The Overlapping Plate Method for Asteroid Astrometry

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The overlapping plate method, first introduced in the early 1960s by Henrich Eichhorn, was originally intended to provide better star catalogs through improved data reduction techniques. Every star, not just the reference stars, contributes to the solution, and a group of overlapping exposures is processed in a simultaneous reduction. This method has become standard for star catalog developers, but only recently has it been applied to asteroid astrometry. Practical techniques for using the overlapping plate method are presented so that asteroid astrometrists can produce improved results.

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- What it is
- Why bother?
- How to do it

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Features of the Overlapping Plate Method

- Developed by Heinrich Eichhorn in early 1960s for star catalog work
- Analyzes multiple plates simultaneously
  - Previous standard practice was to average results from different plates for the same star
- **"Always use all available data!"**
  - There is much more astrometric information in a picture than merely the (x, y) centroids of reference stars
- **"A star can have only one position at a time!"**
  - Each star that appears on > 1 picture helps to constrain the relative pointing of those pictures
  - Plate constants are determined not only by the reference stars but also by the behavior of uncatalogued field stars
  - Must solve for the positions of all field stars
  - Stars that appear only once have no effect on the plate constants

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Advantages of the Overlapping Plate Method

- In one word: **accuracy**
  - Highly accurate measurements are vital for predicting occultation paths, not to mention spacecraft with asteroid encounters
- Taking overlapping fields increases the effective field of view, so you can get more reference stars into the solution
- Higher-order distortion terms are trivial since the uncatalogued stars are useful
- Systematic errors in the reduction are likely to be much lower
- The focal plane map is better, so the target need not be centered
- Four measurements at $0''.05$ have the same weight as sixteen at $0''.1$ or 100 at $0''.25$

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How to Use the Overlapping Plate Method

- Take at least two pictures of the target asteroid, **using a different R.A. and Decl. for each picture.**
  - If you're working in a sparse field, you'll need the freedom to offset the asteroid in order to bring in more reference stars.
  - If you're in a crowded field, you can get higher-order terms directly from your data without resorting to calibration frames. Take one picture centered on the asteroid, one offset 1/4 field N or S, and one offset E or W.

- Centroid every image in each picture, not just the asteroid and the reference stars.
  - My s/w goes through each picture line by line looking for local maxima and attempts to centroid each of them.
  - Close doubles often produce multiple copies of (nearly) the same (x, y); look out for these and eliminate duplicate entries.
  - The accidental bogus centroid (cosmic rays or whatnot) doesn't usually matter, since it won't match up with another star image. If it does get matched with a real star, you'll get huge residuals for that star.

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How to Use the Overlapping Plate Method

- Find the reference stars among the list of centroids. Here's my technique:
  - — Calculate the separation in pixels between each pair of reference stars.
  - — Find the N brightest images and get their pairwise separations too.
  - — For each pair of reference stars, is there a pair of bright images at the right separation (and optionally position angle)? If so,
    - — Solve for the four-constant mapping from sky to picture;
    - — Predict the location of each other reference star in the picture;
    - — Count the number of matches with actual star images.
  - — The pair that gives the most matches is nearly always the correct answer. (I can tell the program which two stars to match with which two images if need be.)

How to Use the Overlapping Plate Method

- Perform a standard four- or six-constant solution on each picture separately. Use the results to update the three camera pointing angles, R.A., Decl., and twist.
  - Optionally account for differential refraction (relative to the tangent point) for each image

- Compute the approximate R.A. and Decl. of each image in each picture

- Determine whether each image is a reference star, a field star, or the target asteroid. Keep a catalog of field stars, and check to see if each one has been seen in an earlier image. The ability to identify the same field star in different exposures is vital.

- The result of all this is one file containing the ID, picture number, (x, y) and (α, δ) for each image

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How to Use the Overlapping Plate Method

- **Do the least-squares solution.** I use these solution parameters:
  - — $\Delta\alpha \cos\delta$ and $\Delta\delta$ for every reference star, with the catalog uncertainties treated as 1-*sigma a priori* constraints
  - — $\Delta\alpha \cos\delta$ and $\Delta\delta$ for every field star
  - — $\Delta\alpha \cos\delta$ and $\Delta\delta$ for every *observation* of the target asteroid; each observation is independent of the others
  - — Three pointing angles for each exposure (these are roughly equivalent to B, C, and F in the six-constant model)
  - — Scale for each exposure (equivalent to A)
  - — Aspect ratio for each exposure (equivalent to D)
  - — Nonorthogonality for each exposure (equivalent to E)
  - — Higher-order terms as necessary, modeled as the product of two Legendre polynomials, one in x and one in y

- This can amount to thousands of parameters, since there are two for each star!

- Therefore ...

How to Use the Overlapping Plate Method

- **Filtering.** Borrow a page from Kalman filter theory and treat $\Delta\alpha\cos\delta$ and $\Delta\delta$ as "white noise stochastic parameters":
  - Only 2 position parameters now, not $2N$. Put these at the top of the parameter list, with all the plate constant parameters after them.
  - Group the observations by object (reference stars first, then field stars, then asteroid) and process them individually.
  - When you encounter the next object:
    - Save the current solution vector.
    - Compute the first column of the inverse of the covariance. Divide each element by element 1 to give $\partial(\Delta\alpha\cos\delta)/\partial q$. Save this array of partials.
    - Zero out the first row and column of the covariance, and put the initial variance in $\alpha\cos\delta$ for the next object into element $(1,1)$.
    - Repeat the previous two steps for row and column 2 and $\Delta\delta$.

How to Use the Overlapping Plate Method

- **Smoothing.** When you reach the end of the data, the parameter set contains the same values that you would have obtained with one giant matrix. The first two parameters are the correct answer for the last object. Those results that were saved above are *not* correct—they reflect only data up to that point—so you need a second pass:
  - Go backwards through the observations
  - When you encounter the last observation of the previous object:
    - Read in those saved arrays
    - Apply the δ partials to the *difference* between the final solution and the saved one, giving the tweak to the saved $\Delta\delta$ to yield the final one
    - Repeat for $\Delta\alpha\cos\delta$
    - It's also possible to update the first 2 rows and columns of the covariance, which gives the formal statistics on the measured position for this object
  - Compute the residual for this observation (always a good idea!)

- **Factorized methods.** It is *never* a good idea to deal directly with the covariance matrix. The Kalman filter is numerically unstable since it can involve subtraction of large numbers. Instead, use either a triangular square root of the covariance or a triangular square root of the inverse of the covariance. Numerically stable methods exist for these.