ORIGIN AND BULK CHEMICAL COMPOSITION

OF THE INNER PLANETS

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Introduction

Mercury's mean uncompressed density $\sim$ 5.3 g/cm^3 greatly exceeds that of its neighbour Venus, viz. $\sim$ 3.9 g/cm^3, suggesting bulk metal to rock mass fractions of 0.67:0.33 instead of 0.33:0.67 [1]. The latter values coincide with the ones expected on the basis of solar elemental abundances [2]. Lewis [3,4] discovered that a natural explanation for Mercury's high metal content existed within his equilibrium chemical condensation model applied to the solar nebula. Reconstruction of the mid-plane temperature T and presssure p_{neb} spatial distributions from all of the planetary density data, including the observation of water ice at Jupiter's orbit, suggested that T and p_{neb} vary with heliocentric distance R (in AU) according as $T(R) = 550\ R^{-1.1\ \pm\ 0.2}$ K and $p_{neb}(R) = 10^{-4}$ $R^{-3.85}$ bar [5]. At the orbit of Mercury ($R = R_n$, $n =1$), $p_{neb} = 0.004$ bar and the condensation temperatures of the dominant metal [1: Fe-Ni] and rock [2: $MgSiO_3$] species are $T_{1,1} = 1520$ K and $T_{1,2} = 1490$ K, respectively (see below). Hence if $\hat{T}(R_1)$ lies just below $T_{1,2}$, only a small quantity of $MgSiO_3$ can condense and so a relative enrichment of metals occurs. Unfortunately, numerical studies of planetary accretion in a condensate disc show that Mercury drew its mass from a wide range of orbital radii [6]. Any initial chemical inhomogeneity at Mercury's distance would soon be blended out. Also, we need $T_{1,1} - T_{1,2} \geq 60$ K if the density enhancement is to be noticeable, even in the absence of radial mixing.
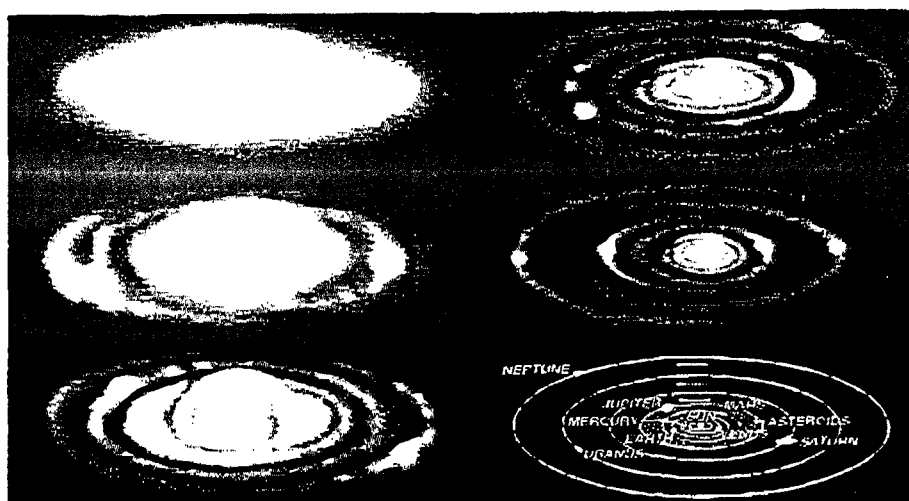

Figure 1

The Modern Laplacian Theory

According to the modern Laplacian theory of solar system origin, the planetary system condensed from a concentric family of orbiting gas rings [7,8]. Figure 1 is a visualization of the original Laplacian hypothesis. The rings were shed from the equator of the rotating proto-solar cloud (PSC) as a means for disposing of excess spin angular momentum during gravitational contraction, starting at the orbit of Neptune. It is proposed that discrete gas ring shedding is achieved by the action of radial supersonic stress $\langle \rho v_r^2 \rangle = \beta \rho GM(r)/r$ arising from supersonic convective motion. Here $\rho = \rho(r)$ is the local gas density, $M(R)$ is the mass inside radius r, G is the gravitation constant and $\beta \sim 0.1$ is a constant parameter. The total radial stress is $p_{tot} = \langle \rho v_r^2 \rangle + p_{gas}$ where $p_{gas} = \rho \Re T/\mu$ is the gas pressure and μ is the mean molecular weight. Now $F_i = \langle \rho v_r^2 \rangle / p_{gas}$ is greatest at the photosurface (ph). If $F_{i,ph} \gg 1$, a steep density inversion accompanies the degeneration of turbulence in the radiative outer layers of the PSC. The cloud now stores a dense shell of non-turbulent gas above its photosurface whose base density is a factor $(1+F_{i,ph})$ times that of the non-turbulent cloud of the same temperature T_{ph}.
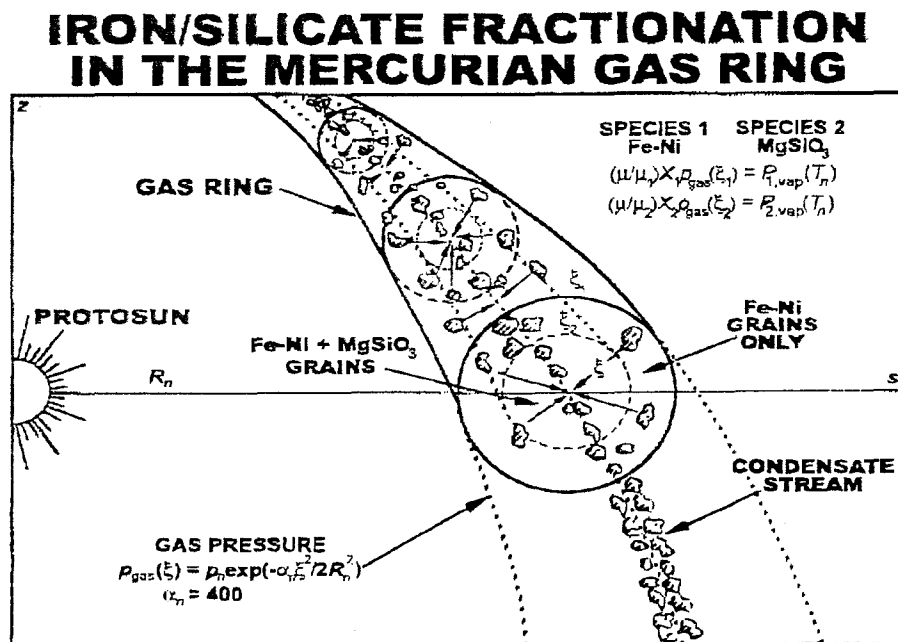
IRON/SILICATE FRACTIONATION IN THE MERCURIAN GAS RING

Figure 2

Choosing $F_{t,ph} \sim 35$, the orbital radii of successively detached rings match the observed mean geometric spacings of the planets. And as the gas pressure on the mean orbit ($s = R_n$, $z = 0$) of the Mercurian ring is now $p_1 = 36p_{neb} \approx 0.14$ bar, for which $T_{1,1} - T_{1,2} \approx 85$ K, a strong fractionation between iron and silicates takes place.

The Gas Ring Condensation Model

Figure 2 is a schematic view of the gas ring cast off at Mercury's orbit. If the gas has uniform temperature T_n and specific angular momentum $(GM_nR_n)^{0.5}$, where M_n is the PSC mass, the ring has a toroidal structure with gas pressure distribution given by $p_{ring}(\xi) = p_n\exp(-f)$, where $f \equiv \alpha_n\xi^2/2R_n^2$. Here $\alpha_n = \mu GM_n/\Re T_nR_n \approx 400$ is a dimensionless constant and ξ is the local minor radius.

If $X_i(i=1,2)$ and μ_i are the mass fractions and molecular weights of species 1 and 2, the partial pressures prior to any condensation are $p_{i,n}(\xi) = (\mu/\mu_i)X_ip_{ring}(\xi)$. At any radius ξ, condensation occurs provided that $p_{i,n}(\xi) \geq P_{i,vap}(T_n) \equiv \exp(B_i - A_i/T_n)$, where $P_{i,vap}$ are the thermodynamic vapour pressures and A_i, B_i are empirical constants. Typical values are $X_1/\mu_1 = 2.348 \times 10^{-5}$, $X_2/\mu_2 = 2.523 \times 10^{-5}$, $\mu = 2.379$ and (at 1600 K), $A_1 = 47820$, $B_1 = 16.130$, $A_2 = 63770$, $B_2 = 27.481$. Next, we define $T_{i,n}$ to be the condensation temperatures on the mean orbit where the partial pressures $p_{i,n}(0)$ are a maximum. We have $T_{i,n} = A_i/(B_i - \log_e(p_{i,n}(0))$. It then follows that condensation is restricted to an inner torus $\xi \leq \xi_i$, where ξ_i is the solution of the equation $p_{i,n}(\xi_i) = P_{i,vap}(T_n)$. It is a simple matter now to show that the condensed mass fraction of species i is given simply by $X_{i,cond} = X_i\{1 - (1 + f_i)\exp(-f_i)\}$, with $f_i = A_i(1/T_n-1/T_{i,n})$. This equation is also valid for gas rings which have a non-uniform orbital angular momentum distribution $h(s) = h(R_n)\cdot(s/R_n)^{2-q}$, where $h(R_n) = (GM_nR_n)^{0.5}$. Here s is the major cylindrical polar radius and $1.5 < q < 2$. Such rings have an elliptic minor crosssection.

After condensation has occurred, the solid grains settle onto the mean orbit R_n to form a concentrated stream. This 'focussing' property of the gas ring is a basic feature of the modern

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Laplacian theory [9]. Next, as there is no exchange of condensate material between adjacent gas rings, the chemical compostion of each planet is uniquely determined by the thermal properties of its own formative ring. Mercury was thus able to enjoy the full benefit of the iron/silicate fractionation process that took place in its ring.

The Predicted Bulk Chemical Compositions

The table below gives the temperatures T_n and mean orbit pressures p_n which come from a representative simulation of the contraction of the PSC which (1) leads to a sun having the observed mass, (2) accounts for the mean planetary spacings, and (3) leads to a bulk chemical composition for Mercury whose mean density matches the observed uncompressed value. If the PSC is assumed to contract homologously, then $T_n \propto R_n^{-0.9}$ [10,11]. The planet Mercury thus provides a valuable marker for calibrating the run of temperatures. Once this calibration is achieved, the bulk chemical composition of the other planets, including the cores of the gas giant planets, are effectively locked into place.

The last 2 columns of the table are the Fe-Ni-Cr-Co-V alloy mass fraction X_{metal} and the condensate mean density $\langle \rho_s \rangle$, in g/cm^3. Aside from metal, the main chemical constituents of Mercury are $Ca_2Al_2SiO_7$ (mass fraction: 0.254) and $MgAl_2O_4$ (0.041). $MgSiO_3$–Mg_2SiO_4 makes up only 0.009 of the mass. The main non-metal constituents of Venus are $MgSiO_3$–Mg_2SiO_4 (0.499), SiO_2 (0.076), $Ca_2MgSi_2O_7$ (0.041) and $MgAl_2O_4$ (0.039). Earth's orbit sees the appearance of (Fe-Ni)S (0.087) and tremolite (0.102). At Mars, nearly all Fe is tied up in (Fe-Ni)S (0.205) and Fe_2SiO_4 (0.180), whilst for the asteroids it is in (Fe-Ni)S (0.200) and Fe_3O_4 (0.193). Lastly, we note that Jupiter's orbit marks the initial condensation of H_2O ice. This makes up 0.341 of the condensate mass, along with 'asteroidal' rock (0.651) and graphite (0.008). Only 33% of the available H_2O vapour can solidify since T_n lies just 5.2 K below the H_2O condensation temperature on the gas ring mean orbit. This feature accounts for the 50%:50% rock:ice mass percents of Ganymede and Callisto [10,11].

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Table: Gas ring properties for the sub-Jovian planets

Planet	R_n/AU	T_n/K	p_n/bar	X_{metal}	$\langle \rho_s \rangle$
Mercury	0.387	1632	0.1334	0.670	5.29
Venus	0.723	911	0.0124	0.326	3.89
Earth	1.000	674	3.6×10^{-3}	0.257	3.83
Mars	1.524	459	7.2×10^{-4}	0.054	3.72
Asteroid	2.816	274	5.9×10^{-5}	0.009	3.67
Jupiter	5.203	163	4.9×10^{-6}	0.006	1.83

Conclusions

The original idea of Lewis, coupled with the gas ring model of planetary formation presented here, suggests that iron/silicate fractionation may indeed provide a valid explanation of the anomalously high metal content of Mercury. This model was first put forward one decade ago [12,13].

Acknowledgements

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