DTFM Modeling and Analysis Method for Gossamer Structures

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Introduction

What is DTFM?
--Distributed Transfer Function Method
Why DTFM is unique?
--In the Laplace domain
--Using Distributed Transfer Function instead of Shape Function
What are advantages
--Exact and closed form solutions for 1-d components
--Deals with very small matrices and is very computational efficiency
--Capable to handle properties which are frequency or rotating speed related
--Capable to handle very slim inflatable booms with surface and material imperfections
Distributed Transfer Function Method (DTFM)

DTFM has been successfully developed to obtain exact frequency and time-domain solutions for control problems of one-dimension (1-D) distributed systems involving:
- Multi-body Coupling
- Damping and gyroscopic forces
- Feedback control systems
- Structures with embedded sensors and actuators
Distributed Transfer Function Method (DTFM)

DTFM has also been used to obtain exact solutions for general 1-D structures:
- Strips, bars, beams and beam-columns
- Rotating shafts
- Axially moving continua
- Pipes conveying fluids
- Flexible robots
- Beams with embedded constrained damping layers

Strip Distributed Transfer Function Method (SDTFM) has been developed to obtain semi-exact solutions for general 2-D structures and components.
Process of Distributed Transfer Function Method

1. Decomposition of a complex structure
2. State Space Form of a Component
3. Distributed Transfer Function of a Component
4. Dynamic Stiffness Matrix of a Component
5. Applications of the Distributed Transfer Function Method
   - Natural Frequencies of the Structure
   - Mode Shapes
   - Frequency Responses
   - Static Analysis
   - Time Domain Responses
Process of DTFM: Step 1--Decomposition
Process of DTFM: Step 1--Decomposition (cont.)
Process of DTFM: Step 2--State Space Form

A group of partial differential equations:

\[
\sum_{j=1}^{n} \sum_{k=0}^{N_j} \left( a_{ijk} + b_{ijk} \frac{\partial}{\partial t} + c_{ijk} \frac{\partial^2}{\partial t^2} \right) \frac{\partial^k u_j(x,t)}{\partial x^k} = f_i(x,t)
\]

\[
x \in (0,L), \quad t \geq 0, \quad i = 1, \ldots, n
\]

\[\downarrow\]

Example: a beam component

\[
EI \frac{\partial^4 v}{\partial x^4} + \rho A \frac{\partial^2 v}{\partial t^2} = p
\]
Process of DTFM: Step 2--State Space Form (cont.)

Laplace transformation with respect to time (t):

\[ \sum_{j=1}^{n} \sum_{k=0}^{N_j} D_{ijk} \frac{d^k \bar{u}_j(x,t)}{d x^k} = \bar{f}_i(x,t) \]

\[ D_{ijk} = (a_{ijk} + b_{ijk}s + c_{ijk}s^2) \]

Example: a beam component

\[ EI \frac{d^4 \bar{v}}{d x^4} + \rho A s^2 \bar{v} = \bar{p} \]
Process of DTFM: Step 2--State Space Form (cont.)

State space form:
\[
\frac{d}{dx} \eta(x,s) = F(s)\eta(x,s) + q(x,s)
\]

\[
\eta = \{ \eta_1^T, \eta_2^T, \ldots, \eta_j^T, \ldots, \eta_n^T \}^T
\]

\[
\eta_i = \left\{ u_i, \frac{d\bar{u}_i}{dx}, \ldots, \frac{d^{N_i-1}\bar{u}_i}{dx} \right\}^T
\]

Example: a beam component

\[
F(s) = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
-\rho As^2 & 0 & 0 & 0
\end{bmatrix}
\]

\[
\eta(x,s) = \left\{ \bar{v}(x,s), \bar{v}'(x,s), \bar{v}''(x,s), \bar{v}'''(x,s) \right\}
\]

\[
\dot{q}(x,s) = \begin{bmatrix}
0 \\
0 \\
0 \\
\frac{p(x,s)}{EI}
\end{bmatrix}
\]
Process of DTFM: Step 3--DTF

A boundary value problem:

\[
\frac{d}{dx} \eta(x,s) = F(s) \eta(x,s) + q(x,s) \quad x \in (0, L)
\]

\[M \eta(0,s) + N \eta(L,s) = r(s)\]

The solution:

\[
\eta(x,s) = \int_0^L G(x, \zeta, s) q(\zeta, s) d\zeta + H(x, s) r(s) \quad x \in (0, L)
\]

\[G(x, \zeta, s) = \begin{cases} 
  e^{F(s)x} (M + Ne^{F(s)L})^{-1} Me^{-F(s)\zeta} & \zeta \leq x \\
  -e^{F(s)x} (M + Ne^{F(s)L})^{-1} Ne^{F(s)(L-\zeta)} & \zeta \geq x
\end{cases}\]

\[H(x,s) = e^{F(s)x} (M + Ne^{F(s)L})^{-1}\]
Process of DTFM: Step 3--DTF (cont.)

State space vector: \( \eta_i(x,s) = [\alpha_i^T(x,s) \quad \varepsilon_i^T(x,s)]^T \)

Displacement vector: \( \alpha(x,s) = [\alpha_1^T(x,s) \quad \alpha_2^T(x,s) \cdots \quad \alpha_n^T(x,s)]^T \)

Strain vector: \( \varepsilon(x,s) = [\varepsilon_1^T(x,s) \quad \varepsilon_2^T(x,s) \cdots \quad \varepsilon_n^T(x,s)]^T \)

Force vector: \( \sigma(x,s) = \bar{E}\varepsilon(x,s) \)

Example: a beam component

\[ \alpha(x,s) = \begin{Bmatrix} V(x,s) \\ V'(x,s) \end{Bmatrix} \quad \varepsilon(x,s) = \begin{Bmatrix} V''(x,s) \\ V'''(x,s) \end{Bmatrix} \]

\[ \sigma(x,s) = \begin{Bmatrix} Q(x,s) \\ M_f(x,s) \end{Bmatrix} = \bar{E}\varepsilon(x,s) = \begin{bmatrix} 0 & EI \\ EI & 0 \end{bmatrix} \begin{Bmatrix} V''(x,s) \\ V'''(x,s) \end{Bmatrix} \]
Process of DTFM: Step 4--Dynamic Stiffness Matrix

Force vectors at tow ends of the component:

\[
\begin{bmatrix}
\sigma(0,s) \\
\sigma(L,s)
\end{bmatrix} =
\begin{bmatrix}
\bar{E}H_{\sigma_0}(0,s) & \bar{E}H_{\sigma_0}(0,s) \\
\bar{E}H_{\sigma_0}(L,s) & \bar{E}H_{\sigma_0}(L,s)
\end{bmatrix}
\begin{bmatrix}
\alpha(0,s) \\
\alpha(L,s)
\end{bmatrix} +
\begin{bmatrix}
p(0,s) \\
p(L,s)
\end{bmatrix}
\]

\[\text{Dynamic stiffness matrix}\]

\[\text{Transformed from distributed external forces}\]

Systematically assemble all component dynamic stiffness matrices

\[\text{Dynamic stiffness matrix of the whole system}\]

\[K(s_i) \times U(s_i) = P(s_i)\]
Process of DTFM: Step 5--Applications

Natural frequencies of the structure
\[ \det[K(s_i)] = 0 \quad s_i = \sqrt{-1} \times \omega_i \]

Mode shapes--nontrivial solutions
\[ K(s_i) \times U(s_i) = 0 \]

Frequency responses
\[ U(s) = K^{-1}(s) \times P(s) \]

Static analysis
\[ K(0) \times U(0) = P(0) \]

Time domain responses

Inverse Laplace transform
Example--two elastically coupled beams

\[ \mathbf{K}(s) \begin{bmatrix} \bar{\nu}_2(s) \\ \bar{\nu}'_2(s) \\ \bar{\nu}_3(s) \\ \bar{\nu}_4(s) \\ \bar{\nu}_5(s) \\ \bar{\nu}'_5(s) \end{bmatrix} = \begin{bmatrix} \bar{Q}_2(s) \\ \bar{M}_{f2}(s) \\ \bar{Q}_3(s) \\ \bar{Q}_4(s) \\ \bar{Q}_5(s) \\ \bar{M}_{f5}(s) \end{bmatrix} \]

EI=40 \quad \rho A=0.5

EI=50 \quad \rho A=0.5
**Example--two elastically coupled beems (cont.)**

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<th>Mode number</th>
<th>DTFM 6*6 matrix</th>
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Gossamer Structures

Gossamer structures:

» Mostly composed of highly flexible, long tubular components and pre-tensioned thin-film membranes.

» Offer order-of-magnitude reductions in mass and launch volume

» Revolutionize the architecture and design of space flight systems with large in-orbit configurations.
Disadvantages of FEM for Gossamer Structures

Major shortcomings of general finite element analysis:
1) Tens of thousands elements are needed due to:
   - Accuracy requirements
   - Aspect ratio (a/b) limitations
2) Time-domain solutions require small time steps for convergence
   - excessive computation time
3) Unable to investigate the effect of surface imperfection
DTFM for Gossamer Structures

» Using a couple of large components instead of numerous tiny elements.
» Dealing with very small matrices.
» Very computational efficient.
» Capable to handle no-uniform long booms.
» Capable to study surface and material imperfections.
» Very easy to incorporate with control systems (Laplace domain).
» Capable to handle properties which are frequency or rotating speed related. Able to handle damping forces.
» Able to handle spinning space structures (gyroscope forces).
Computational Efficiencies Between DTFM and FEM

(Strip-Discretization vs. Finite-Element Discretization)

<table>
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<tr>
<th>DTFM</th>
<th>FEM</th>
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<tr>
<td>Applied Load</td>
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Future Tasks

Goal--test-correlated modeling/analysis methods and user-friendly computer software that can be directly employed for the development of flight gossamer systems

- To develop DTFM-based approach for solving structural problems related to gossamer structures.
- To develop analysis capabilities for studying design perturbations, geometric and material imperfections, long booms of non-uniform and non-axisymmetry cross-sections.
- To develop synthesis and assembly processes for modeling and analyzing general 2-dimensional and 3-dimensional gossamer structures formed by multiple long booms and membranes.
- To incorporate the developed DTFM into a selected general-purpose finite-elements code to be user-friendly to all engineers.
THE END

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