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APPLICATION OF A NOVEL OPTIMAL CONTROL ALGORITHM TO LOW-THRUST TRAJECTORY OPTIMIZATION

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An application of the new optimization algorithm called Static/Dynamic Control (SDC) to the design of low-thrust interplanetary trajectories is presented. SDC is a general, gradient based optimization method that is distinct from both parameter optimization and the calculus of variations. Interplanetary trajectories are integrated with a multi-body force model and may include gravity assists. Engine operation is modeled as finite burns. A feature of the SDC approach is its ability to locate favorable intermediate flybys. It is not necessary to specify which intermediate flyby bodies will be used for gravity assists. This is in contrast to many existing optimization methods. SDC does not require a good initial trajectory guess to begin the optimization. SDC's ability to begin with poor guesses and locate favorable intermediate flybys results in the identification of non-obvious, yet highly efficient trajectories. Results produced by SDC are compared to results produced by a program based on the calculus of variations and a program based on parameter optimization. The test problems feature solar electric propulsion with a specific impulse that is a function of the engine throttle level. The objective is to maximize final mass taking into account a launch vehicle performance curve and propellant usage.

INTRODUCTION

Low-thrust electric propulsion is increasingly being selected as the propulsion system of choice for future interplanetary missions. The higher efficiency of electric propulsion compared to traditional chemical propulsion results in larger payload delivered or shorter flight times. The successful Deep Space 1 mission demonstrated the reliability of electric propulsion.

Low thrust engines typically operate for days, months or even years. This is in contrast to chemical systems that operate for minutes. The continuous operation associated with low thrust significantly increases the optimization complexity and renders approximations used for chemical propulsion trajectories inaccurate. For example, the instantaneous impulse engine model is inaccurate or requires a fine discretization to represent low-thrust arcs. While a discretized patch-conic method is adequate for low fidelity preliminary design, it cannot be used for high fidelity simulation or design optimization. What is needed is a robust high fidelity optimization method for low-thrust trajectories. Optimizing a high fidelity low-thrust formulation was a main objective of this research.

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Existing methods for optimizing low-thrust trajectories are classified as either direct or indirect. Direct approaches parameterize the trajectory and solve the parameterized problem using a gradient based nonlinear programming method, or a hueristic method such as simulated annealing. Direct methods typically remove the explicit time dependence in the optimal formulation by parametrizing the trajectory as a series of impulse burns and conic coasts. Indirect approaches are based on the calculus of variations, resulting in a two point boundary value problem[1]. Indirect methods do no remove the explicit time dependence of the trajectory problem, rather it is solved as an optimal control problem. Calculus of variations methods are limited to a single body (Sun) force model due to the sensitivity of the method. Intermediate planetary flybys are modeled by instantaneous rotations of the velocity vector.

The optimization method used in this research is called Static/Dynamic Control or SDC. SDC is a new, general optimization algorithm which was derived to address a general class of problems with the same structure as low-thrust optimization. SDC best fits into the direct method category. However, unlike other direct methods, the explicit time dependence of the optimization problem is not removed by parameterization. The SDC optimization algorithm is a form of optimal control. Unlike many other optimization approaches, SDC can be used with the highest fidelity space flight simulators available.

In addition to SDC's capability as a high fidelity design tool, SDC optimization provides several important features which are useful for preliminary trajectory design. A novel feature of the SDC approach is its ability to locate favorable intermediate flybys. It is not necessary to specify which intermediate flyby bodies will be used for gravity assists. This is in contrast to many existing optimization methods. Another useful feature is that SDC does not require a good initial trajectory guess to begin the optimization. SDC's dual ability to begin with poor guesses and locate favorable intermediate flybys results in the identification of non-obvious, highly efficient trajectories.

APPROACH

The General SDC Problem Structure

SDC is a general optimization method designed to solve a class of mathematical problems. The SDC optimization algorithm is based in part on the Hamilton, Bellman, Jacobi dynamic programming equation[2]. Unlike traditional differential dynamic programming methods, SDC is constructed to solve highly nonlinear and non-convex problems with a dual dynamic and parametric structure. Optimal solutions generated by SDC satisfy both the necessary and sufficient conditions of optimality.

Three distinct classes of variables are recognized by SDC. The first is the *dynamic control* which are functions of time. Dynamic control variables are analogous to control variables in optimal control theory. The vector $v(t)$ is used to represent the dynamic control at time t. The second variable class is the *static control* which can be thought of as parameters in the ordinary parameter optimization sense. The vector w is used to represent the static control. Both the static and dynamic control variables encompass design variables that are under direct control by the engineer. In addition to the static and dynamic controls, SDC recognizes time dependent *state* variables. The state vector encompasses variables not under the direct control of the engineer. The vector $x(t)$ is used to represent the state at time t.

The general objective or cost function of SDC can be written as the addition of a time-integrated cost and a sum of point-in-time costs:

$$J = \int_{t_0}^{t_N} F(x(t), v(t), w, t)dt + \sum_{i=1}^{N} G(x(t_i), v(t_i), w, t_i, i). \tag{1}$$

The goal of SDC is to optimize J by choosing the optimal or "best" dynamic control vector $v(t)$ at all time instants $t \in (t_0, t_N)$ simultaneously with the optimal static parameter vector w. The objective J can be either minimized or maximized in value. The general functions F and G in Eq. (1) are selected to best represent the design and control objectives for a specific application. The times t_i are assumed to lie between t_0 and t_N for $i = 1, 2, ..., N-1$. The functions F and G are required to be twice continuously differentiable with respect to x, v, and w. The functions F and G do not need to be continuous in time t.

In addition to the objective Eq. (1), SDC requires an ordinary differential equation which describes the time evolution of the state vector x, and a function that specifies the initial state $x(t = t_0)$:

$$\frac{dx(t)}{dt} = T(x(t), v(t), w, t) \qquad x(t = t_0) = \Gamma(w), \tag{2}$$

The state function T is required to be twice continuously differentiable with respect to x, v, and w. However, the state function T does not need to be continuous in time t. The vector function T is selected to best represent the time evolution of the state vector $x(t)$ under the influence of the current state, the current dynamic control vector $v(t)$, and the static parameter vector w at time instant t. The initial condition can be given and fixed, or it can be a function of the static control vector w. The function Γ is required to be once continuously differentiable with respect to w.

SDC optionally allows two types of constraints on the formulation Eqs. (1) and (2). The first type are ordinary constraints of the general form:

$$L(x(t), v(t), w, t) \geq 0 \quad \textit{and/or} \quad K(x(t), v(t), w, t) = 0, \tag{3}$$

The linear or nonlinear vector functions L and K are selected to represent practical or physical constraints on the engineering problem. An example of a constraint of this type is a minimum allowed distance between the Sun and a spacecraft to avoid spacecraft overheating.

The second type of constraint SDC allows are "control dynamic" constraints. Control dynamic constraints represent any physical or practical engineering constraints on the *time evolution* of the dynamic control vector $v(t)$. The control dynamic constraints have the general form:

$$v(t) = \left| \begin{array}{ll} f(u_1, w, t, 1) & \textit{for } t = t_0 \textit{ to } t_1 \\ f(u_2, w, t, 2) & \textit{for } t = t_1 \textit{ to } t_2 \\ \vdots & \vdots \\ f(u_N, w, t, N) & \textit{for } t = t_{N-1} \textit{ to } t_N. \end{array} \right| \tag{4}$$

The vector functions $f(u_i, w, t, i)$ are selected to properly represent the limitations on the time evolution of $v(t)$. The number of periods N may be chosen arbitrarily. The functions f are parameterized by a parameter vector u_i, the static parameter vector w, and time t. The functions f can be used to effectively limit or constrain the SDC algorithm to consider only solutions $v(t)$ which are of the form of Eq. (4). The time intervals t_i to t_{i+1} are called "periods." The dynamic parameter vector u_i is constant within each period i, $i = 1, 2, ..., N$. For example, the simplest useful set of functions f is $f(u_i, w, t, i) = u_i$. The dynamic control vector $v(t)$ may be optimized such that $v(t)$ is constant over each period, allowing changes only at period interfaces t_i. Alternatively, $v(t)$ may be subject to a dynamic limitation that allows $v(t)$ to vary within each period, either continuously or discontinuously.

If SDC is used with control dynamic constraints, then the algorithm is called the *period formulation of the SDC*. If no control dynamic constraint is used then the algorithm is called the *fully continuous formulation of SDC*. In this research, the period formulation of SDC was used to constrain the trajectory optimization to only allow changes in the thrust direction and magnitude at regular time intervals. The regular time intervals represent the practical limitations of spacecraft control resulting from communications and/or duty cycles.

Application of SDC to Trajectory Optimization

The first step in applying SDC to the problem of low-thrust trajectory optimization requires defining the state and control variables. The state vector $x(t)$ is defined to be the spacecraft state at any given time t. The components of the state vector $x(t)$ are defined as follows,

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \\ x_6(t) \\ x_7(t) \end{bmatrix} = \begin{bmatrix} x \; coordinate \; of \; spacecraft \\ y \; coordinate \; of \; spacecraft \\ z \; coordinate \; of \; spacecraft \\ x \; velocity \; of \; spacecraft \\ y \; velocity \; of \; spacecraft \\ z \; velocity \; of \; spacecraft \\ mass \; of \; the \; spacecraft. \end{bmatrix} \tag{5}$$

The dynamic control $v(t)$ is defined to be the electric propulsion thrust vector as a function of time. The components of the dynamic control vector $v(t)$ are defined as follows,

$$v(t) = \begin{bmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \end{bmatrix} = \begin{bmatrix} x \; component \; of \; thrust \\ y \; component \; of \; thrust \\ z \; component \; of \; thrust. \end{bmatrix} \tag{6}$$

The components of the static control vector w are defined to be

$$w = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \\ \vdots \\ w_{5+n} \end{bmatrix} = \begin{bmatrix} Date \; of \; Earth \; launch \\ total \; flight \; time \\ x \; component \; of \; launch \; V_\infty \\ y \; component \; of \; launch \; V_\infty \\ z \; component \; of \; launch \; V_\infty. \\ Date \; of \; first \; intermediate \; flyby \\ \vdots \\ Date \; of \; n^{th} \; intermediate \; flyby. \end{bmatrix} \tag{7}$$

The SDC algorithm is not limited to the definition Eqs. (5), (6), and (7). These definitions were used to perform comparisons between SDC and existing programs. Additional control and state dimensions can be added. For example, the static control w could be augmented with design parameters like solar array size. The state vector x could be augmented with a state representing the total spacecraft radiation dose.

The second step in applying SDC is to provide an initial condition function $x(t_0) = \Gamma(w)$. The following definition is used to provide a launch from Earth as the initial condition. The initial mass of the spacecraft is obtained from a launch vehicle performance curve depending on the launch $V_\infty = (w_3, w_4, w_5)$.

$$\Gamma(w) = \begin{Bmatrix} initial \; position \\ initial \; velocity \\ initial \; mass \end{Bmatrix} = \begin{Bmatrix} X_e(w_1) \\ V_e(w_1) + w_{3:5} \\ mlv_c(w_{3:5}) \end{Bmatrix} \tag{8}$$

The vector functions $X_e(w_1)$ and $V_e(w_1)$ are the Earth's center location and velocity at the launch date w_1. The function $mlv_c(w_{3:5})$ is the launch mass for the launch energy $C_3 = ||w_{3:5}||^2$. The JPL

Lunar and Planetary Ephemerides are used to define $X_e(w_1)$ and $V_e(w_1)$. Note that the trajectory begins at the center of a massless Earth. This simplification was necessary to make comparisons with existing low thrust optimization programs which make the same launch approximation. More realistic launch conditions involving multi-body propagation have been successfully incorporated using SDC.

The state equation used to describe the time evolution of the state is

$$\frac{dx}{dt} = T(x, v, w, t) = \begin{bmatrix} x_4(t) \\ x_5(t) \\ x_6(t) \\ \frac{v_1(t)}{x_7(t)} + \sum_{i=1}^{11} \frac{\mu_i r_i(1)}{\|r_i\|^3} \\ \frac{v_2(t)}{x_7(t)} + \sum_{i=1}^{11} \frac{\mu_i r_i(2)}{\|r_i\|^3} \\ \frac{v_3(t)}{x_7(t)} + \sum_{i=1}^{11} \frac{\mu_i r_i(3)}{\|r_i\|^3} \\ \dot{m}(\|v\|) \end{bmatrix} = \begin{bmatrix} x \; velocity \; of \; spacecraft \\ y \; velocity \; of \; spacecraft \\ z \; velocity \; of \; spacecraft \\ x \; acceleration \; of \; spacecraft \\ y \; acceleration \; of \; spacecraft \\ z \; acceleration \; of \; spacecraft \\ mass \; flow \; rate \end{bmatrix}, \quad (9)$$

The mass flow rate $\dot{m}$ is provided by a polynomial fit to the performance of the NSTAR 30-cm ion thruster[3], a version of which is in operation on Deep Space 1. The specific impulse is not constant, but depends on the engine throttle level.

Constraints of the form of Eq. (3) are used to constrain the thrust and reach intermediate and final target bodies. The maximum thrust is constrained by the performance of the thruster(s) and the power available from the solar array at a given heliocentric radius. Target constraints include flyby or rendezvous state constraints.

The objective used for all comparisons was to maximize the final spacecraft mass or net mass, i.e. *maximize* $x_7(t_N)$. This objective takes into account the launch vehicle performance and propellant usage.

RESULTS

Results produced by SDC for several test problems are compared to results produced by two other optimization programs. One program is based on the calculus of variations, the other is based on parameter optimization. The two programs used for comparison are SEPTOP[1] and CLSEP[4], both developed by the Jet Propulsion Laboratory. The program SEPTOP is a low-thrust optimization program based on the calculus of variations. SEPTOP is the successor program to the well known program VARITOP[5]. Both SEPTOP and VARITOP have been used extensively by the Jet Propulsion Laboratory to design a variety of low-thrust missions. The program CLSEP is a low-thrust optimization program based on nonlinear parameter optimization. CLSEP parameterizes the problem by dividing the trajectory into a series of legs. CLSEP uses the nonlinear programming software SNOPT[6] to solve the resulting problem. Both SEPTOP and CLSEP propagate the trajectory assuming the only gravitating body is the Sun. An intermediate flyby is modeled as an instantaneous rotation of the V_∞ vector at the planet's center. This is in contrast to SDC which is based on multi-body propagation. In order to optimize the trajectory, SDC requires that the Sun and at least the intermediate flyby planet(s) be gravitating. SDC was applied with a non-gravitating Earth launch and final target body in order to make the comparisons with SEPTOP and CLSEP as close as possible.

SEPTOP and CLSEP require intermediate flyby sequences to be specified before the optimization begins. Both parameter optimization and calculus of variations methods require the intermediate flyby sequence to be given and fixed. The SDC method does not require the intermediate flyby sequence to be given and fixed. Since SDC does not require the flyby sequence to be fixed, SDC can locate favorable intermediate flyby sequences on its own. The particular intermediate flyby sequence that SDC will converge to will depend on the initial trajectory supplied to SDC to begin the iterations. SDC does not require a good, or even feasible, trajectory to begin with. Examples of starting trajectories are given with some of the test cases that follow. A constraint was added to the SDC formulation to fix the intermediate flyby sequence for comparison to SEPTOP and CLSEP.

The test problems include an Earth launch to Mars flyby; Earth launch to Mars flyby, to a flyby of the asteroid Vesta; Earth launch to Venus flyby to Mercury rendezvous; and Earth launch to Venus flyby to Mars flyby to Jupiter flyby.

Earth to Mars Flyby

The Earth to Mars flyby is the simplest and most direct comparison possible because SDC can be used with only the Sun gravitating, similar to both SEPTOP and CLSEP. The flyby of Mars test problem fixed the launch date to be May 20, 2003, the arrival date to be December 6, 2003, the launch mass to be 585.0 kg, and the launch V_∞ magnitude to be 1.66 km/s. The direction of the V_∞ vector is free. For SDC, the flight time was divided into 200 periods (1 day per period) during which the thrust magnitude and direction was constrained to be constant. The solar array power at 1 AU is 6.0 kw. The spacecraft carries a single thruster with a maximum thrust of 92.3 mN when supplied 2.6 kw. The optimization variables include the launch V_∞ direction and the solar electric thrust sequence. Since the launch mass is fixed in this problem, maximizing the final mass at Mars is the same as minimizing the propellant mass. The results for SDC, CLSEP, and SEPTOP are provided in Table 1.

Table 1

EARTH - MARS FLYBY

Program	SDC	CLSEP	SEPTOP
Propellant (kg)	30.61	30.67	30.65

The propellant mass results for all three programs agree closely. The larger difference between SDC and SEPTOP verses CLSEP and SEPTOP is likely a result of slightly different final locations for the Sun used by the three programs. Unlike SDC, SEPTOP does not use the JPL Ephemerides at all times to locate the planets and the Sun. SEPTOP uses a single epoch at which the state returned from the Ephemerides is used to define fixed classical orbital elements for the planets. Unlike SDC, both CLSEP and SEPTOP do not account for the motion of the Sun relative to the solar system barycenter.

The initial guess supplied to SDC was launch V_∞=(1.66,0,0) km/s, and zero thrust at all times. This initial guess misses Mars by 75 million kilometers. Despite the poor starting trajectory, SDC locates Mars and converges rapidly. SDC runtime for this problem is on the order of 30 seconds using a Sun Ultra 10 workstation.

Earth to Mars Flyby to Vesta flyby

The Earth to Mars flyby to Vesta flyby involves a single thruster and a solar array which produces 10 kw at 1 AU. The base case launch date is October 4, 2009, the Mars flyby date is May 2, 2010, and the Vesta flyby date is January 27, 2011. The base case launch mass is fixed at 545.0 kg, and the launch V_∞ magnitude is fixed at 2.80 $\frac{km}{s}$. Other comparisons involve optimizing the flight time, the launch date, the Mars flyby date, the magnitude of V_∞, and the initial mass.

SDC must use multi-body propagation for the Earth to Mars Flyby to Vesta flyby problem. SDC requires Mars to be gravitating to correctly represent the intermediate flyby. SEPTOP and

CLSEP propagate the trajectory under the influence of only the Sun to the center of Mars and then instantaneously rotate the V_∞ vector. This is a fairly significant difference computationally and is expected to result in somewhat different results. Table 2 provides the results of the comparison. Figure 1 provides a plot of the optimal trajectory obtained by SDC.

Table 2

EARTH - MARS - VESTA FLYBY*

Launch V_∞ Magnitude	Earth Launch Date	Mars Flyby Date	Quantity to be Compared	SEPTOP	CLSEP	SDC
fixed	fixed	fixed	Final mass (kg)	493.74	493.73	494.05
			Mars radius (km)	6260	6215	6212
free	fixed	fixed	Final mass (kg)	503.38	503.46	503.31
			Mars radius (km)	6756	6691	6643
			Launch C_3 ($\frac{km^2}{s^2}$)	6.768	6.764	6.811
fixed	fixed	free	Final mass (kg)	503.46	NA	503.36
			Mars radius (km)	5496		5497
			Date at Mars	312.92		313.17
free	free	fixed	Final mass (kg)	503.39	503.46	503.32
			Mars radius (km)	6685	6682	6645
			Launch C_3 ($\frac{km^2}{s^2}$)	6.778	6.769	6.811
			Launch date	108.79	108.68	108.31
free	free	free	Final mass (kg)	504.36	504.43	504.14
			Mars radius (km)	6082	6039	6071
			Launch C_3 ($\frac{km^2}{s^2}$)	7.283	7.317	7.300
			Launch date	109.97	110.07	110.15
			Date at Mars	315.35	315.05	315.45

* The "Mars radius" is the distance of closest approach to Mars measured to the center of Mars. The "Date at Mars" is the time of closest approach. All dates are the last three digits plus two decimals of the Julian date: 2,455,xxx.xx. "NA" indicates results that are not available.

Table 2 demonstrates very good agreement between all three methods - despite the fact SDC integrates through the Mars flyby and CLSEP and SEPTOP do not. The approximation used by CLSEP and SEPTOP to model flybys will be worse for larger planets.

The initial guess supplied to SDC was launch V_∞=(0,2.8,0) km/s, and zero thrust at all times. The initial trajectory is never closer than 23 million kilometers to Mars and misses Vesta by 375 million kilometers. Despite this poor guess, SDC locates the Mars flyby and Vesta and converges rapidly. The runtime for this problem is on the order of 3 minutes using a Sun Ultra 10 workstation.

Earth to Venus Flyby to Mercury Rendezvous

The Earth to Venus flyby to Mercury rendezvous involves a single thruster and a solar array which produces 1.5 kw at 1 AU. The spacecraft bus consumes a constant 200 watts leaving only 1.3 kw available for the thruster at 1 AU. With this arrangement, the thruster can operate at its maximum rated thrust only below heliocentric radius 0.6455 AU. At radii greater than 0.6455 AU, the engine must be throttled. The base case launch date is August 29, 2002, the Venus flyby date is February 11, 2003 and the Mercury arrival date is December 24, 2004. Other comparisons involve freeing the launch, Venus flyby, and arrival dates. The launch vehicle used is a Delta 7326 with a

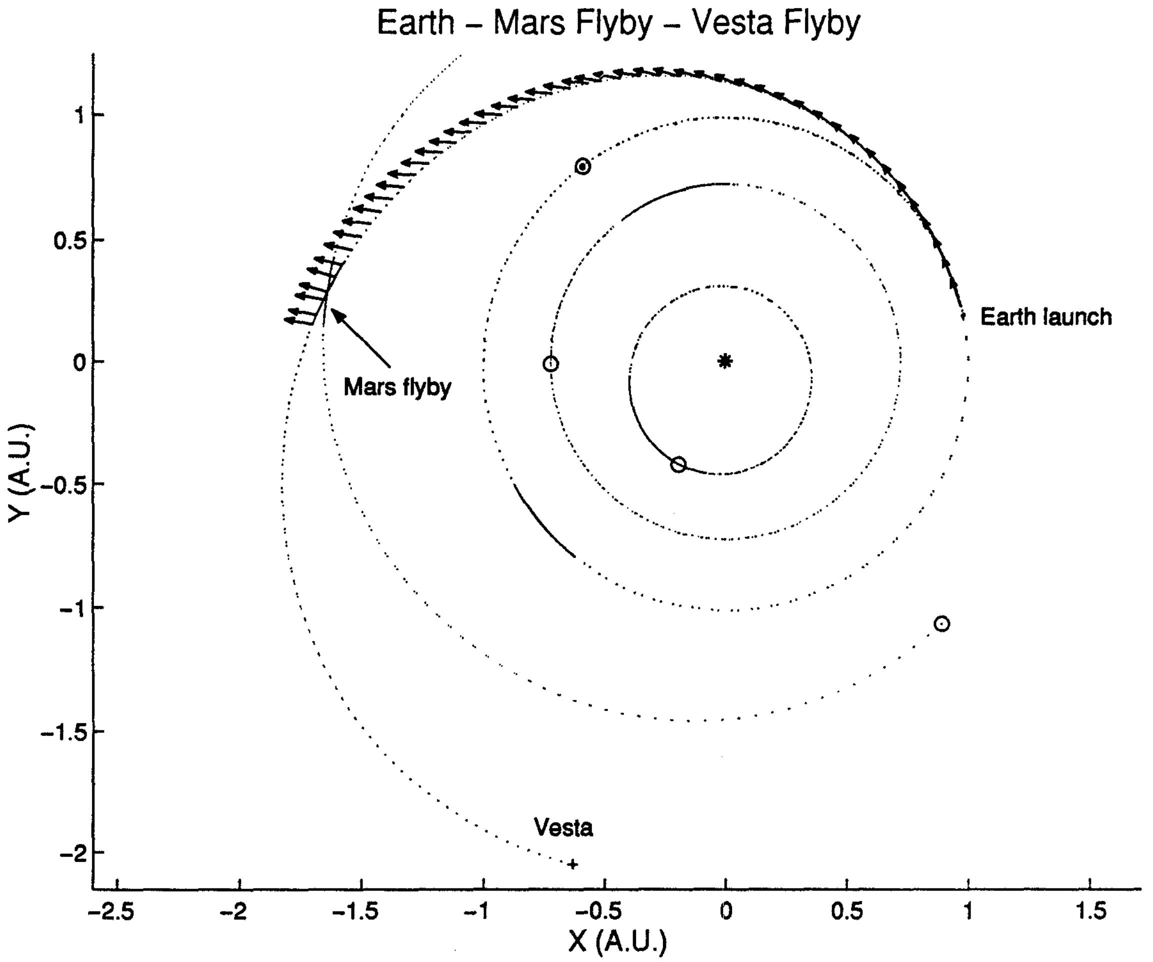

Figure 1: SDC Optimal trajectory for the Earth launch - Mars flyby - Vesta flyby problem. The arrows along the spacecraft trajectory indicate the thrust direction. The lack of arrows along the trajectory indicate coasting periods.

10% launch vehicle contingency. The launch V_∞ magnitude and direction are free. The objective is to maximize final mass. This problem is known to be very difficult.

The results for the Earth launch to Venus flyby to Mercury rendezvous problem are provided in Table 3. The optimal trajectory for this test problem as determined by SDC is plotted in Figure 2.

Table 3 indicates that all three programs produce similar optimal values. The SDC solution is closer to the SEPTOP solution than to the CLSEP solution in terms of both final mass and the trade off between propellant usage and launch energy C_3. The three programs are not expected to produce identical results due to differences in the way each method represents engine operation, flybys, and planet locations.

Table 3

EARTH - VENUS - MERCURY RENDEZVOUS*

Earth Launch Date	Venus Flyby Date	Mercury Arrival Date	Quantity to be Compared	SEPTOP	CLSEP	SDC
			Final mass (kg)	316.01	312.76	315.34
fixed	fixed	fixed	Venus radius (km)	6951	6352	6953
			Launch C_3 ($\frac{km^2}{s^2}$)	7.421	8.769	7.539
			Final mass (kg)	316.90		317.74
			Venus radius (km)	9703		7591
free	free	fixed	Launch C_3 ($\frac{km^2}{s^2}$)	6.721	NA	7.288
			Launch date	2,496.8		2,507.9
			Date at Venus	2,672.8		2,677.4
			Final mass (kg)	328.12		328.77
			Venus radius (km)	9031		7857
free	free	free	Launch C_3 ($\frac{km^2}{s^2}$)	6.099	NA	6.469
			Launch date	2,502.4		2,507.4
			Date at Venus	2,678.8		2,680.6
			Mercury arrival	3,373.5		3,373.5

* The "Venus radius" is the distance of closest approach to Venus measured to the center of Venus. The "Date at Venus" is the time of closest approach. All dates are the last four digits plus one decimal of the Julian date: 2,45x,xxx.x. "NA" indicates results that were not available. The arrival date at Mercury was free in the last row subject to the range constraint 2,453,353.5 $\leq$ Arrival $\leq$ 2,453,373.5 Julian Date.

The starting trajectory provided to the SDC optimization program was poor (refer to Figure 3). The starting trajectory consisted of a simple inward spiral (thrust directed nearly opposite velocity). The spiral results in a spacecraft location more than 20 million kilometers from Venus on the nominal Venus flyby date. In addition, the spiral fails to match Mercury's position at the arrival date by more than 100 million kilometers (refer to Figure 3). Despite the poor initial trajectory, SDC converges readily – demonstrating the robustness of the method. SDC runtime for this problem is on the order of 40 minutes using a Sun Ultra 10 workstation. Convergence for SEPTOP was very difficult for this problem, requiring several weeks of user intervention.

Earth to Venus to Mars to Jupiter Flyby

The Earth to Venus flyby to Mars flyby to Jupiter flyby involves two thrusters with a 90% duty cycle and a solar array which produces 7.0 kw at 1 AU. The spacecraft bus consumes a constant 100

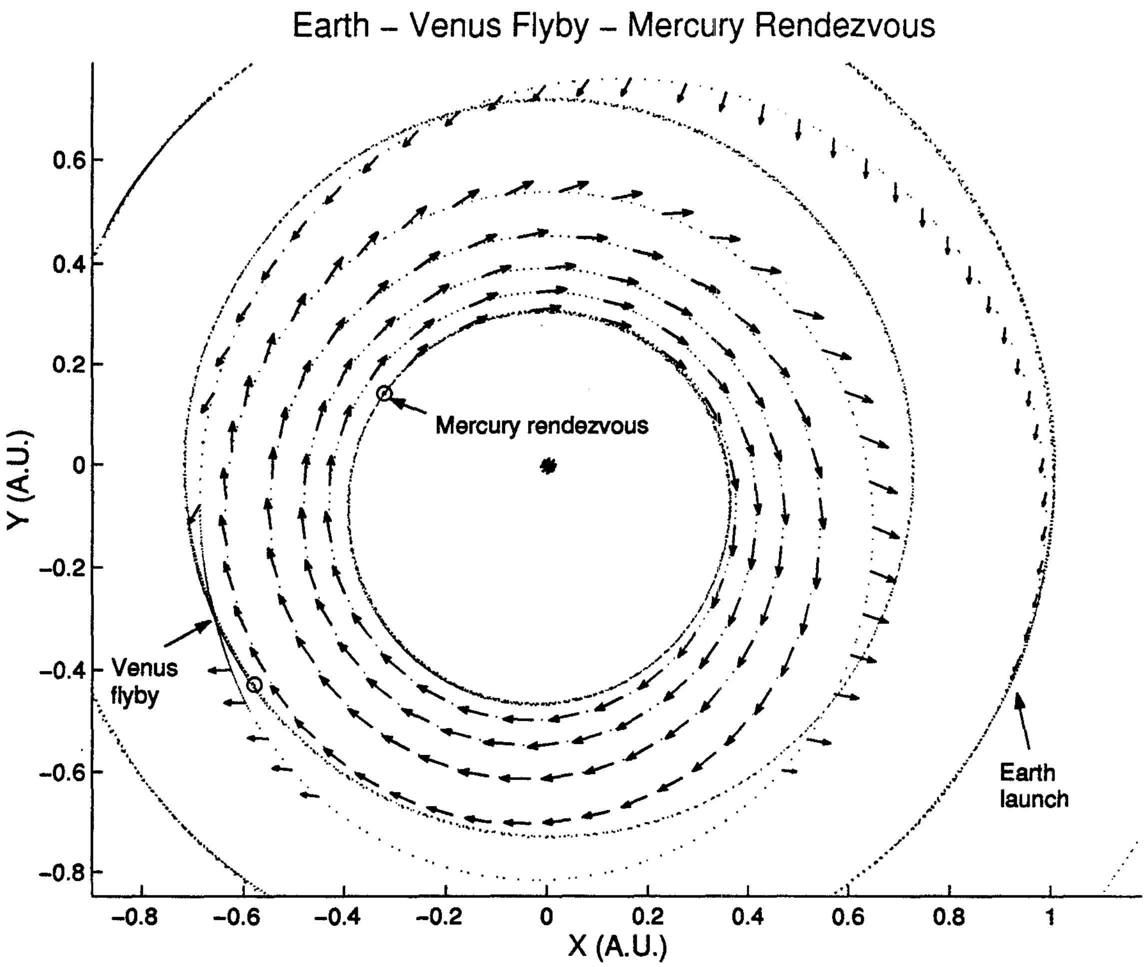

Figure 2: Optimal trajectory for the Earth launch - Venus flyby - Mercury rendezvous problem. The arrows along the spacecraft trajectory indicate the thrust direction.

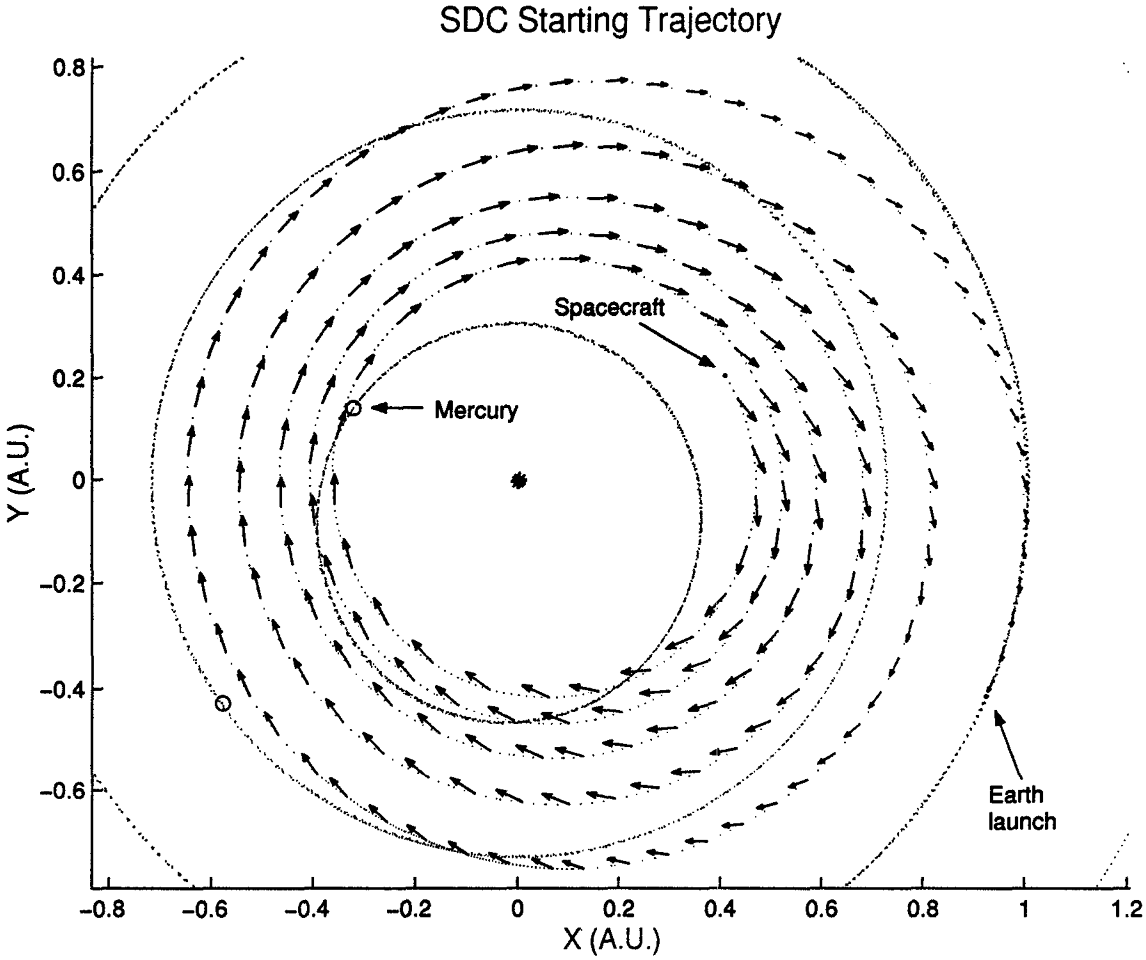

Figure 3: The initial trajectory given to SDC for the Earth launch - Venus flyby - Mercury rendezvous problem. The arrows along the spacecraft trajectory indicate the thrust direction. The spiral results in a spacecraft location more than 20 million kilometers from Venus on the nominal Venus flyby date. In addition, the spiral fails to match Mercury's position at the arrival date by more than 100 million kilometers. This demonstrates the ability of SDC to converge with a simple starting guess.

watts leaving only 6.9 kw available for the thrusters at 1 AU. The base case launch date is December 28, 2004, the Venus flyby date is September 14, 2006, the Mars flyby date is sometime at the end of 2006 (free) and the Jupiter arrival date is February 14, 2009. The launch vehicle used is a Delta 7325 with a 7% launch vehicle contingency. The launch V_∞ magnitude and direction are free.

This test problem used the most intermediate flyby bodies in this comparison. The program SEPTOP is programmed for at most two intermediate flybys due to the sensitivity created by flybys. Table 4 provides the results for SDC and SEPTOP

Table 4

EARTH - VENUS - MARS - JUPITER FLYBY*

Program	**C_3** ($\frac{km^2}{s^2}$)	**Final Mass** (kg)	**Venus Flyby Radius** (km)	**Mars Flyby Radius** (km)	**Mars Date**
SEPTOP	9.96	820.94	6604	3657	Dec. 4, 16:34
SDC	10.05	820.54	6506	3607	Dec. 4, 18:05

* The flyby radii are measured to the centers of Mars and Venus. The Mars flyby dates are in the year 2006.

Table 4 demonstrates excellent agreement between SDC and SEPTOP for a complex problem of multiple intermediate flybys. Figure 4 provides a plot of the optimal trajectory obtained using SDC.

Earth to Mars Flyby Revisited

The focus of this paper has been to verify that SDC is an accurate optimization method. In particular, it has been shown that SDC can reproduce the results of existing, accepted methods of trajectory optimization. So far in this paper, the SDC method has only been used to replicate the functionality of parameter optimization and the calculus of variations methods. However, SDC can solve problems that the previous methods cannot. Future papers will present the unique abilities of SDC. One simple example of SDC's ability to locate favorable intermediate flybys is presented in this subsection.

SDC does not require the prespecification of which intermediate bodies to use for gravitational assists. Since SDC is based on a gradient method of optimization *and* multi-body propagation, SDC can locate flybys based on the time changing derivatives of the gravitational field produced by all bodies included in the gravitational terms in Eq. (9).

As an example, reconsider the Earth to Mars flyby problem. If SDC is used to solve this problem with only the Earth and the Sun gravitating, then no intermediate flyby possibilities exist. The launch condition used for a gravitating Earth is multi-body propagated escape hyperbola, beginning at the hyperbola's periapse. If the same problem is solved using SDC with the Sun, Earth, and Moon gravitating, then SDC quickly finds a trajectory which includes a lunar gravity assist and improves the performance. Table 5 provides a comparison of the result with or without the Moon gravitating. No constraint was used to make SDC use the Moon as a gravity assist; SDC locates the flyby and optimizes it automatically. Figure 5 provides a plot of the optimal Earth to Moon flyby to Mars flyby trajectory. Figure 6 provides a plot of the same trajectory centered on the Earth to show the lunar flyby.

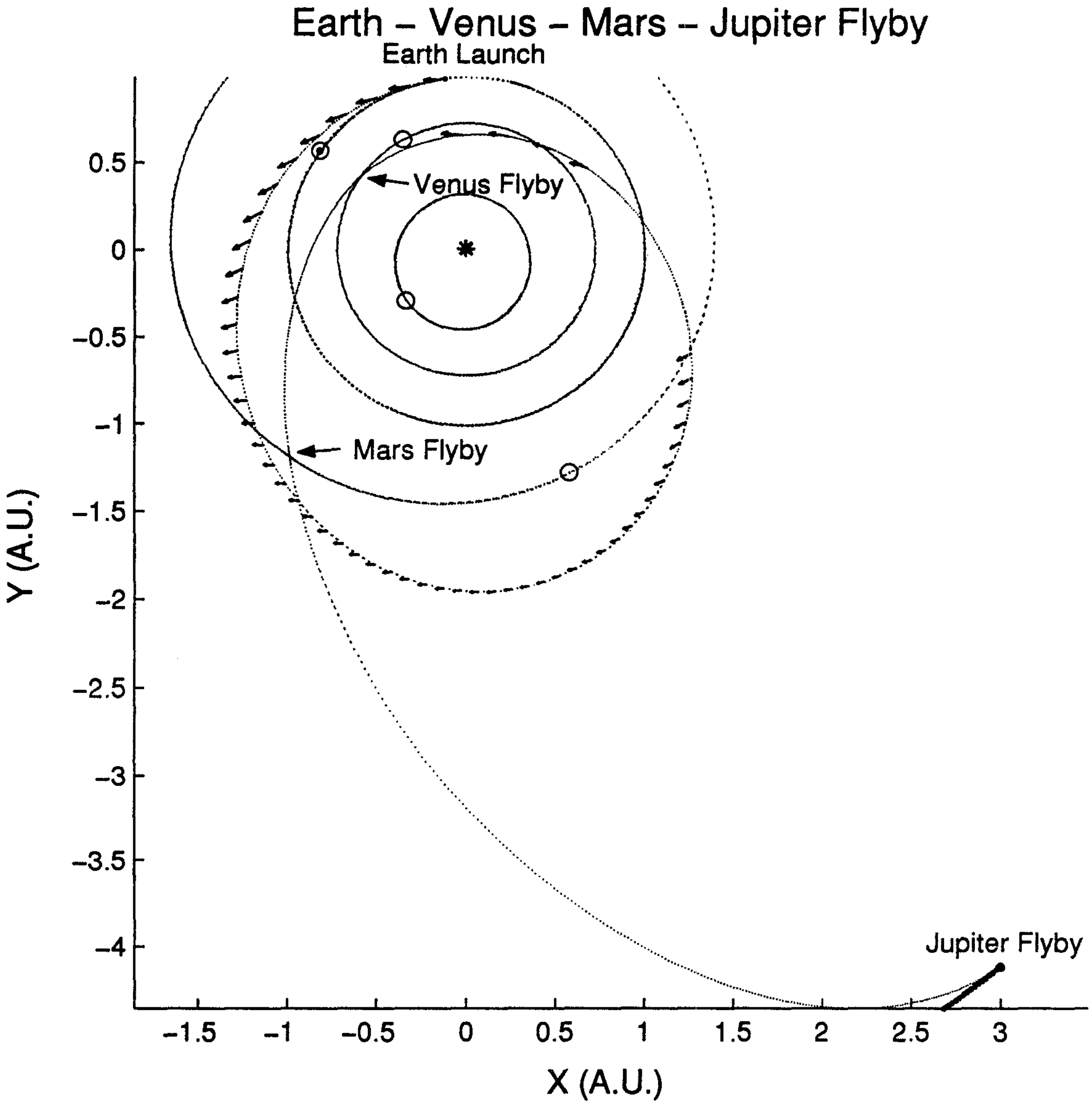

Figure 4: Optimal trajectory for the Earth launch - Venus flyby - Mars flyby - Jupiter flyby problem. The arrows along the spacecraft trajectory indicate the thrust direction. The lack of arrows indicate coasting periods

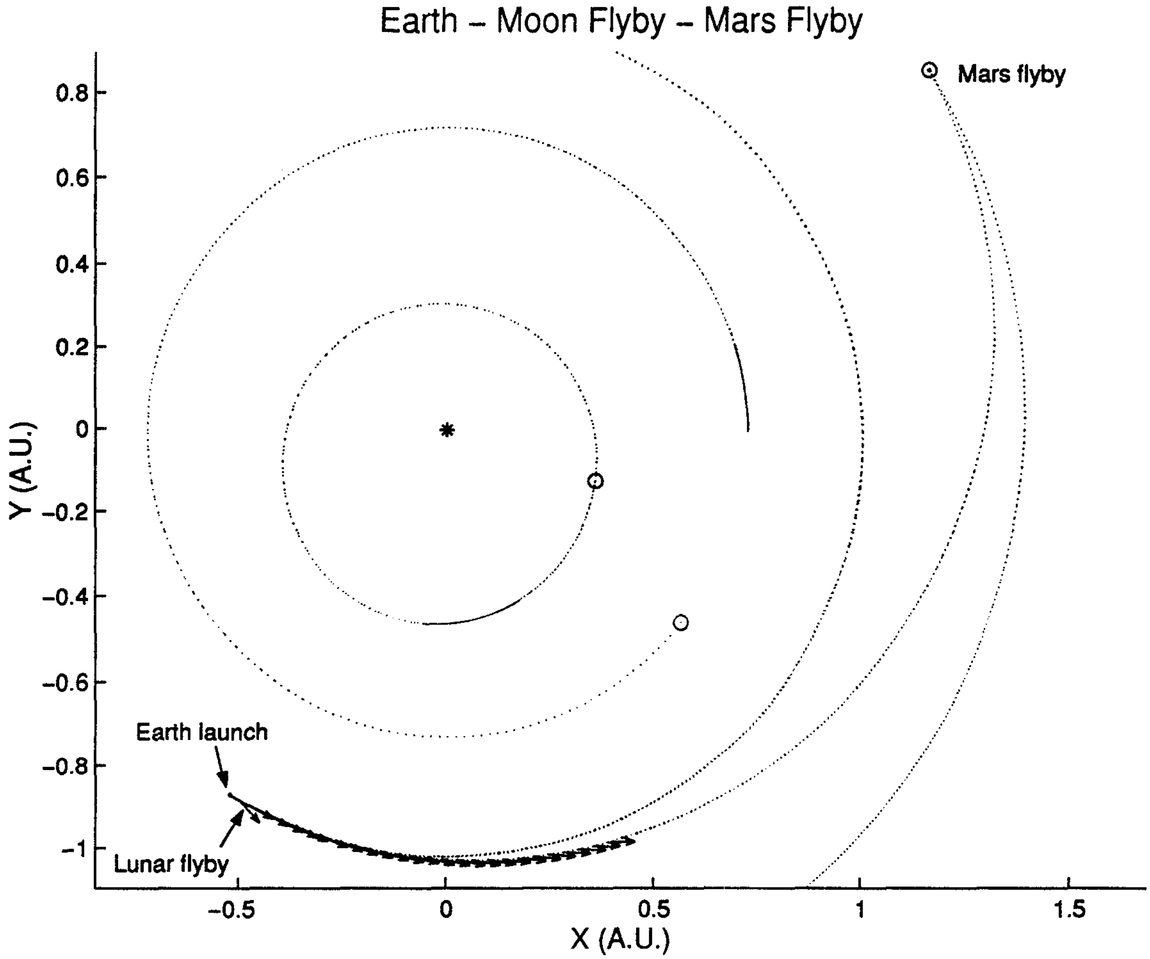

Figure 5: Optimal trajectory for the Earth launch - Moon flyby - Mars flyby problem. The arrows along the spacecraft trajectory indicate the thrust direction. The lack of arrows indicate coasting periods

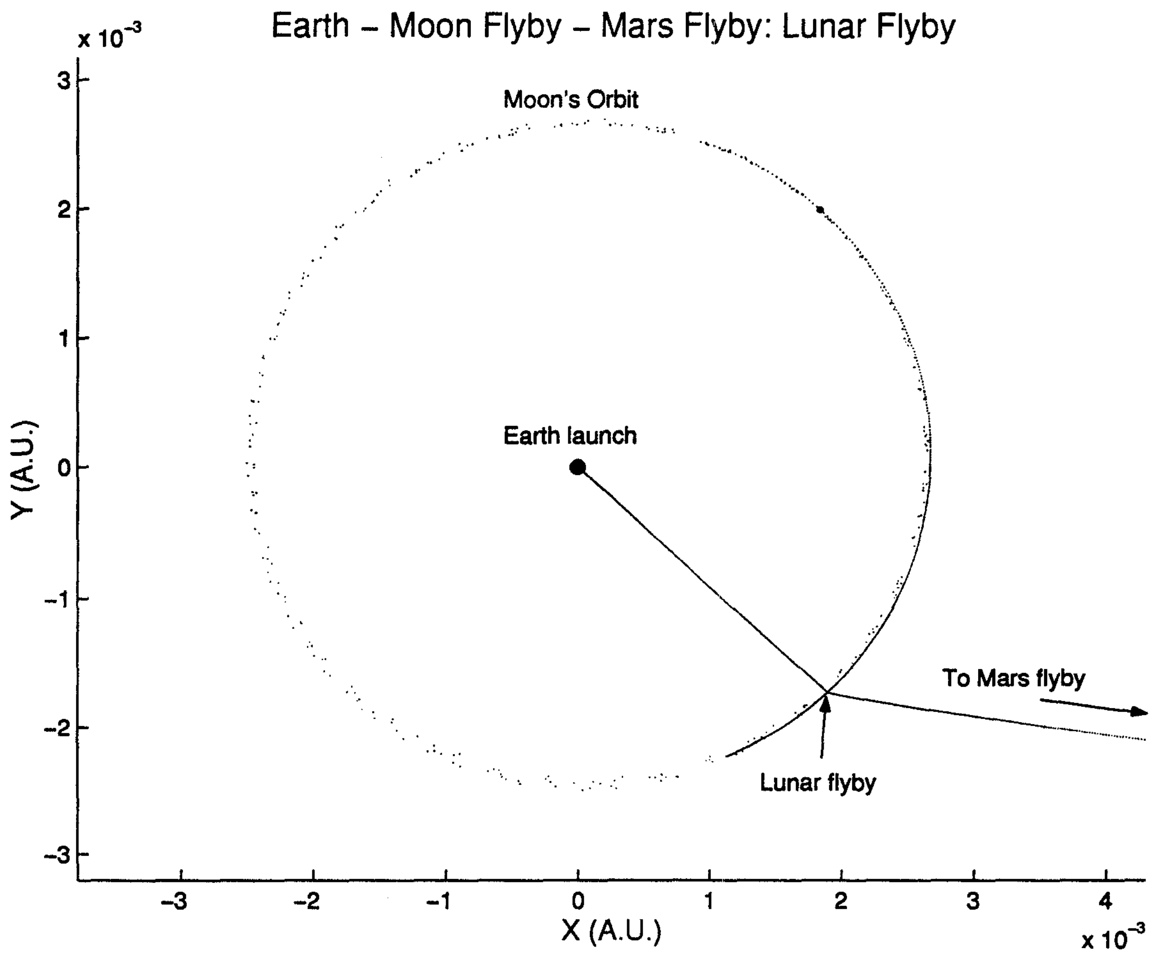

Figure 6: Optimal trajectory for the Earth launch - Moon flyby - Mars flyby problem centered on the Earth.

Table 5

EARTH - MARS FLYBY

Gravitating Terms	Final Mass at Mars (kg)	Closest approach to the Moon (km)
Sun, Earth	555.90	122,700
Sun, Earth, Moon	571.68	1,836

SDC also found alternate solutions to the Earth to Venus flyby to Mercury rendezvous problem. Depending on the initial trajectory supplied to SDC, SDC converged to different trajectories. For example, a double flyby of Venus, followed by the Mercury rendezvous was discovered. When Mercury is given mass, then SDC converges to a flyby of Mercury before the rendezvous with Mercury.

CONCLUSION

The accuracy of the SDC algorithm applied to low-thrust trajectory optimization was verified by comparison to two programs based on parameter optimization and the calculus of variations. The results of the three optimization programs SDC, SEPTOP, and CLSEP agree closely. SDC was demonstrated to be an effective method for low-thrust trajectory design. Since SDC is based on multi-body propagation, SDC can provide a high fidelity design tool. Both SEPTOP and CLSEP are limited to low fidelity preliminary design optimization.

Future papers will present the abilities and characteristics of SDC which cannot be reproduced with other methods of optimization.

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