CHAMP, Mass Displacements, and the Earth's Rotation

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Abstract. The angular momentum of the atmosphere and oceans changes as both the distribution of mass within the atmosphere and oceans changes and as the direction and speed of the winds and currents change. Since, in the absence of external torques, the angular momentum of the solid Earth-atmosphere-ocean system is conserved, the changing atmospheric and oceanic angular momentum will cause the solid Earth's angular momentum to change, or, in other words, will cause the Earth's rotation to change. The changing distribution of mass within the atmosphere and oceans also causes the Earth's gravitational field to change, an effect being measured by the CHAMP satellite mission. By measuring changes in the second-degree spherical harmonic coefficients of the Earth's gravitational field, which are related to the elements of the Earth's inertia tensor, CHAMP is, in effect, directly measuring changes in the Earth's rotation caused by mass redistribution. Thus, independent measurements of the Earth's rotation can, in principle, be used to validate the time-varying second-degree coefficients of the Earth's gravitational field measured by CHAMP after the effects of winds and currents on the Earth's rotation are removed. In addition, and perhaps surprisingly, the effects of sufficiently large earthquakes on the trace of the Earth's inertia tensor must also be removed from length of day measurements before comparing them to observed changes in the second-degree zonal gravitational field coefficient.

Introduction

The Earth is a dynamic system — it has a fluid, mobile atmosphere and oceans, a continually changing global distribution of ice, snow, and ground water, a fluid core that is undergoing some type of hydromagnetic motion, a mantle both thermally convecting and rebounding from the glacial loading of the last ice age, and mobile tectonic plates. These dynamic processes change the mass distribution of the Earth (on vastly disparate time scales) and hence change its gravitational field.

Any dynamic Earth process that rearranges the Earth's mass will, in general, change the Earth's inertia tensor, and hence change the Earth's rotation (Munk and MacDonald 1960). As discussed below, in terms of a spherical harmonic expansion of the Earth's gravitational potential, changes in the degree two terms are related to changes in the Earth's inertia tensor (Chao and Gross 1987) and hence to

changes in the Earth's rotation. Thus, many (if not all) of the dynamic Earth processes that cause temporal variations of the Earth's gravitational field also cause variations of the Earth's rotation.

Since mass displacements change both the Earth's rotation and gravitational field, independent observations of the Earth's rotation can be used to assess measurements of the changing second-degree coefficients of the gravitational field (Chen et al. 2000) such as those being taken by the CHAMP satellite mission. Because changes in the angular momentum carried by the atmospheric winds and oceanic currents affect the Earth's rotation, but not the gravitational field, the effects of these and other motion terms must be removed from the Earth rotation observations before they can be used to assess the CHAMP measurements. In addition, it is shown here that the effects on the trace of the Earth's inertia tensor of sufficiently large earthquakes must also be removed from length of day observations before they are used to assess CHAMP measurements of changes in the second-degree zonal gravitational field coefficient.

Earth Rotation and Gravitational Field

Gross (2001) has recently derived from first principles the relationship between changes in the second-degree gravitational field coefficients and changes in: (1) the x- and y-components, $\chi_x(t)$ and $\chi_y(t)$ respectively, of the polar motion excitation functions, and (2) the length-of-day $\Delta\Lambda(t)$:

$$\chi_x(t) = \frac{1.61}{\Omega\,(I_{zz} - I_{xx})}\left[\Delta h_x(t) - \frac{\sqrt{5/3}}{1.44}\,\Omega\, M a^2\, \Delta C_{21}(t)\right] \tag{1}$$

$$\chi_y(t) = \frac{1.61}{\Omega\,(I_{zz} - I_{xx})}\left[\Delta h_y(t) - \frac{\sqrt{5/3}}{1.44}\,\Omega\, M a^2\, \Delta S_{21}(t)\right] \tag{2}$$

$$\Delta\Lambda(t) = \frac{\Lambda_o}{C_m\,\Omega}\left[\Delta h_z(t) + \frac{0.756}{3}\,\Omega\,\Delta\mathrm{Tr}(\mathbf{I}) - \frac{0.756}{3}\,\sqrt{20}\,\Omega\, M a^2\, \Delta C_{20}(t)\right] \tag{3}$$

where Λ_o is the nominal length-of-day of 86400 seconds, $\mathbf{I}$ is the inertia tensor of the Earth with trace $\mathrm{Tr}(\mathbf{I})$ and elements I_{xx} and I_{zz}, C_m is the greatest principal moment of inertia of the Earth's crust and mantle, Ω is the mean rotation rate of the Earth, M is the mass of the Earth, a is the radius of the Earth, the factors of 0.756 and 1.44 account for the yielding of the crust and mantle to imposed surface loads and the factor of 1.61 additionally accounts for the decoupling of the core and mantle. Changes in the x-, y-, and z-components of the angular momentum due to motion relative to the rotating, body-fixed terrestrial reference frame, such as that due to atmospheric winds and oceanic currents, are denoted by $\Delta h_x(t)$, $\Delta h_y(t)$ and $\Delta h_z(t)$, respectively.

From Eqs. (1) and (2) it is seen that the polar motion excitation functions $\chi_x(t)$ and $\chi_y(t)$ are linearly proportional to both: (1) changes in the relative angular mo-

mentum $\Delta h_x(t)$ and $\Delta h_y(t)$, and (2) changes in the normalized second-degree gravitational field coefficients $\Delta C_{21}(t)$ and $\Delta S_{21}(t)$. From Eq. (3), it is seen that length-of-day changes are proportional to: (1) changes in the z-component $\Delta h_z(t)$ of the relative angular momentum, (2) changes in the fully normalized second-degree zonal gravitational field coefficient $\Delta C_{20}(t)$, and (3) changes in the trace of the inertia tensor. Thus, it is straightforward to use observed polar motion excitation functions to assess CHAMP measurements of $\Delta C_{21}(t)$ and $\Delta S_{21}(t)$ once the effects of the relative angular momentum $\Delta h_x(t)$ and $\Delta h_y(t)$ have been removed from the polar motion excitation functions. Models of the atmospheric and oceanic general circulation can be used to compute the angular momentum of the winds and currents in order to remove these effects from the observed polar motion excitation functions.

The situation is a bit more complicated for length-of-day because of its dependence on changes in the trace of the Earth's inertia tensor. Length-of-day measurements, from which motion effects have been removed, can be used to assess the fidelity of CHAMP measurements of $\Delta C_{20}(t)$ only if the mass displacements that are changing the length-of-day and $\Delta C_{20}(t)$ do not change the trace of the inertia tensor. As discussed by Rochester and Smylie (1974), the trace of the inertia tensor is invariant under a wide class of deformation processes. They show that it is invariant under any process whose deformation potential has no spherical harmonic degree l=0 term. Thus, it is invariant under tidal deformations as well as under deformations due to surface loading as long as the loading is not accompanied by a net change in mass. However, the trace of the inertia tensor is not invariant under processes whose deformation potential includes a degree l=0 term or that have radial body forces such as earthquakes.

Earthquakes

Since the trace of the inertia tensor does not vanish for mass displacements caused by earthquakes (Chao and Gross 1987), care must be taken when using length-of-day observations to assess CHAMP measurements of $\Delta C_{20}(t)$. If the change in the trace of the Earth's inertia tensor due to earthquakes is large enough, it must be modeled and removed from the length of day observations before comparing them to measurements of $\Delta C_{20}(t)$. Chao and Gross (1987) modeled the cosesimic effect of earthquakes on the length of day itself, finding that a great earthquake such as the 1964 Alaskan event should have changed the length of day by 6.8 microseconds (μs). They also studied the effect on the length of day of the 2146 largest earthquakes that occurred during 1977–1985, finding a cumulative change in the length of day due to these earthquakes of –0.9 μs. They also found that the greatest change in the length of day during 1977–1985, amounting to 0.33 μs, was caused by the 1977 Sumba event which had a moment magnitude M_w of 8.3.

In order to assess the relative importance of the $\Delta \mathrm{Tr}(\mathbf{I})$ term to earthquake-induced changes in the length of the day, the coseismic change in the trace of the Earth's inertia tensor due to earthquakes is modeled here following the procedure

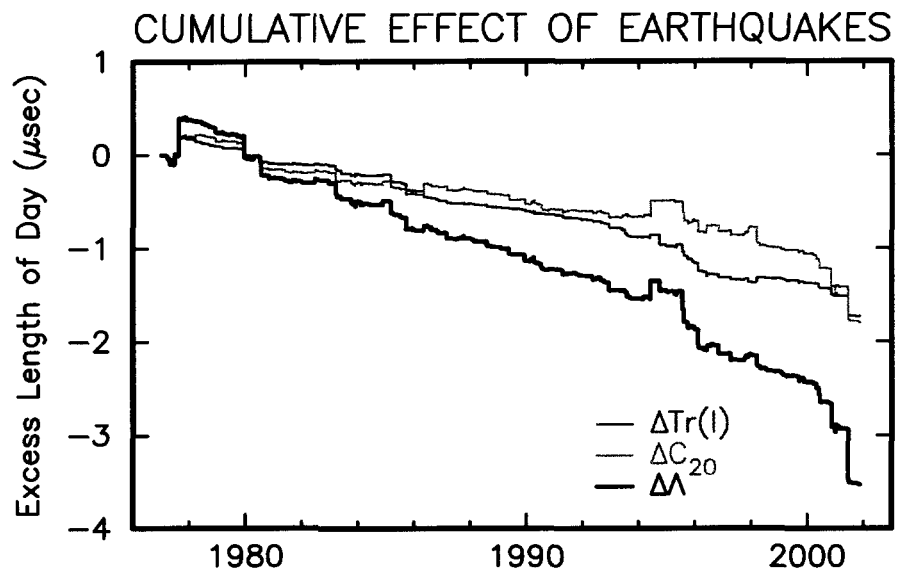

Fig. 1. Cumulative earthquake-induced change in: (1) the trace of the Earth's inertia tensor $\Delta\mathrm{Tr}(\mathbf{I})$ (middle thin black curve), (2) the second-degree zonal coefficient of the Earth's gravitational potential $\Delta C_{20}(t)$ (top gray curve), and (3) the length of day $\Delta\Lambda(t)$ (bottom thick black curve) due to the largest 18553 earthquakes that occurred during 1977–2001

of Chao and Gross (1987) as updated by Gross and Chao (2001). Fig. 1 shows the modeled change in the trace of the inertia tensor (middle thin black curve), in equivalent length of day units, for the 18553 earthquakes spanning January 1977 through November 2001 currently contained in the Harvard centroid-moment tensor catalog. The Harvard centroid-moment tensor catalog (Dziewonski et al. 2001 and references therein) contains the source properties of all earthquakes having moment magnitude M_w greater than about 5.5 that have occurred since 1977. Also shown in Fig. 1 is the contribution of the $\Delta C_{20}(t)$ term to earthquake-induced changes in the length of day (top gray curve), as well as the total change in the length of day (bottom thick black curve), being the sum of the $\Delta\mathrm{Tr}(\mathbf{I})$ and $\Delta C_{20}(t)$ terms. Since the earthquake is modeled as occurring instantaneously, there is no relative angular momentum associated with it (Chao and Gross 1987). During this longer time interval, the cumulative earthquake-induced change in the length of day is seen to be about −4 μs, with the largest change, amounting to −0.5 μs, being due to the 2001 Peruvian event which had a moment magnitude M_w of 8.4.

From Fig. 1 it is seen that the contribution of the $\Delta\mathrm{Tr}(\mathbf{I})$ term to earthquake-induced changes in the length of day is comparable to the contribution of the $\Delta C_{20}(t)$ term. However, the size of these effects is rather small, each of their cumulative effects during 1977–2001 being only about −2 μs. Thus, although in principle earthquake-induced changes in the trace of the Earth's inertia tensor should be removed from length of day observations before comparing them to measured changes in the second-degree zonal gravitational field coefficient, in

practice this is not likely to be necessary since the size of the effect, amounting to at most a few microseconds, is much smaller than the current length of day measurement uncertainty of about 20 μs. It would be necessary to remove the effect of earthquakes on the trace of the Earth's inertia tensor from length of day observations only if a truly great earthquake, such as the 1960 Chilean event, were to occur during the lifetime of the CHAMP mission.

Summary

Changes of both the Earth's rotation and gravitational field arising from mass displacements caused by the same underlying dynamic process must, of course, be consistent with each other. Thus, independent observations of the Earth's rotation can be used to assess the fidelity of measurements of changes in the second-degree gravitational field coefficients. Of course, motion effects such as those due to atmospheric winds and oceanic currents must first be removed from the Earth rotation observations since they do not affect the Earth's gravitational field. In addition, the length of day is also affected by processes that change the trace of the Earth's inertia tensor, notably earthquakes. So, at least in principle, the effect of earthquake-induced changes in the trace of the Earth's inertia tensor on the length of day must also be removed before comparing length of day observations to measured changes in the second-degree zonal gravitational field coefficient. The importance of this earthquake effect was evaluated by modeling the coseismic change in the trace of the inertia tensor due to the 18553 largest earthquakes that occurred during January 1977 through November 2001. It was shown that the contribution of earthquake-induced changes in the trace of the Earth's inertia tensor to length of day changes during this time period is much smaller than the length of day measurement uncertainty. Thus, this effect can probably be neglected when comparing length of day observations to measured changes in the second-degree zonal gravitational field coefficient. It need only be considered if a truly great earthquake, such as the 1960 Chilean event, were to occur.

Acknowledgements. The work described in this paper was performed at the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration. Support for this work was provided by the Solid Earth and Natural Hazards Program of NASA's Office of Earth Science.

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