Improving Turbo-Like Codes using Iterative Decoder Analysis

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Abstract — **The density evolution method is used to analyze the performance and optimize the structure of parallel and serial turbo codes, and generalized serial concatenations of mixtures of different outer and inner codes. Design examples are given for mixture codes.**

I. Decoder Convergence for Turbo-Like Codes

An iterative decoder for a parallel or serial turbo code with two constituent codes can be viewed as a nonlinear dynamical feedback system. Extrinsic information messages λ with probability densities $f(\lambda)$ for symbol=+1, and $f(-\lambda)$ for symbol=−1, are passed between decoders. With large interleavers, the extrinsic messages are independent and identically distributed, given, say, that the all-zero codeword is selected (corresponding to transmission of +1's on the channel). At a given iteration, each input or output message λ has a probability density with mean $\bar{\lambda}$ and variance σ_λ^2. Furthermore, this density is *consistent* [1], satisfying $\lambda = \ln[f(\lambda)/f(-\lambda)]$. A signal-to-noise ratio (SNR) for the random variable λ can be defined as SNR $= \bar{\lambda}/2$, which is equivalently interpreted as one-half of the *discrimination* between the extrinsic densities $f(\lambda)$ and $f(-\lambda)$.

If the probability density $f(\lambda)$ is both consistent and Gaussian, then this definition of SNR is equivalent to $\bar{\lambda}^2/\sigma_\lambda^2$. A Gaussian approximation enables quick evaluation of the transfer characteristics of each component code. However, more accurate results are obtained through density evolution without the Gaussian approximation.

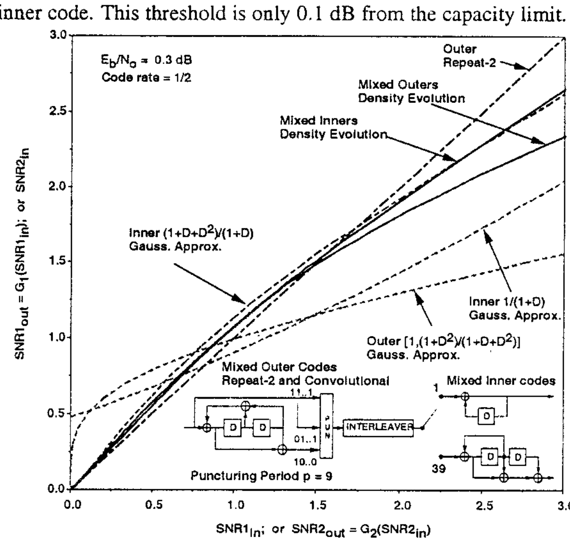
For a given bit-energy-to-noise ratio, E_b/N_0, on an assumed additive white Gaussian noise channel, let G_1, G_2 represent the transfer characteristics for one iteration of the two component decoders of a parallel or serial concatenation, i.e., the SNR of the ith decoder's output messages is $\mathrm{SNR_{out}} = G_i(\mathrm{SNR_{in}})$ whenever the SNR of its input messages is $\mathrm{SNR_{in}}$. With G_1 and G_2^{-1} plotted on the same graph, a condition for iterative decoding convergence is that the two curves do not cross. The iterative decoding threshold is the value of E_b/N_0 at which the "iterative decoding tunnel" between the two curves just closes.

In [2] we have shown that the asymptotic slope of G_1 for a recursive convolutional inner code is 1, and that the asymptotic slope of G_2 for an outer code with minimum distance d_{min} is $d_{min} - 1$. Repetition codes have constant slope even for low SNR. But the slope of G_2 for convolutional codes at low SNR is much less than the asymptotic slope. This accounts for the weakness of convolutional outer codes at the initial iterations.

II. Improvements with Mixed Outer/Inner Codes

The analytical method can be applied to discover combinations of constituent codes whose individual strengths and weaknesses complement each other. Turbo-like concatenated codes can then be constructed using a mixture of such complementary constituent codes that outperform codes formed from either constituent alone.

For example, a very low complexity rate-1/2 code with low iterative decoding threshold can be constructed as follows. For the inner code, use a mixture of the 2-state rate-1 accumulator code, octal (1/3), and a 4-state rate-1 recursive convolutional code, octal (7/3). For the outer code, use the terminated 4-state rate-1/2 recursive convolutional code, octal (1,5/7), but, for $p - 1$ of every p bit times, replace the parity symbol with a repetition of the systematic symbol. This configuration is diagrammed in the figure. An iterative decoding threshold E_b/N_0 just under 0.3 dB is obtained by using a period $p = 9$ for the outer code, interleaving all the output bits together, and then sending 39 of every 40 bits from the interleaver to the octal (7/3) convolutional inner code and 1 of every 40 to the octal (1/3) accumulator inner code. This threshold is only 0.1 dB from the capacity limit.



Solid lines in the figure show SNR transfer characteristics obtained by density evolution for the mixed inner and outer codes, which almost touch each other at $E_b/N_0 = 0.3$ dB. Dashed lines show SNR transfer characteristics obtained by Gaussian approximation for each component of the inner and outer code mixtures. The outer code in this case is effectively a mixture of the octal (1,5/7) convolutional code with a repetition-2 code, except that the termination of the convolutional code's systematic component produces $d_{min} = 4$ instead of $d_{min} = 2$ for the overall outer code.

Of the four possible serial concatenations of the pure (unmixed) inner and outer code components, only the octal (1,5/7) convolutional outer code concatenated with the octal (1/3) accumulator code yields a reasonable threshold E_b/N_0 (1.0 dB). However, this threshold is 0.7 dB above that of the mixture code, because the sharp curvature of the SNR characteristic of the outer (1,5/7) convolutional code requires the inner accumulator code to have a high output SNR at the start, corresponding to a higher E_b/N_0 on the channel.

References

[1] T. Richardson and R. Urbanke, "Analysis and Design of Iterative Decoding Systems," 1999 IMA Summer Program: Codes Systems and Graphical Models, Minnesota, USA, August 2-6, 1999.

[2] D. Divsalar, S. Dolinar, and F. Pollara, "Iterative Turbo Decoder Analysis based on Density Evolution," *IEEE Journal on Select Areas in Communications*, special issue on "The Turbo Principle: from Theory to Practice," to be published, 2001.

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