Information Theoretic Approach to Bayesian Inference

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Abstract

An increasingly important problem in fields such as astrophysics is the explanation of observed morphological patterns of varying degrees of complexity within specific physical scenarios. From a Bayesian perspective, the ultimate goal is to compute the posterior probability for a theory given the observations. A direct approach to Bayesian inference can be prohibited when the likelihood itself is not known (i.e. non-linear processes with stochastic initial conditions). An information theoretic approach to this problem is developed in which a sequence of exponential family Gibbs densities are used as likelihoods. The sequence of Gibbs densities are constructed by successively matching expectation value constraints, giving a positive *Gibbs Information Gain*, or rate of convergence to the limiting density in the Kullback-Leibler distance sense. Evaluating the sequence of Gibbs densities for the observed data gives a sequence of Bayesian posterior densities which are successively more concentrated. It is shown that the successive confidence intervals form a decreasing sequence of subsets which include, and converge to, the true Bayesian confidence intervals. This provides a justifiable approach for extending Bayesian inference to problems where the likelihood is unknown, as "error bars" are simply larger. Furthermore, the *Bayes Information Gain*, defined as the rate at which the confidence intervals contract to the limiting interval, is shown to be maximized by a "greedy" approach to inference when constructing the Gibbs likelihoods.