Overview of SIM Wide Angle Astrometric System Calibration Strategies

L. Sievers*, R.P. Korechoff, M.H. Milman, S. Shaklan, J. Catanzarite, I. Basdogan, M.V. Papalexandris, R. Swartz; Jet Propulsion Laboratory and the California Institute of Technology

ABSTRACT

This paper summarizes two very different strategies envisioned for calibrating the systematic field dependent biases present in the Space Interferometry Mission (SIM) instrument. The *Internal* Calibration strategy is based on pre-launch measurements combined with a set of on orbit measurements generated by a source *internal* to the instrument. The *External* Calibration strategy uses stars as an *external* source for generating the calibration function. Both approaches demand a significant amount of innovation given that SIM's calibration strategy requires a post-calibration error of 100 picometers over a 15 degree field of regard while the uncalibrated instrument introduces 10's-100's of nanometers of error. The calibration strategies are discussed in the context of the Wide Angle Astrometric mode of the instrument, although variations on the Internal Calibration Strategy may be used for doing Narrow Angle Astrometry.

1. INTRODUCTION

1.1. Overview of the SIM measurement

SIM is designed to measure the positions of the stars at the microarcsecond level. The SIM instrument [ref 1] uses three Michelson white light interferometers to make the measurement. Figure 1 shows how a single interferometer makes an astrometric measurement.

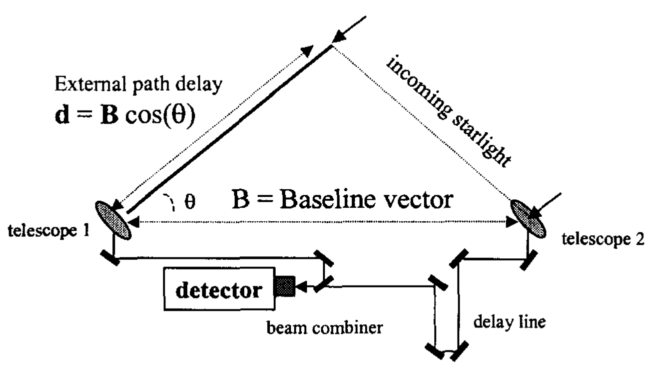

Figure 1. Single interferometer configuration while making a measurement

The interferometer uses two separated collectors to view light from a single star. The line connecting the fiducial points at the center of the two collectors is defined as the interferometer baseline. After the starlight arrives at each collector, it is directed to the beam combiner where the starlight from the two arms is interfered. Interference fringes can only occur when the optical path traversed from the star through the left interferometer arm is equal to the optical path from the star through the right interferometer arm. In order to achieve this, a delay line is used in the optical train to introduce additional optical path into one of the arms. The instrument is in its operational state when the starlight interference fringe contrast is maximized, thus the optical path from the star through each of the interferometer arms is equal.

As shown in Figure 1, if the star being observed is not normal to the interferometer baseline, then light incident on one of the collectors will arrive before the other collector. The extra distance traveled, $d = B\cos(\theta)$, is known as the external delay and is the fundamental instrument measurement. This quantity cannot be measured directly, but can be deduced from measuring the difference in the optical path between the center of each of the collectors to a common fiducial in the interferometer beam combiner using a metrology system. This quantity is called the internal path delay and should be exactly equal to the external path delay, d. If the baseline vector B were known perfectly, the astrometric quantity of interest, θ, could be computed using the internal path measurement and B.

* lisa.a.sievers@jpl.nasa.gov; phone: 1-818-393-1990; fax:1-818-393-4440; http://www.jpl.nasa.gov

In space, the orientation of the science baseline can change due to a drift in the attitude of the spacecraft. In order to monitor the baseline orientation in a fixed reference frame, SIM uses two additional interferometers to measure the angle between the science target star and two "fixed" guide stars. Changes in the attitude of the science interferometer baseline with respect to the guide star baselines can be updated continuously (assuming the relative baseline orientation of the three interferometers remains fixed). The effect of attitude changes in the spacecraft on the science baseline estimate can then be removed in the ground data processing of the astrometry data.

If the instrument were perfectly rigid, the baseline orientation of the two guide interferometers and the science interferometer would be fixed relative to one another. Unfortunately due to thermal and dynamic effects, the baselines move relative to one another, thus creating the necessity for adding an external metrology system (truss) that monitors the motion of the science baseline relative to the guide baselines. The external metrology system triangulates to each of the interferometer fiducials from a common set of fiducials. Similar to the spacecraft attitude, the effect of thermal and dynamic deformation of the SIM instrument on the baseline estimate is also removed in the ground data processing of the astrometry data.

1.2. Description of calibration problem

1.2.1. Requirements

SIM measures the optical path delay d when a star s is observed with baseline B. Ideally, the delay is given by

$$d = |B| \cos(\theta) = \langle s,B \rangle \quad \text{Equation 1}$$

where < > indicates the inner product. The star position is expressed on the unit sphere with coordinates $s(x,y) = [x,y,(1-x^2-y^2)^{1/2}]$ as shown in Fig. 2. The baseline is given by the 3-vector $B = [B_x,B_y,B_z]$.

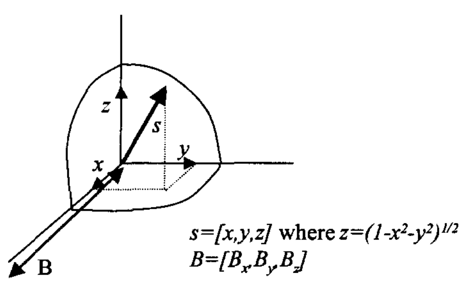

Figure 2. Star position defined on the unit sphere.

Like other astrometric instruments, the SIM instrument is not ideal and suffers from systematic field dependent biases in the measurement. In practice, the delay measurement contains an unknown constant term C_o and a bias term $C(x,y)$ so that SIM actually measures

$$d = \langle s,B \rangle + C_o + C(x,y) \quad \text{Equation 2}$$

The calibration problem is to identify/minimize the systematic field dependent bias term C(x,y) so that the calibrated delay has an error due to bias of < 100 pm. This is a stringent requirement given the un-calibrated bias levels are in the 10's-100's of nanometers (see Figure 3). We ignore any time variation in the calibration function since we assume that the variation is slow relative to the time interval between calibrations.

Another set of key requirements is that the time to calibrate must be <5 % of the mission time. It is also desirable for the calibration strategy to work any time of day as well as any time of year.

1.2.2. Sources of Bias

When an interferometer observes a star at a certain point in its field of regard, the field point is uniquely defined by a position of the delay line and corresponding articulation angle of its collectors. Hence, when the instrument slews to a new star, the collectors and delay line move to new locations. Systematic bias is introduced into the measurement due to the delay line translation and the collector articulation necessary to view multiple stars over the field of regard of the instrument. A siderostat is used as the collector (see Figure 1) in the SIM design. A corner cube is mounted on the center of the siderostat and defines the interferometer fiducial location.

As the delay line translates along its rails there is a bias introduced in both the starlight and metrology paths from diffraction. The Fresnel number for the starlight and internal metrology paths are not only different but change as the delay line translates, resulting in a different bias function due to diffraction for each source.

If the delay line rails are slightly misaligned with the beam, the metrology and starlight will walk on the retro-reflector of the delay line as it translates, creating a beamwalk on the retro-reflector as well as a beamwalk on all of the downstream optics thus introducing an OPD bias term.

As the siderostat/corner cube articulates during a slew to a new target star, biases from beamwalk are introduced in both the metrology and starlight paths. Dihedral errors on the corner cube show up as systematic biases in the metrology measurement as the corner cube articulates. There is a change in reflection phase shift dependent on the angle of incidence of the metrology beam on the corner cube that also contributes to a bias term during articulation. The offset between the siderostat face and the corner cube vertex can also introduces a geometric bias.

1.2.3. Bias terms linear in field angle don't require calibration

A significant revelation occurred early on in this work. It was realized that linear bias terms were eliminated in the data post-processing of the delay measurements. This means that all bias terms that are linear in field angle can be ignored as far as calibration is concerned. This has quite an impact on the level at which we are required to calibrate since the dominant term in the instrument bias function is linear in field (see Figures 3 and 4). A more detailed discussion of this result is contained in Shaklan, et.al. [ref. 2]

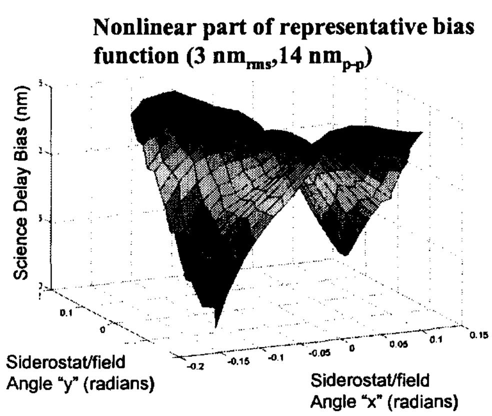

Figure 3. Example bias function of SIM instrument

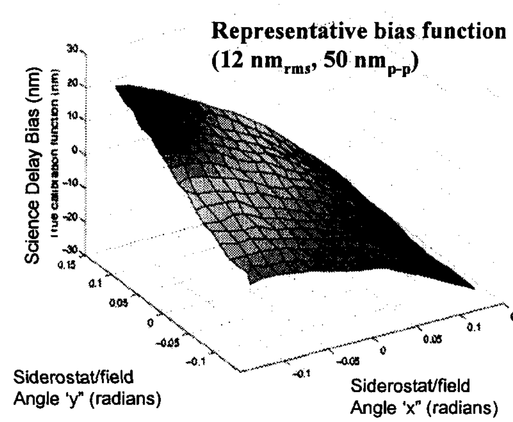

Figure 4. Bias function in figure 3 with linear term removed.

2. TWO APPROACHES FOR CALIBRATION

Two very different approaches have been proposed for calibrating SIM's systematic field dependent biases. The first approach is an indirect approach called *Internal Calibration*. The way SIM works is that multiple measurements (e.g. internal metrology, external metrology, starlight detection) are combined to create the fundamental measurement; the science delay, *d*. Internal calibration relies on a set of individual component/subsystem calibrations, not a direct calibration of the science delay. The theory says that when the calibrated component/subsystem measurements are combined to form the science delay, there will be no bias since it will have been taken out at the component/susbsystem level. The instrument is initially calibrated during a ground test and then updated on orbit with component/subsystem measurements that rely on sources internal to the instrument.

External Calibration directly measures the bias in the science delay on orbit using a field of stars as the reference system. A ground test is not feasible, although ground tests would be done to establish whether the instrument should be calibratable once it is on orbit.

2.1. *Internal* calibration strategy

2.1.1. Internal calibration process

The internal calibration scheme envisioned for SIM can be divided into two principal types of measurements, namely ground and on orbit. The ground measurement, which requires a source usually called a pseudostar, establishes the calibration function. If we were confident that this calibration function would remain valid once the SIM instrument were launched and continued to be valid over the five-year duration of the mission no on orbit calibration would be necessary. Because this is not likely to be the case, the ground calibration must be updated on orbit. Thus, the on orbit calibration procedure is concerned with *changes* to the ground calibration function. This on orbit calibration is accomplished using an internal monochromatic source, referred to as a full-aperture metrology (FAM) beam, to simulate starlight. The calibration scheme described here derives its name from the fact that only an internal source is used on orbit; that is, no use is made of real stars (see discussion below of external calibration). In order to use an internal source it is necessary to change the siderostat/corner cube element to a retro-reflection configuration. This allows the internal source and metrology to double pass the optical system.

As described above, when the science interferometer changes target objects there are two macro changes in the optical system. To view a new field point the siderostat mirror with its embedded corner cube articulates around two gimbal axes. In response to this change in field angle the optical delay line (ODL) translates to match the internal and external delays. As noted above, the on orbit calibration does not permit simultaneous translation of the ODL and articulation of the corner cube because the siderostat/corner cube must be fixed in the retro configuration. For this reason it is necessary to split the calibration function into three pieces; this concept is referred to as superposition and is discussed below. Hence, internal calibration requires deriving a separate calibration function for each macro change to the optical system. This is accomplished on the ground by translating the ODL while leaving the corner cube fixed and likewise fixing the ODL and articulating the corner cube. On orbit we may repeat the ODL translation measurement but the calibration function associated with the articulation of the corner cube must be obtained by indirect methods that are described later in this paper.

If the internal metrology system accurately reported the change in optical path experienced by the starlight when the system macro changes occurred there would be no need for calibration of field dependent systematic bias terms.[1] The reasons that the metrology system does not track changes in the starlight optical path to the accuracy required by the SIM error budget can be summarized as follows.

- Starlight (450 – 950 nm) and metrology (1.3 µm) cover different wavelengths
- Starlight and metrology have different beam sizes, beam geometries, and intensity profiles
- Starlight passes through the optical system once while the metrology double passes the optical system starting in the opposite direction from starlight
- The starlight and metrology beam footprints are mutually exclusive on a given optic and in some cases they use different optical elements

When considering a given calibration scheme we must be sensitive to all the factors listed above

2.1.2. Total calibration function

To understand the form of the total calibration function we begin by considering the ideal case where all changes to optical phase only result from changes in geometric path length through the relationship $\Delta\phi = 2\pi\Delta z/\lambda$. Referring to Fig. 1, the starlight fringe pattern for the *i*th target object measures the difference in optical path from the source to the beam splitter (B/S in Fig. 1).

$$STL^i = L_2^i - L_1^i - d^i$$

Equation 3

[1] This statement holds for measurements of the external delay. However, calibration of the corner cube is still required for the external metrology system whose function is described in section 2.1.4.

The internal metrology beams measure the path length between the fiducials (corner cubes) cc1 and cc2 and the beam splitter, B/S.

$$MET_k^i = 2L_k^i + C_k \qquad k = 1,2$$

Equation 4

The quantity C_k represents an unknown constant that is of no consequence because we are only interested in differential metrology measurements between two objects. We define the change in the metrology beam measurement as $\Delta MET_k = MET_k^i - MET_k^j$. From the above equations we have that the *change* in the external delay between object *i* and *j* is given by:

$$\Delta d \equiv d^i - d^j = \frac{1}{2}(\Delta MET_2 - \Delta MET_1) - (STL^i - STL^j)$$

Equation 5

It is important to note that even in the ideal case we are able to determine Δd only because all changes to the geometric path are common mode between starlight and metrology and thus cancel out. If this were not the case we would have to hold the geometric path length stable at the picometer level, a condition that would be very challenging on the ground to satisfy and virtually impossible on orbit. The key point is that all SIM measurements involving macro system changes must be the difference of two independent simultaneous measurements.

To transition from the ideal case described above to the real experiment we must include changes in phase introduced by diffraction, beam walk, reflection phase shifts, dihedral errors in the corner cubes, and corner cube vertex/siderostat reflecting surface separation. When we do so we get the following expression for the estimate of the change in external delay.

$$\Delta\hat{d} = \frac{1}{2}(\Delta MET_2 - \Delta MET_1) - (STL^i - STL^j) = \Delta d$$

$$+\frac{1}{2}(\Delta u_2 + \Delta\mu_2 + \Delta\omega_2 - \Delta\omega_1) - \Delta t_2 - \Delta\tau_2 - \Delta\eta_2 + \Delta\eta_1$$

Equation 6

where $\Delta\hat{d}$ is the estimate of the change in delay. The total calibration function, f_{cal}, is just the difference between the estimated change in delay and the true change in delay.

$$f_{cal} \equiv \Delta\hat{d} - \Delta d = \frac{1}{2}(\Delta u_2 + \Delta\mu_2 + \Delta\omega_2 - \Delta\omega_1) - \Delta t_2 - \Delta\tau_2 - \Delta\eta_2 + \Delta\eta_1$$

Equation 7

The various bias terms are defined below according to their associated optical system macro change. Note that the subscript number indicates the interferometer arm for which the term contributes a bias (see Fig. 1).

- ODL translation: Δt_2 = change in starlight diffraction
  $\Delta\tau_2$ = starlight beam walk
  Δu_2 = change in metrology diffraction
  $\Delta\mu_2$ = metrology beam walk
- Cornercube articulation: $(\Delta\omega_2 - \Delta\omega_1)/2$ = dihedral errors + reflection phase shifts + beam walk
- Siderostat articulation: $\Delta\eta_2 - \Delta\eta_1$ = foot print change + corner cube vertex/siderostat separation

2.1.3. Superposition

Superposition refers to the concept of splitting the total calibration function, f_{cal}, into the sum of three functions associated with macro changes to the ODL, corner cubes, and siderostats. The bias terms constituting each of these functions are given above. The validity of the superposition concept requires weak coupling between the bias terms originating with different macro system changes. One example of this cross coupling which has been studied involves the effect of changes in the metrology beam wavefront, resulting from articulation of a corner cube with dihedral errors, on the metrology beam diffraction term. R. Benson and E. Rayna of Lockheed-Martin Company compared the metrology beam diffraction bias term, as a function of ODL position, with a fixed wavefront and with wavefronts produced by a corner cube oriented to different targets locations in the field of regard (FOR). Corner cubes with dihedrals errors of (1,0,0), (1,1,0), and (1,1,1) were used where each component of the ordered triplet of numbers refers to the deviation from 90°, in units of arc seconds, of a particular interface between two facets. For example, (1,0,0) means the angle between facets 1 and 2 was set to 90° plus one arc second while the angles between facets 1 and 3 and 2 and 3 were exactly 90°. The results of this study showed that the cross coupling between corner cube induced wavefront changes and diffraction is at the sub-picometer level and, thus, not a concern for SIM.

While this cross coupling study only includes interaction between dihedral errors and diffraction, we will assume that superposition is valid for the rest of our discussion.

2.1.4. Ground calibration function

Calibration of the SIM instrument on the ground requires a source (pseudostar) that is monitored at least as well as the flight article (FA) under calibration. That is, one must essentially use an interferometer with many of the features of the FA. For this reason the source is called an inverse interferometer pseudostar (IIPS). Figure 3 shows how the IIPS and FA are configured for generating the ODL portion of the calibration function. The procedure consists of translating the ODL over the range required by the FOR and recording the difference between the starlight and metrology measurements. In Fig. 3 the position of the ODL is given by the parameter ζ. It can be seen from Eq. (6) that this measurement gives the ODL bias terms in the absence of an external delay and with fixed corner cubes/siderostats.

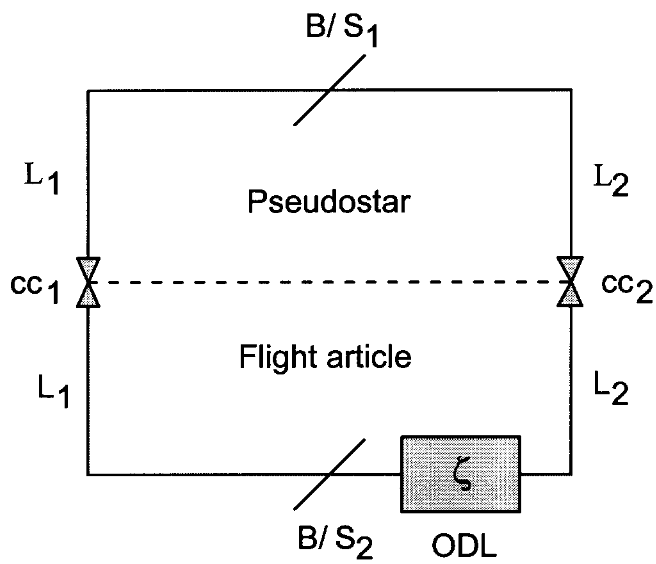

Figure 3. Flight article and pseudostar configuration calibration of the optical delay line.

The key features of the ODL calibration are as follows. (1) No IIPS macro system changes. This implies unequal path lengths between arms 1 and 2 as the ODL translates because there is no compensation in the IIPS. (2) The IIPS starlight beam is a monochromatic source ($\lambda = 0.66$ μm). This follows from item (1). The choice of wavelength is based on the fact that 0.66 μm is approximately in the middle of the starlight band and this wavelength can be easily generated by frequency doubling the highly stabilized metrology source. (3) The IIPS has an independent internal metrology system to monitor geometric path length changes at the picometer level. This feature follows from our previous discussion regarding cancellation of geometric path length perturbations. (4) The IIPS starlight beam should be oversized compared to the FA collectors to obviate the effects of IIPS diffraction on the starlight wavefront. The quantitative meaning of this statement (i.e., how big is oversized?) has not been determined as yet. (5) Implicit in item (2) is the assumption that the diffraction bias term for $\lambda = 0.66\ \mu m$ is approximately the same as the intensity weighted average of the diffraction bias term over the band 450 to 950 nm. This has been shown to be true at the 10 pm level by R. Benson.

In addition to the ODL measurement described above, we must also measure the ODL calibration function using the internal FAM source with corner cube/siderostats in the retro configuration. This measurement will be used in conjunction with a similar measurement on orbit to update the ODL calibration function derived from using the IIPS as a source.

The determination of the corner cube and siderostat calibration functions is not as direct a procedure as that for the ODL function. The calibration is performed indirectly without articulating the corner cubes/siderostats. The reason for this is two-fold. First, covering the entire FOR (7.5° in radius)) would require significant translation of the pseudostar optical system producing unwanted IIPS contributions to the bias terms. Second, articulation of the corner cube/siderostats is not an option on orbit so it makes sense to use a ground procedure that is similar to the one that will be employed in space. The calibration concept is to characterize certain parameters of the corner cubes and the siderostats and then input these parameters into a physics-based model that will predict the bias terms. The parameters of interest for the corner cubes are dihedral errors between each pair of facets; the complex index of refraction[2] for each facet; the rms wavefront error and possibly a surface phase map and/or power spectral density function for each facet. For the siderostat mirrors we need the separation between the corner cube vertex and the siderostat front (reflecting) surface and the rms wavefront error, surface phase map, and power spectral density function.

There is an alternative technique for determining the corner cube parameters. The SIM instrument utilizes an optical truss to measure the positions of its three interferometer-baselines (one for science measurements and two that act as fine-pointing sensors) relative to each other. This truss is essentially a web of one-dimensional measurements between corner cubes marking the termini of the baselines and two additional corner cubes that provide additional measurements to "stabilize" the truss. This optical truss is referred to as the external metrology system. In addition to measuring the positions of the SIM baselines relative to each other, the external metrology beams can also be used to determine corner cube parameters of interest. In order to carry out this function, each corner cube must be simultaneously interrogated by four or more beams. This lower bound on the number of beams is set by the fact that for each measurement three beams are required just to establish the spatial location of the corner cube vertex. A set of one-dimensional distance measurements are built up for each metrology beam by articulating the corner cube over the entire FOR. A least-squares formalism is then implemented to extract the corner cube parameters of interest. A simulation of this procedure is currently being developed to determine the expected accuracy of this technique as a function of the number and angle diversity of the measurements in the presence of metrology system noise.

It is important to perform both types of measurements – component characterization and truss measurements – for several reasons. The two measurements act as a consistency check that will give an indication of the expected accuracy in the determination of the bias terms. These measurements will provide an opportunity to debug data analysis software that will be needed to process on orbit data. Finally, the so-called alternative method is the only one available in space. Therefore, this measurement forms part of the baseline function update and thus is required as part of the ground calibration procedures.

2.1.5. On orbit calibration function update

The ground calibration function is considered the baseline. That baseline function is used on orbit with appropriate updates. The idea is to perform measurements on the ground that can be repeated on orbit. The differences between those ground and on orbit measurements become the updates to the baseline function.

We perform the ODL calibration by turning the corner cube/siderostats to the retro configuration and use the internal FAM source to simulate starlight. As with the ground measurement, the calibration function is the difference between the FAM and metrology measurements. Combining the on orbit and ground FAM measurements with the ground baseline measurement yields the following expression:

---

[2] The current design calls for the corner cube facets to be coated with unprotected gold.

$$f_{ODL}^{b} + f_{ODL}^{o} - f_{ODL}^{g} = \frac{1}{2}\left(\Delta u_2 + \Delta \mu_2\right)_o - \left(\Delta t_2 + \Delta \tau_2\right)_g$$
$$-\frac{1}{2}\left(\Delta t_2^{*} + \Delta \tau_2^{*}\right)_o + \frac{1}{2}\left(\Delta t_2^{*} + \Delta \tau_2^{*}\right)_g \quad \text{Equation 8}$$

The designations *b, o*, and *g* stand for baseline, on orbit, and ground. The first term on the right hand side is exactly one of the quantities of interest, namely the on orbit bias terms for metrology diffraction and beam walk. Ideally, the next three terms should give the on orbit equivalent of the starlight diffraction and beam walk. Instead, we have diffraction and beam walk bias terms for starlight measured with the IIPS and for the FAM on the ground and on orbit. The FAM terms are indicated by a superscript asterisk. Note that they are also multiplied by a factor of 1/2 because they represent a double pass through the optical system. Although the three starlight bias terms do not give us the desired quantity, all is not lost if the starlight bias terms in Eq. (8) are small compared to the calibration error budget. At this time it is difficult to estimate the size of these terms because their magnitude will strongly depend on the degree to which the optical system alignment changes between ground and space. This analysis has not yet be carried out with the SIM reference design.

Updating the corner cube bias terms is accomplished by using the external metrology truss to determine the pertinent corner cube parameters. As a departure point we assume that the parameters retain their values that were measured on the ground. The least-squares solution on orbit determines the deviation of each parameter from it ground value. Because we expect these deviations to be small, a linearized approach to the solution should be sufficiently accurate.

The final portion of the total calibration function that potentially needs to be updated is the siderostat bias terms that consist of changes to the starlight average phase as a result of a beam footprint change with articulation angle and corner cube vertex/siderostat surface separation. The internal calibration scheme described here cannot monitor either of these bias terms on orbit. Therefore, these terms must remain stable within a yet to be determined fraction of the total calibration error budget. This stability condition imposes a set of design requirements on the corner cube/siderostat assembly.

2.2. *External* calibration strategy

External calibration uses Eq. (2) to estimate the instrument bias function $c(x,y)$. Note that if s, B, and c_0 are known precisely, then (2) produces a direct measurement of $c(x,y)$. However, the star positions can only be known to catalogue accuracy (10mas--100mas), and knowledge of the baseline vector is limited by the attitude determination system accuracy and the on--board alignment subsystem (about 1 as). These *a priori* accuracies are several orders of magnitude insufficient for determining $c(x,y)$ to the required 100pm precision. Hence, (2) is inadequate as a direct measurement.

To circumvent this difficulty the external calibration scheme leverages on the main strength of the instrument -- the ability to make highly accurate *differential* measurements. This leads to a somewhat circuitous approach for estimating c by way of first making approximate measurements of ∇c at sampled points in the field of regard. Each measurement is produced by a *manuever* of the instrument while observing a single star located at a given position in the field of regard. These stars will henceforth be referred to as "calibration" stars.

2.2.1. The basic estimation problem for the instrument bias function

We will discuss later the details of these maneuvers and how they manage to get us around the problems of the insufficient initial conditions of the direct approach above. But first suppose these approximate measurements have been obtained at various points in the field, $y_{ij} \approx c(x_i,y_j)$ corresponding to the star positions $s(x_i,y_j)$. The continuous analogue for estimating c from the measurement data y is the least squares problem

$$\frac{\min}{c} J(c) = \int_D \left|\nabla c - y\right|^2 . \quad \text{Equation 9}$$

The solution to this problem is given by the solution to the elliptic boundary value problem [*]:

$$\nabla c - \operatorname{div} y = 0, \text{ inside } D, \quad \nabla_n c = \langle c, n \rangle \text{ on } \partial D.$$

Equation 10

Observe that if c is a solution, then so is $c+\alpha$ for any constant α; hence c is only determined modulo a constant. This does not present any difficulty, however, since as discussed earlier the instrument bias function only needs to be determined modulo a constant and linear term. We remark that the association of the least squares problem with the boundary value problem is very useful in the error analysis [*].

The discretized least squares problem uses finite differences to approximate the gradient. This problem arises in many applications, including the standard adaptive optics problem of wavefront reconstruction from slope measurement data (e.g., Hartmann sensor data) [*]. In this context the inability to uniquely determine the solution results in a piston error of the reconstructed wavefront; which is also generally tolerable in the AO application.

Assuming the gradient has been sampled at a finite number of points in the field, there are two important sources of error that contribute to the error in reconstructing c. The first is the error produced by the measurement error. This is essentially a stability issue, and the noise propagation properties of the least squares approach are well understood [*]. The second error is the discretization error due to the finite sampling of the field. The resulting error is a function of the mesh size, which for us is governed by the number of calibration stars used in the field of regard and their distribution, and importantly, the smoothness of the underlying instrument bias function. The basic relationship is that the smoother the function (as defined by the magnitude of the higher order derivatives of the function), the larger the mesh size, or equivalently the fewer calibration stars required. A priori knowledge of the properties of the instrument bias function will be garnered from a combination of modeling and testing. As described in the previous section, effects due to polarization, dihedral error, diffraction and corner cube/siderostat offset all produce behaviors that can be modeled by low order polynomials, which portends well for the overall external calibration approach.

2.2.2. The gradient approximation

The measurement of the gradient uses a central difference approximation constructed from delay measurements made in conjunction with specific maneuvers of the instrument. These maneuvers involve canting and rolling the interferometer baseline a known magnitude to produce small known changes in the star positions relative to the instrument. The associated observed change in delay is then attributed to a change in the instrument bias function.

The geometry of this process is based on the following observation. Let R denote a rotation matrix, and suppose the baseline vector B is rotated by the inverse of this matrix R^T (recall that the inverse of an orthogonal matrix is its transpose). Then

$$\begin{aligned}\langle s(x,y), R^T B \rangle &= \langle Rs(x,y), B \rangle \\ &= \langle s(x',y'), B \rangle\end{aligned}$$

Equation 11

where

$$\begin{bmatrix} x' \\ y' \\ \gamma' \end{bmatrix} = R \begin{bmatrix} x \\ y \\ \gamma \end{bmatrix}$$

Equation 12

For example, when R^T represents a rotation about the y-axis of α radians, then

$$x' = x\cos(\alpha) - \gamma\sin(\alpha), \quad y' = y.$$

Equation 13

From the perspective of the instrument the star has moved to *(x',y')*. Correspondingly the associated instrument bias term is *c(x',y')*, and consequently the measured delay due to the rotation of the baseline vector is

$$\begin{aligned} d' &= \langle s(x',y'), B \rangle + c(x',y') \\ &= \langle s(x,y), R^T B \rangle + c(x',y') \end{aligned}$$

Equation 14

In particular, if the baseline is canted $\pm\alpha$ radians about the y-axis, the observed delays are (assuming small $|\alpha| \approx 1^\circ$)

$$d \pm \langle s(x,y), R^T(\pm\alpha)B \rangle + c(x \pm \alpha, y).$$

Equation 15

The central difference approximation to $\partial c/\partial x$ is

$$\frac{\partial c}{\partial x}(x,y) = \frac{1}{2\alpha}\left[c(x+\alpha,y) - c(x-\alpha,y)\right] + O(\alpha^2).$$

Equation 16

where the magnitude of the constant term associated with $O(\alpha^2)$ is governed by $|\partial c^3(x,y)/\partial x^3|$ This underscores again the importance of the underlying smoothness of the bias function to the ability of obtaining accurate estimates of it using the external calibration approach.

Equating the difference approximation with the delays we have

$$\frac{\partial c}{\partial x}(x,y) \approx \frac{1}{2\alpha}\left[d_+^x - d_-^x - \left\langle s(x,y), \left(R_y^T(\alpha) - R_y^T(-\alpha)\right)B\right\rangle\right]. \quad \text{Equation 17}$$

By performing the same maneuver about the x-axis an approximation to $\partial c(x,y)/\partial y$ is constructed in the same fashion:

$$\frac{\partial c}{\partial y}(x,y) \approx \frac{1}{2\alpha}\left[d_+^y - d_-^y - \left\langle s(x,y), \left(R_x^T(\alpha) - R_x^T(-\alpha)\right)B\right\rangle\right]. \quad \text{Equation 18}$$

Obtaining the approximate gradient requires the four delay measurements and a priori knowledge of the star position and the baseline orientation and length. The reliance on a priori baseline and star position vector knowledge was the downfall of the direct measurement approach. However, in [*] it is shown that the sensitivities to these errors are dramatically reduced in the gradient approach. The relationship between the reconstruction error in *c* and star position error is demonstrated to be attenuated by the square of the mesh size. For a 15°x15° field of regard, by placing a calibration star within every square degree, errors of about 3 mas in star position are tolerated. Similarly, using an analysis based on the elliptic equation (*), it is shown that a baseline orientation/length error contributes principally to a linear error in the estimation of *c*. But linear errors are again tolerable. The result of this analysis is that 100 mas orientation and .5 um length a priori knowledge is adequate.

There is still a small gap remaining between our attainable a priori knowledge of the star positions and baseline vector using catalogue values and the attitude/alignment determination system and the requirements imposed above. To bridge this gap we use the instrument itself to improve on the a priori knowledge. By making delay measurements on the 15°x15° calibration field of stars, it is possible to improve the knowledge of both the baseline vector and star positions significantly enough to reach these objectives. The companion paper [*] describes this process.

2.2.3. Comparison of approaches

Each strategy has its own distinct advantage associated with it. A compelling advantage to Internal Calibration is that the speed of the on orbit calibration procedure would likely be very fast, thus the science observing time would be minimally impacted. Another advantage would be that the instrument can be calibrated pre-launch using a pseudo-star source. The post-launch calibration function will not be identical to the pseudo-star generated calibration function, but it should be very similar. Having access to the pseudo-star-based calibration function will allow SIM to identify possible problems before the instrument is launched.

The External Calibration strategy would be significantly slower than Internal (minutes verses a couple of hours) but the parameter being calibrated is the parameter of interest, the science delay. Having the ability to directly calibrate the science delay is this strategy's strongest feature.

3. SIM'S BASELINE CALIBRATION STRATEGY

The baseline calibration strategy SIM has adopted is Internal. It is believed that this strategy has less risk associated with it since a calibration function can be generated and tested pre-launch. The general procedure is as follows. A set of ground measurements will be made that identifies the various component/subsystem bias functions. The bias functions will be verified using two different/independent methods pre-launch. Ground verification will be concluded when the functions match at an appropriate level. The bias functions generated pre-launch will be updated on orbit using various component measurements.

The on orbit calibration verification procedure is still work in progress. It may end up that some version of the External Calibration Strategy will be used for on orbit verification purposes.

4. SUMMARY

Two different strategies for calibrating the systematic biases present in the SIM instrument have been presented. The *Internal* Calibration strategy is based on pre-launch measurements combined with a set of on orbit measurements generated by a source *internal* to the instrument. This is the strategy that is baselined for SIM. The *External* Calibration strategy uses stars as an *external* source for generating the calibration function. SIM has multiple years before it has to settle on the optimal calibration strategy, but at this stage in the project it was crucial to understand whether it was even feasible to calibrate at the levels needed to reach the Wide Angle Astrometry mission goals. The work done in support of this paper has given the SIM project the confidence that the final goal of calibrating SIM to the 100's of picometer level is achievable.

ACKNOWLEDGEMENTS

This research was performed at the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration.

REFERENCES

1. SIM instrument reference
2. S.B. Shaklan, M.H. Milman, J. Catanzarite, I. Basdogan, M.V. Papalexandris, L. Sievers, R. Swartz, "Overview of SIM external calibration", *Astronomical Telescopes and Instrumentation Conference Proceedings*, Aug, 2002.