Sudden Ionospheric Delay Decorrelation and Its Impact on WAAS

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Abstract

In the absence of selective availability, the ionosphere represents the largest source of positioning error for single-frequency users of the Global Positioning System (GPS). In differential GPS systems such as the Wide Area Augmentation System (WAAS), vertical ionospheric delays are modeled at regularly-spaced intervals in latitude and longitude, *i.e.*, at *ionospheric grid points* (IGPs). The broadcast bound on the error at each of these points is designated the *grid ionospheric vertical error* (GIVE). A critical integrity requirement of WAAS is that the broadcast GIVE bounds residual error with a very high degree of confidence. The threat posed by the ionosphere manifests itself in three ways:

(1) instantaneous residual errors due to mismodeling of the ionosphere at the IGPs;

(2) residual errors that arise when interpolating IGP delays to a user position; and

(3) residual errors that grow over the life-span of the broadcast message.

The broadcast GIVE must protect the user from each of these threats. The rate, both in space and time, at which neighboring measurements of ionospheric delay become decorrelated is a critical component in the calculation of the WAAS GIVE. Under nominal quiet-time conditions, a *planar fit* of slant delay measurements projected to vertical provides estimates of the local vertical delay that are of sufficient accuracy for WAAS operation. In the first phase of WAAS implementation, the uncertainty in the vertical ionospheric delay as modeled by a planar fit is conservatively assumed to be a constant (35 cm), independent of both measurement elevation angle and distance from the IGP.

Subsequent implementations of WAAS that have higher performance requirements will demand a reduction in the magnitude of the GIVE broadcast under nominal conditions. Achieving this reduction will require a better understanding of the decorrelation of ionospheric delay, both in space and time. In this paper we focus on temporal decorrelation. We report a methodology for assessing the impact on WAAS posed by a sudden increase in the level of ionospheric disturbance. The methodology is based upon forming an estimate of the probability P_D that a WAAS user will confront a sudden increase in the level of ionospheric disturbance following a period of relative calm. By restricting the tabulation of fit residuals to only those fits where the spatial coverage of the fit points is sufficiently good, we have determined a limiting upper bound of P_D to be 2×10^{-6}.

I. Introduction

The Wide Area Augmentation System (WAAS) is designed to provide reliable differential GPS corrections for aircraft navigation. In the absence of selective availability, the largest source of positioning error is the radio delay caused by the ionosphere. Since WAAS user measurements generally do not coincide with reference station measurements, it is necessary to rely on ionospheric correlation to infer the state of the ionosphere in regions sampled by the user. Irregularities in the ionosphere, both in space and time, represent a threat to the accuracy of the confidence bounds describing the integrity of the broadcast corrections.

Under nominal conditions the ionospheric delay can be accurately determined using a planar fit of neighboring slant delay measurements projected to vertical. When the ionosphere is disturbed, the residual error associated with the planar fit increases, indicating that delay estimates based on this fit are less reliable. Consequently, the confidence bounds must be increased or the fit declared unusable. As long as the fit residuals accurately reflect the degree of disturbance of the ionosphere, the integrity of the corrections should remain high. Since fits are performed at finite intervals, however, it is possible that significant growth in the degree of disturbance could occur between fit evaluations. In this case the fit residuals no longer accurately reflect the true ionospheric state as encountered by the user.

The purpose of this paper is to establish a methodology for assessing the risk to the WAAS user associated with sudden increases in the level of ionospheric disturbance. In Section II we review the WAAS model for ionospheric delay and related algorithms. In section III we propose a strategy for risk assessment in terms of P_D, the probability that a WAAS user will sample a region of the ionosphere during the onset of a significant level of disturbance. Section IV discusses our method for estimating an upper bound on P_D. Section V presents an iterative method for calculating σ_{decorr}, the standard deviation of the local vertical total electron content of the ionosphere relative to a planar approximation. In

Section VI we describe the manner in which observational data were processed. Section VII reports the results of our analysis and provides justification for a final upper bound on P_D of 2 x 10^{-6}.

II. Review of WAAS ionospheric algorithms

In this section we briefly review algorithms that WAAS uses to estimate the ionospheric delay.

A. Ionospheric delay model

At each ionospheric grid point (IGP), WAAS models the vertical ionospheric delay by constructing a planar fit of a set of slant delay measurements projected to vertical. Each slant delay value is converted to a vertical delay value using the standard *thin-shell* model of the ionosphere: at the ionospheric pierce point (IPP), *i.e.*, the point where the raypath crosses the shell height h_i, the ratio of the slant delay to the vertical delay is approximated as

$$M(\alpha, h_i) = \left[1 - \left(\frac{R_e \cos\alpha}{R_e + h_i}\right)^2\right]^{-1/2},$$

where R_e is the earth radius and α is the elevation angle. All IPPs that lie within a minimum fit radius R_{min} are included in the fit. If the number of IPPs within this minimum radius is less than N_{min}, the fit radius R_{fit} is extended until it encompasses N_{min} points. In this study we do not tabulate data when the fit radius reaches a maximum value of R_{max} without encircling N_{min} points.

Formally the planar fit approximation can be written as

$$\mathbf{y} \approx \mathbf{Gx},$$

where $\mathbf{x}$ is a vector of planar fit parameters, $\mathbf{y}$ is a vector of vertical delay values, and $\mathbf{G}$ is a matrix of partials with each row of the form $[1 \;\; d_E \;\; d_N]$, where d_E and d_N are the distances from the IGP to the IPP in the eastern and northern directions, respectively. The least squares solution $\mathbf{x}$ is obtained by solving the equation

$$\mathbf{G}^T\mathbf{W}(\sigma^2)\mathbf{Gx} = \mathbf{G}^T\mathbf{W}(\sigma^2)\mathbf{y},$$

where

$$\mathbf{W}^{-1}(\sigma^2) \equiv \begin{bmatrix} \sigma^2 + \sigma^2_{IPP,1} & \sigma^2_{bias,1,2} & \cdots & \sigma^2_{bias,1,N} \\ \sigma^2_{bias,2,1} & \sigma^2 + \sigma^2_{IPP,1} & \cdots & \sigma^2_{bias,2,N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma^2_{bias,N,1} & \sigma^2_{bias,N,2} & \cdots & \sigma^2 + \sigma^2_{IPP,N} \end{bmatrix}$$

is the observation weighting matrix, where σ^2 is the local variance in the vertical delay of the ionosphere about a plane, the $\sigma^2_{IPP,i}$ are measurement error variances, and the $\sigma^2_{bias,i,j}$ are covariances that account for the correlation of the bias errors between vertical delay measurements made with common satellites or common receivers.

B. Ionospheric irregularity detector

As discussed above, the WAAS estimation of ionspheric delay and its confidence is based upon a local planar model with uncertainties bounded by a limiting σ. To ensure the integrity of the broadcast delay and confidence values, it is imperative to determine whenever ionospheric behavior cannot be accurately described by such a model. To address this question, WAAS relies on an *irregularity detector* based upon the χ^2 of the planar fit. [A full description of the WAAS irregularity detector has been provided by Walter *et al.* (2000).] The χ^2 of the fit may be written as follows:

$$\chi^2(\sigma^2) = [\mathbf{Gx} - \mathbf{y}]^T \mathbf{W}(\sigma^2)[\mathbf{Gx} - \mathbf{y}]$$

where all quantities were defined in the previous section. In WAAS operation each planar fit is performed with $\sigma = \sigma^{nom}_{decorr}$, where σ^{nom}_{decorr} is the standard deviation for the spatial decorrelation of the vertical total electron content under nominal, *i.e.*, quiet, conditions (this parameter is currently set conservatively to 35 cm). Local storm conditions are declared whenever the χ^2 exceeds a specified threshold that depends upon the number of observations fitted. On such occasions, the ionosphere is no longer assumed to be characterized by nominal behavior, and the bound on the error at the IGP is raised to a maximum limit.

III. The probability of sudden ionospheric decorrelation

Our goal is to assess the risk to the WAAS user posed by a sudden increase in the level of ionospheric disturbance. We will need to address two distinct aspects of this problem: the degree of disturbance and the time-scale of its onset. Let P_D be the probability that a WAAS user will sample a region of the ionosphere experiencing a sudden, significant growth in perturbation following a period of relative calm. To quantify the *period of relative calm*, we define a *non-storm duration* as a period of time T_{ns} during which the local ionospheric storm detector has not tripped. As a measure the magnitude of a disturbance, we define the *decorrelation ratio* to be

$$D_{iono} \equiv \frac{\sigma_{decorr}}{\sigma_{crit}},$$

where σ_{decorr} is the standard deviation for the local spatial decorrelation of the vertical total electron content of the ionosphere, and σ_{crit} is a critical bound required for user safety, that is, the decorrelation ratio must be less than 1 to a very high degree of confidence. In current WAAS algorithms, $\sigma_{crit} \equiv R_{irreg}\sigma_{decorr}^{nom}$, where R_{irreg} is the irregularity inflation factor (Walter *et al.*, 2000) and σ_{decorr}^{nom} is defined in the previous section. If time $t = 0$ corresponds to the conclusion of a non-storm duration of length T_{ns}, then we define $P_D \equiv P_D(t, T_{ns})$ to be the probability at time $t > 0$ that a user samples the ionosphere in a region where the local ionospheric decorrelation ratio exceeds 1.

We anticipate that $P_D(t, T_{ns})$ will tend to be an increasing function of t, becoming flat when t is sufficiently large. In other words, the ionosphere can become progressively more disturbed with the passage of time following a non-storm duration, increasing the likelihood that the decorrelation ratio will exceed unity. This remains true until the time is sufficiently great that the state of the ionosphere is statistically uncorrelated with the prior non-storm duration, at which point the probability of the decorrelation ratio exceeding unity becomes roughly constant. Rather than approximate $P_D(t, T_{ns})$ directly, we will seek an upper bound on $P_D(t, T_{ns})$ for $t \le t_{sample}$, where t_{sample} is a sample period of interest. The non-storm duration period and sample period currently of interest in WAAS are, respectively, $T_{ns} = 900$ seconds (15 minutes) and $t_{sample} = 85$ seconds.

IV. Method of determining an upper bound on $P_D(t, T_{ns})$

For the decorrelation ratio to exceed unity, the ionosphere must be sufficiently disturbed. In ionospheric science, the geomagnetic index K_p is often used to approximate the degree of disturbance of the ionosphere, since ionospheric disturbances are known to be highly correlated with perturbations of the earth's magnetic field. The initial strategy for determining an upper bound on $P_D(t, T_{ns})$ has been to assume $P_D(t, T_{ns})$ to be of the form:

$$P_D(t, T_{ns}) \equiv \sum_{i=0}^{9} P_i^K P_D(t, T_{ns}|K_i)$$

where P_i^K is the probability that the value of the K_p index lies within the range $i - \Delta \le K_p \le i + \Delta$ where $\Delta \equiv 0.3$ (see Table 1), and $P_D(t, T_{ns}|K_i)$ is the conditional probability that a user, at time t following a non-storm duration of length T_{ns}, samples the ionosphere in a region where the local ionospheric decorrelation ratio exceeds 1, given that the K_p index lies within the range $i - \Delta \le K_p \le i + \Delta$. Note that $\sum_{i=0}^{9} P_i^K = 1$. An upper bound on $P_D(t, T_{ns})$ can then be calculated from a determination of upper bounds on the $P_D(t, T_{ns}|K_i)$. The expectation is that, for low values of i (*i.e.*, for low values of K_p), upper bounds on the $P_D(t, T_{ns}|K_i)$ will be small, thereby ensuring that the bound on $P_D(t, T_{ns})$ will be small.

i	0	1	2	3	4	5	6	7	8	9
P_i^K	.0852	.2497	.2556	.2089	.1199	.0525	.0184	.0066	.0026	.0006

Table 1. The probability that the value of the K_p index lies within the range $i - \Delta \le K_p \le i + \Delta$ (where $\Delta \equiv 0.3$) as tabulated from K_p data over the time period 1932-2000 (obtained from the National Geophysical Data Center).

Let $\overline{P}_D(t, T_{ns}|K_i)$ be an estimate of $P_D(t, T_{ns}|K_i)$ determined purely from data collected on days in which ionospheric storms have occurred. On such days the decorrelation ratio is more likely to attain values greater than 1, ensuring that $P_D(t, T_{ns}|K_i) \le \overline{P}_D(t, T_{ns}|K_i)$. An upper bound on $P_D(t, T_{ns})$ may then be determined as follows:

$$P_D(t, T_{ns}) \le \sum_{i=0}^{9} P_i^K \overline{P}_D(t', T_{ns}|K_i) \equiv P_D^{storm}(t', T_{ns})$$

where $t' \approx t_{sample}$. Since $P_D(t, T_{ns}|K_i)$ is roughly an increasing function of time, we can safely assume that $P_D(t, T_{ns}|K_i) \le \overline{P}_D(t', T_{ns}|K_i)$ for $t \le t'$. Note that $P_D^{storm}(t', T_{ns})$ is likely to be a conservative upper bound, *i.e.* the true value of $P_D(t, T_{ns})$ may be considerably less than this bound, depending upon the data sets used to calculate $P_D^{storm}(t', T_{ns})$.

V. Algorithm for calculating σ_{decorr}

The value of σ_{decorr} in the neighborhood of a given ionospheric grid point (IGP) can be defined in terms of the χ^2 associated with the planar fit at that IGP. Let us define σ_{decorr} to be the value of σ such that the χ^2 per degree of freedom is unity. Since the planar ionospheric model has three fit parameters, the χ^2 per degree of freedom is $\chi^2(\sigma^2)/(N-3)$, where N is the total number of points in the fit. Thus, to obtain an estimate of σ_{decorr}, we solve

$$f(\sigma^2) \equiv 1 - \frac{\chi^2(\sigma^2)}{N-3} = 0,$$

using a Newton-Raphson method iteration:

$$\sigma_{n+1}^2 = \sigma_n^2 - \frac{f(\sigma_n^2)}{\left.\dfrac{\partial f}{\partial \sigma^2}\right|_{\sigma=\sigma_n^2}}.$$

It can be shown that

$$\begin{aligned}\frac{\partial f}{\partial \sigma^2} &= -\frac{1}{N-3}\frac{\partial \chi^2}{\partial \sigma^2} \\ &= \frac{1}{N-3}\mathbf{y}^T\left[\mathbf{I} - \mathbf{WG}(\mathbf{G}^T\mathbf{WG})^{-1}\mathbf{G}^T\right]\mathbf{W}^2[\mathbf{Gx}-\mathbf{y}],\end{aligned}$$

where **I** is the identity matrix. The initial guess for σ_{decorr} is set according to the equation

$$\sigma_1^2 \equiv \left\{\left[(\sigma_{decorr}^{RMS})^2 + (\sigma_{decorr}^{nom})^2 + (\sigma_{bias}^{satellite})^2 + (\sigma_{bias}^{receiver})^2\right]\chi^2\left((\sigma_{decorr}^{nom})^2\right)\right\} - (\sigma_{decorr}^{RMS})^2,$$

where $(\sigma_{decorr}^{RMS})^2 \equiv \frac{1}{N}\sum_{i=1}^{N}(\sigma_{IPP,i})^2$, $(\sigma_{bias}^{satellite})^2$ is the variance of the hardware bias for each satellite, and $(\sigma_{bias}^{receiver})^2$ is the variance of the hardware bias for each receiver (the latter two variances are assumed to be constant for all satellites and receivers, respectively). We use a convergence criterion of

$$\left|\frac{\sigma_{n+1}^2 - \sigma_n^2}{\sigma_n^2}\right| < 10^{-6}.$$

which is generally found to be satisfied within 3-8 iterations. Occasionally, we find that the Newton-Raphson iteration produces a negative estimate for σ^2 (usually when χ^2 is anomalously small). When this occurs, we replace the Newton-Raphson iteration with a false position search (see Press *et al.*, 1988). The false position method converges more slowly to the root of $f(\sigma^2)$, but the search interval can be constrained so that $\sigma^2 \ge 0$. This iteration is stopped when

$$\sigma_{n+1}^2 - \sigma_n^2 < 10^{-4}\, m$$

is satisfied. Thereafter, the Newton-Raphson iteration is resumed.

Note that when satellite and receiver biases are neglected and $\sigma_{IPP,i}^2 << \sigma_{decorr}^2$,

$$\frac{\chi^2(\sigma_{decorr}^2)}{(N-3)} \approx 1 \Rightarrow \sigma_{decorr}^2 \approx \sigma_{fit}^2.$$

where $\sigma_{fit}^2 \equiv \frac{1}{(N-3)}\sum_{i=1}^{N}\left(y_i - \bar{y}_i\right)^2$ with $\bar{\mathbf{y}} \equiv \mathbf{Gx}$. For the data sets processed in this study, we find that this approximate equation σ_{decorr}^2 generally holds.

VI. Data processing

To determine the $\bar{P}_D\left(t, T_{ns}\middle|K_i\right)$, we have tabulated values of the decorrelation ratio as a function of the K_p index for delay data from days on which moderate to severe ionospheric storms occurred. These data consist of post-processed slant delay measurements collected by the existing 25 WAAS Reference Stations. The intent of the post-processing is to eliminate interfrequency biases, to remove the effects of cycle slips in carrier phase measurements, to level the carrier phase measurements to the corresponding range measurements, and to filter spurious measurements using the redundancy provided by the presence of multiple receivers at each station. Such data contain minimal error due to noise.

Data from the following 17 dates have been processed: 1/11/00, 2/12/00, 4/6/00, 4/7/00, 5/25/00, 6/8/00, 7/15/00, 7/16/00, 8/11/00, 8/12/00, 3/19/01, 3/20/01, 3/28/01, 3/29/01, 3/30/01, 3/31/01, and 4/01/01. The following parameters have been used in the analysis:

Ionospheric reference height (h_i):	350 km
Minimum permitted fit radius (R_{min}) :	800 km
Maximum permitted fit radius (R_{max}):	2100 km
Minimum number of points in fit (N_{min}):	30
Maximum number of points in fit when $R_{fit} > R_{min}$:	30
Standard deviation of nominal ionosphere (σ_{decorr}^{nom}):	35 cm
Data epoch interval (t')	100 seconds
Non-storm duration (T_{ns})	900 seconds

Fit residuals are tabulated for each epoch of data that follows a non-storm duration of at least 900 seconds.

VII. Results

The distributions for individual storms are found to be highly varied. The results combined for all data sets are presented in Fig. 1. Note that K_p never falls below 2 on any of the days in question. Figure 2 shows the accumulated results of tabulating σ_{decorr} for all 17 storms.

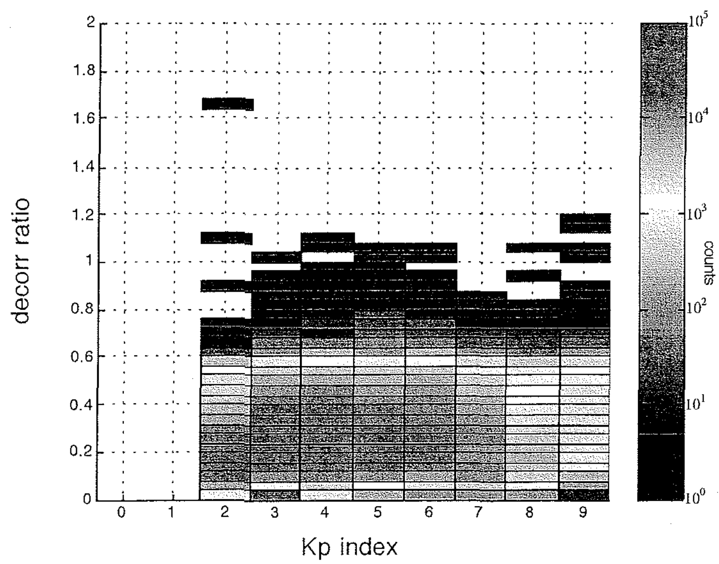

Figure 1. Decorrelation ratio tabulated as a function of K_p index for all 17 storms combined.

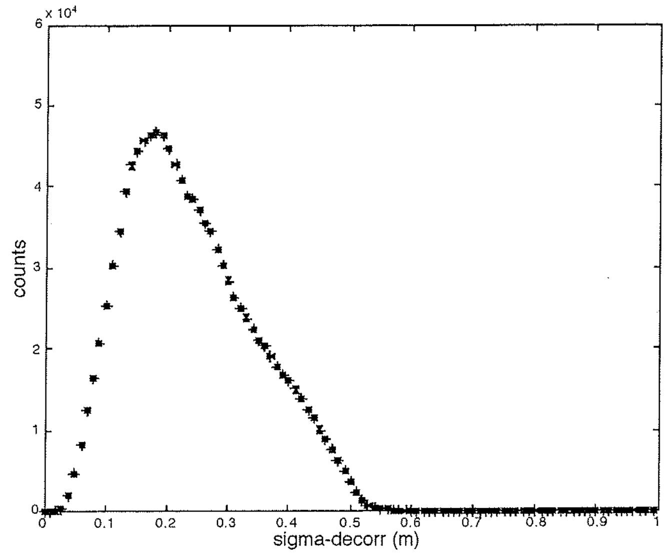

Figure 2. Histogram of σ_{decorr} for all 17 storms combined.

Figure 3 displays the cumulative probability distribution for each K_p column in Fig. 1. Each curve represents the probability of exceeding a given decorrelation ratio magnitude as a function of that magnitude. Note that for a decorrelation ratio of 1, only one curve – with $K_p = 9$ – has a probability of exceeding 1 that is greater than 10^{-4}. Since the probability that $K_p = 9$ is very small (0.0006), this figure indicates clearly that P_D^{storm} will be significantly less than 10^{-4}. Using the K_p probabilities listed in the table in Table 1, the upper bound on P_D is found to be 9.4 x 10^{-6}.

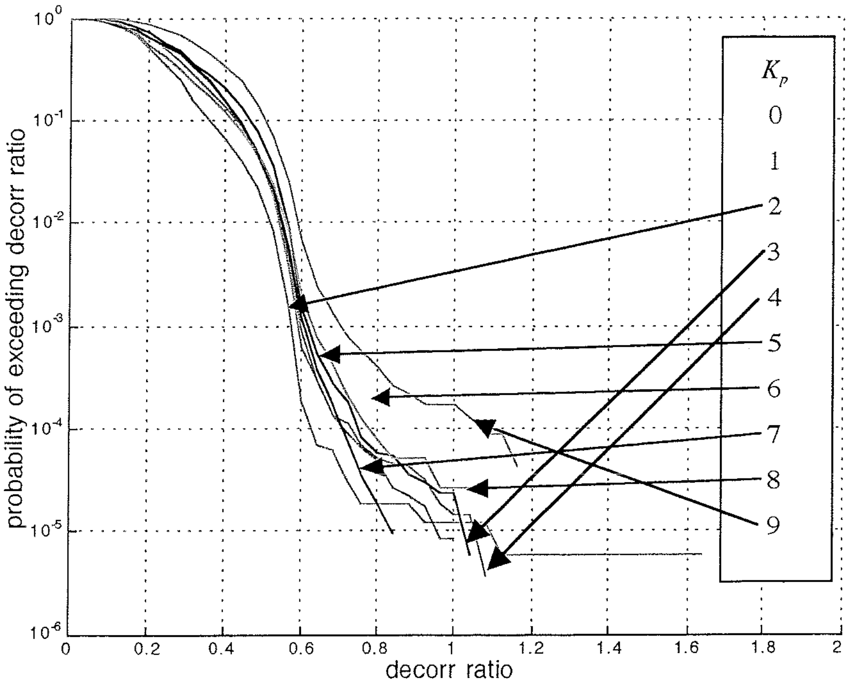

Figure 3. Cumulative probability distribution for exceeding a given decorrelation ratio.

In calculating this bound, $\overline{P}_D(t', T_{ns}|K_0)$ and $\overline{P}_D(t', T_{ns}|K_1)$ have been set identically to zero since the decorrelation ratio has not been observed to exceed 1 when $K_p \leq 1$. It may be argued that this is not sufficiently conservative. If finite values of $\overline{P}_D(t', T_{ns}|K_0)$ and $\overline{P}_D(t', T_{ns}|K_1)$ were to be observed when processing another data set (*e.g.*, days when the ionosphere was less disturbed), however, we can anticipate that these values would be less than $\overline{P}_D(t', T_{ns}|K_2)$. To be safe therefore, we choose to augment our calculated bound by the amount $P_0^K\overline{P}_D(t', T_{ns}|K_2) + P_1^K\overline{P}_D(t', T_{ns}|K_2)$, where $\overline{P}_D(t', T_{ns}|K_2) \approx 10^{-5}$. This increases the upper bound by 3 x 10^{-6}, resulting in a upper bound of 1.2 x 10^{-5}.

One limitation of our analysis is that our results are contaminated by large residuals that arise, not due to irregularities in the ionosphere, but rather due to poor spatial coverage of the IPPs. To address this question, we have repeated our analysis, restricting the tabulation of residuals to planar fits where the spatial coverage provided by the fit points is deemed *good.* As a criterion for goodness of coverage, we require that the centroid of the IPP distribution lie less than 840 km form the IGP (which is 0.4 times the maximum fit radius of 2100 km). The results are displayed in Fig. 4. This restriction is found to eliminate all events where the decorrelation ratio exceeds unity, indicating a strong correlation between such events and poor spatial coverage.

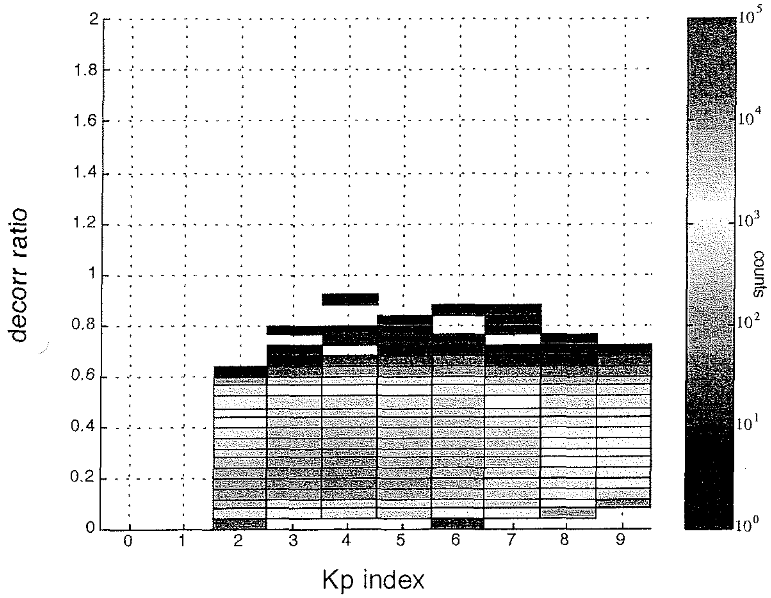

Figure 4. Decorrelation ratio tabulated as a function of K_p index for all 17 storms combined, where the centroid radius is less than 840 km.

Since no events with a decorrelation ratio greater than unity are found in the reanalysis, it is somewhat problematic what upper bound is assigned to the probability of such events occurring in general. Figure 5 shows the cumulative probability distribution for the decorrelation ratio exceeding a given value, where we have combined data from all 17 storm days. Based upon this curve alone, one may conclude that a value of 2 x 10^{-6} is a reasonable upper bound for P_D. It could be argued that this value is derived from an insufficient amount of data and that we should process up to 10 times more data to be confident that this value is truly an upper bound. However, we can also argue that this value is conservative since we have only looked at data collected on days when storms have occurred. Our results imply that the decorrelation ratio will never rise above unity *on a quiet day* and that processing data from a representative number of quiet days will simply ensure that the upper bound of 2 x 10^{-6} remains valid.

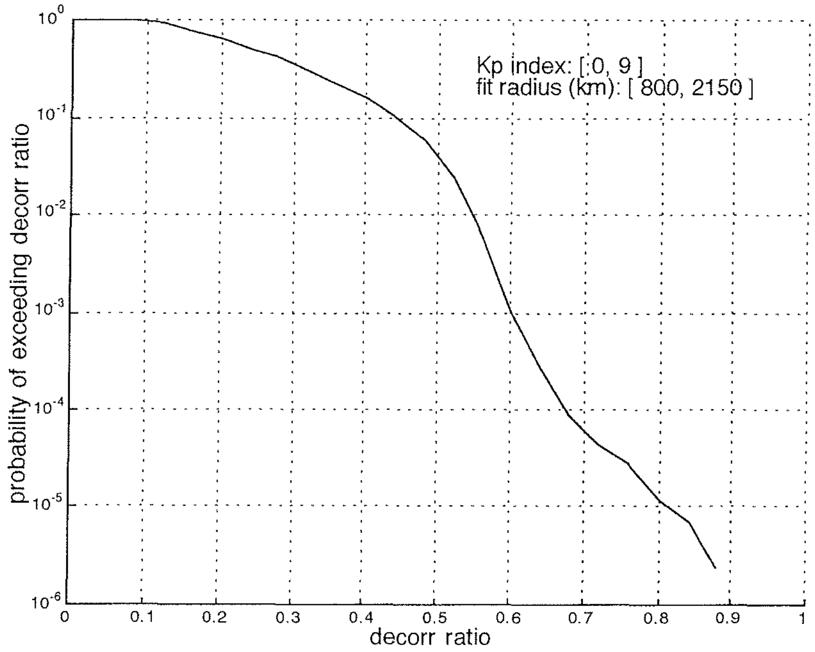

Figure 5. Cumulative probability distribution for exceeding a given decorrelation ratio based on data from all 17 storm days, where the centroid radius is less than 840 km.

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