP – 70 K (see Figure 15),
3. Full Aperture/Stitched Using WaRPP – 293 K (see Figure 16).
Figure 15: Point-by-Point Difference Between Stitched Zernike Surfaces and Full Aperture Zernike Surface for 70 K Surface Data

Figure 16: Point-by-Point Difference Between Stitched Zernike Surfaces, using WaRRP and Full Aperture Zernike Surface for 293 K Surface Data (0.45/5.0 μm RMS/PV)

Estimates of the errors involved in stitching are challenging due to the complexity of the processes involved. Error propagation is not simply a set of independent probabilities. However, a few estimates can be posed for the errors. Individual interferograms for this test have been demonstrated to have an accuracy of ±0.4 μm RMS. If these errors were merely added as an RSS, the result would yield an error of ±1.5 μm RMS. Another estimation of errors can be made by
weighting the errors in the full aperture by the proportion of area and the number of overlapping surfaces. Areas with no overlap are assigned an error of ±0.4 μm RMS. Areas with multiple overlapping surfaces are weighted with 0.4√N were N is the number of overlapping surfaces. The terms are summed as:

\[ \sigma_{\text{total}} = 0.4 \mu m \times \sqrt{\sum_{n=1}^{3} \left( \frac{A_n}{A} \right)^n}, \]

where \( A_n \) is the area of each subaperture containing \( n \) overlapping measurements and \( A \) is the area of a subaperture. Using this procedure, the error in the full aperture is approximated as ±0.94 μm RMS.

7. DISCUSSION

This comparison provides some insight into the variables involved in sub aperture stitching. The following comments serve as guidance for future work in this area:

- Although an attempt was made to maximize the overlap area (18% to 31%), a larger overlap area is expected to produce a better comparison with full aperture measurements.
- The reference optic for ensuring the same focus for each sub aperture measurement proved to be invaluable.
- The measurement of fiducials via theodolite prior to testing provided a valuable global coordinate system for registration. Local registration by matching fiducials in overlapping sub apertures did not prove nearly as accurate. At least four (4) fiducials for each sub aperture are recommended.
- Close monitoring of the registration process is essential to avoiding non-physical transformations. Constraining the variables during the registration process can avoid transformations such as skew or anamorphism from distorting the data when they did not actually occur during data collection.
- Improvements in the software include diagnostic data to view mismatched regions and least squares fitting for surface matching.

Several variables distinguish this comparison and have yet to be understood. The speed of the mirror (F/1) is suspected to have an adverse effect on accuracy. The presence of high spatial frequencies from quilting is also expected to have an adverse effect on accuracy. However, neither of these was studied with this comparison.

8. SUMMARY

A motivation for investigating sub aperture stitching has been presented. The motivation suggests that sub aperture stitching may be useful to other large aperture cryogenic telescopes. The methodology for sub aperture stitching has been discussed, describing the steps involved and two commercially available software packages capable of implementing the algorithms. Data collected from a large, fast (F/1), spherical, and quilted surface were stitched and compared with full aperture surface maps. In some cases, an excellent comparison (λ/10 where λ is the test wavelength) was achieved. Suggestions for directions to advance the state of the art have been made.

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10. REFERENCES


