CL02-0833

Final version

Sensitivity analysis of radiative heating and cooling rates in planetary atmospheres: general linearization and adjoint approaches

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April 23, 2002

Radiative heating and cooling provide primary source and ultimate sink of energy driving lower planetary atmospheres. Corresponding radiative heating and cooling rates represent important inputs to models of atmospheric dynamics. In turn, these rates are outputs of corresponding radiative transfer models where the inputs are fields of primary atmospheric parameters, such as temperature, mixing ratios of greenhouse gases, etc. Evaluating the sensitivities of atmospheric dynamics models on these primary atmospheric parameters requires knowing how heating and cooling rates depend on these same parameters. We discuss two approaches that make it possible to directly compute the sensitivities of heating and cooling rates in parallel with evaluation of heating and cooling rates themselves.

For non-scattering atmospheres, the heating and cooling rates can be evaluated analytically. In this case, sensitivities to atmospheric parameters are obtained by straightforward linearization of corresponding analytic expressions. For scattering atmospheres (e.g., with scattering aerosol and/or clouds) the heating and cooling rates can, in general case, be obtained only from the numerical solution of the corresponding forward problem of radiative transfer. It turns out that the solution of the corresponding adjoint problem, obtained in parallel with its forward counterpart, can be used to evaluate sensitivities of heating and cooling rates with respect to atmospheric parameters.

An analogy with evaluation of weighting functions in remote sensing is pointed out and applications to the sensitivity analysis of models of atmospheric dynamics are discussed.

CONTENTS

1. **Motivation**

3. **Quantitative analysis of sensitivities**

2. **Format and "design" of sensitivities**

3.1. *Non-scattering atmospheres: Linearization approach*

3.2. *Scattering atmospheres: Adjoint approach*

4. **Discussion and conclusion**

Motivation

Sensitivity analysis: Analysis of responses of output parameters of models to perturbations of input parameters.

Linearized models of atmospheric dynamics: Description of perturbations of state due to perturbations of input parameters.

Radiative heating and cooling of the atmosphere and surface: ultimate source and sink of energy of atmospheric motion. Their perturbations *are* important.

Example: Radiative heating and cooling of the atmosphere enter via forcing term of the thermodynamics equation. In the linearized form:

$$\frac{\partial T'}{\partial t} + (\mathbf{V}', \nabla T) + (\mathbf{V}, \nabla T') = \frac{1}{c_v} J' \tag{1}$$

Vertical profile $J(z)$ depends on vertical profiles of temperature $T(z)$ and on some of atmospheric parameters $X_i(z)$. Thus

$$J'(z) = \int_{z_0}^{\infty} \left(K^{(T)}(z,\zeta)\, T'(\zeta) + \sum_i K^{(X_i)}(z,\zeta)\, X_i'(\zeta) \right) d\zeta \tag{2}$$

This means that the linearized thermodynamics equation actually contains an extra perturbation term due to T' and a right-hand term due to the rest of parameters X_i':

$$\frac{\partial T'}{\partial t} + (\mathbf{V}', \nabla T) + (\mathbf{V}, \nabla T') - \frac{1}{c_v} \int K^{(T)} T' d\zeta = S' \tag{3}$$

For complete linearization of the thermodynamics equation we need to know sensitivities to temperature $K^{(T)}$ and to all relevant parameters $K^{(X_i)}$.

Format and "design" of sensitivities

Complete set of sensitivities: Four types: those of net heating rates of the atmosphere $J(z)$ and surface J_s to the model parameters of the atmosphere $Y(z)$ and surface Y_s:

$$\frac{\delta J(z)}{\delta Y(\zeta)}, \quad \frac{\delta J_s}{\delta Y(\zeta)}, \quad \frac{\partial J(z)}{\partial Y_s}, \quad \frac{\partial J_s}{\partial Y_s} \tag{4}$$

Variational derivatives are represented by corresponding kernels:

$$\frac{\delta J(z)}{\delta Y(\zeta)} = K^{(Y)}(z,\zeta), \quad \frac{\delta J_s}{\delta Y(\zeta)} = K_s^{(Y)}(\zeta) \tag{5}$$

Net atmospheric and surface heating rates: differences of corresponding solar heating and thermal IR cooling rates:

$$J(z) = Q_\odot(z) - Q_{IR}(z), \quad J_s = Q_{s\odot} - Q_{sIR} \tag{6}$$

Sensitivities of net atmospheric and surface heating rates: differences of corresponding sensitivities of solar heating and thermal IR cooling rates. Thus, in general case, we need to be able to evaluate eight different entities

$$\begin{aligned} &\frac{\delta Q_\odot(z)}{\delta Y(\zeta)}, \quad \frac{\delta Q_{s\odot}}{\delta Y(\zeta)}, \quad \frac{\partial Q_\odot(z)}{\partial Y_s}, \quad \frac{\partial Q_{s\odot}}{\partial Y_s} \\ &\frac{\delta Q_{IR}(z)}{\delta Y(\zeta)}, \quad \frac{\delta Q_{sIR}}{\delta Y(\zeta)}, \quad \frac{\partial Q_{IR}(z)}{\partial Y_s}, \quad \frac{\partial Q_{sIR}}{\partial Y_s} \end{aligned} \tag{7}$$

for all relevant atmospheric and surface parameters.

Straightforward evaluation of sensitivities Eq.(8) involves monochromatic computations with subsequent integration over the solar or, respectively, thermal IR spectral regions $\Delta\nu_\odot$, $\Delta\nu_{IR}$.

Solar heating rates. Dependence on all atmospheric parameters is "forwarded" via atmospheric absorption coefficient $\kappa_\nu(z)$. Dependence on all parameters of the surface is "forwarded" via surface albedo A_ν.

For both $Q_\odot(z)$ and $Q_{s\odot}$ we have respectively [below $Y(\zeta)$ is any atmospheric, and Y_s is any surface parameter; "kernel" notations as in Eq(5) are used henceforth where appropriate, and symbol "·" means a presence of the argument z in case of atmospheric radiative heating rate]:

$$K_\odot^{(Y)}(\cdot,\zeta) = \int\limits_{\Delta\nu_\odot} K_{\odot\nu}^{(\kappa)}(\cdot,\zeta)\,\frac{\partial\kappa_\nu}{\partial Y}\bigg|_{Y(\zeta)} d\nu \tag{8}$$

$$\frac{\partial Q_\odot}{\partial Y_s} = \int\limits_{\Delta\nu_\odot} \frac{\partial Q_{\odot\nu}}{\partial A_\nu(\zeta)}\,\frac{\partial A_\nu}{\partial Y_s}\,d\nu \tag{9}$$

Thermal IR cooling rates. Dependence on all atmospheric parameters is "forwarded" via atmospheric absorption coefficient $\kappa_\nu(z)$. Dependence on all parameters of the surface is "forwarded" via surface emissivity ε_ν. Besides, dependence on atmospheric and surface temperatures is "forwarded" through Planck function $B_\nu(T)$.

For both $Q_{IR}(z)$ and Q_{sIR} we have (X is any parameter besides temperature):

$$K_{IR}^{(X)}(\cdot,\zeta) = \int\limits_{\Delta\nu_\odot} K_{IR\nu}^{(\kappa)}(\cdot,\zeta)\,\frac{\partial\kappa_\nu}{\partial X}\bigg|_{X(\zeta)} d\nu \tag{10}$$

$$\frac{\partial Q_{IR}}{\partial X_s} = \int\limits_{\Delta\nu_{IR}} \frac{\partial Q_{IR\nu}}{\partial\varepsilon_\nu(\zeta)}\,\frac{\partial\varepsilon_\nu}{\partial X_s}\,d\nu \tag{11}$$

$$K_{IR}^{(T)}(\cdot,\zeta) = \int\limits_{\Delta\nu_{IR}} \left(K_{IR\nu}^{(B)}(\cdot,\zeta)\,\frac{\partial B_\nu}{\partial T}\bigg|_{Y(\zeta)} + K_{IR\nu}^{(\kappa)}(\cdot,\zeta)\,\frac{\partial\kappa_\nu}{\partial T}\bigg|_{T(\zeta)} \right) d\nu \tag{12}$$

$$\frac{\partial Q_{IR}}{\partial T_s} = \int\limits_{\Delta\nu_{IR}} \left(\frac{\partial Q_{IR\nu}}{\partial B_{\nu s}}\,\frac{\partial B_{\nu s}}{\partial T_s} + \frac{\partial Q_{IR\nu}}{\partial\varepsilon_\nu(\zeta)}\,\frac{\partial\varepsilon_\nu}{\partial T_s} \right) d\nu \tag{13}$$

Quantitative analysis of sensitivities

Partial derivatives of the absorption coefficient κ_ν, surface albedo A_ν and Planck function B_ν with respect to corresponding atmospheric and surface parameters are assumed to be known.

Evaluation of variational and partial derivatives of the solar heating and thermal IR cooling rates with respect to κ_ν, A_ν, and B_ν represent the problem which, in its general form, still has to be addressed.

Two approaches, applicable, respectively, to the non-scattering and scattering atmospheres are presented below for the baseline example: sensitivities atmospheric thermal IR cooling rate to the atmospheric parameters $\delta Q_{IR}(z)/\delta Y(\zeta)$.

Non-scattering atmospheres: Linearization approach

Theoretical foundations for this case are formulated in different forms, starting from [1]. For the non-scattering atmospheres, the radiative heating and cooling rates can be analytically expressed through relevant atmospheric and surface parameters.

Suitable set of expressions to be linearized using a general linearization approach [2] is as follows. Atmospheric thermal IR cooling rate (frequency ν is implied):

$$Q_{IR}(z) = 4\pi\kappa(z)B(z) - 2\pi\kappa(z)\Big(\langle t_\uparrow(z,z_0)\rangle\,\varepsilon B_s + \int_{z_0}^{z} B(\zeta)\,d\langle t_\uparrow(z,\zeta)\rangle + \int_{\infty}^{z} B(\zeta)\,d\langle t_\downarrow(z,\zeta)\rangle\Big) \tag{14}$$

In this expression, cooling by proper thermal radiation from level z, and heating from the surface and from the atmosphere below and above this level, are explicitly separated.

In Eq.(14) $Q_{IR}(z)$is expressed through Planck function $B(z)$ and angular averages of transmittance for upwelling ($\uparrow$) and downwelling ($\downarrow$) radiation

$$t_{\uparrow\downarrow}(\tau, \tau'; u) = \exp\left(-\frac{\tau - \tau'}{u}\right) \tag{15}$$

where optical depth

$$\tau(z) = \int_z^\infty \kappa(\zeta)\, d\zeta \tag{16}$$

is directly linked to the absorption coefficient $\kappa(z)$. (Monochromatic frequency ν is implied).

Intermediate sensitivities $K_{IR}^{(B)}(z,\zeta)$ and $K_{IR}^{(\kappa)}(z,\zeta)$ are obtained from the set of Eqs.(14)–(16) in a unified form

$$\begin{aligned} K_{IR}(z,\zeta) &= \delta(z-\zeta)\, K_\delta(z) \\ &+\theta(\zeta - z)\, K_\downarrow(z,\zeta) + \theta(z-\zeta)\, K_\uparrow(z,\zeta) \end{aligned} \tag{17}$$

containing terms due to cooling by proper radiation and heating by radiation from outside atmosphere. Kernels in separate terms have the form:

$$K_\delta^{(B)}(z) = 4\pi\delta(z-\zeta) \tag{18}$$

$$K_{\uparrow\downarrow}^{(B)}(z,\zeta) = 2\pi\kappa(z)\left\langle \frac{1}{|u|}\, t_{\uparrow\downarrow}(z,\zeta;u)\right\rangle \tag{19}$$

$$K_\delta^{(\kappa)}(z) = \frac{Q_{IR}(z)}{\kappa(z)} \tag{20}$$

$$K_{\uparrow\downarrow}^{(\kappa)}(z,\zeta) = 2\pi\kappa(z)\left\langle \frac{1}{|u|}\, t(z,\zeta;u)(B(\zeta)) - I_{\uparrow\downarrow}(\zeta,u)\right\rangle \tag{21}$$

These expressions are substituted in Eqs.(8), (10), (12) to obtain resulting sensitivities to the atmospheric parameters.

Scattering atmospheres: Adjoint approach

There is no analytic solution of RT equation in the general case of scattering atmospheres. As a result, no analytic expressions are available for radiative heating and cooling rates through the atmospheric and surface parameters.

Instead, radiative heating and cooling rates can be analytically expressed through the solution of the corresponding RT equation with boundary conditions $I(z,u)$. For atmospheric IR cooling rate we have:

$$Q_{IR}(z) = 2\pi \int_{-1}^{1} I(z,u)\, du \tag{22}$$

or, in general form:

$$Q_{IR}(z) = \int_{0}^{\infty} d\zeta \int_{-1}^{1} du\, W(z,\zeta;u) I(\zeta,u) = (W,\, I) \tag{23}$$

where the function

$$W(z,\zeta;u) = 2\pi\delta(z-\zeta) \tag{24}$$

describes here the procedure of converting $I(\zeta,u)$ into $Q_{IR}(z)$ and an inner product notation was used.

It turns out that sensitivities of radiative heating and cooling rates can also be expressed analytically through $I(z,u)$ and the solution of the corresponding *adjoint* RT problem $I^*(z,u)$ in a fashion similar to Eq.(23).

This problem is analogous to that of evaluation of weighting functions in remote sensing of scattering atmospheres, first considered using an adjoint approach in [3]. Essentially, these functions are sensitivities of observed radiances to the atmospheric and surface parameters.

The adjoint approach can be summarized as follows. The forward RT equation with boundary conditions is combined (see, e.g., [4]) into a single linear operator equation:

$$LI = S \tag{25}$$

where operator L combines all linear homogeneous operations on $I^*(z, u)$ in, and right-hand term S combines all right-hand terms of, the RT equation and boundary conditions.

The adjoint problem

$$L^* I^* = W \tag{26}$$

involves the operator L^*, adjoint to the operator L, and the right-hand term W derived from procedures converting $I(z, u)$ into the atmospheric and surface heating/cooling rates as in Eq.(23).

The nature of radiative output, be it radiative heating/cooling rates or observable radiances, is captured by the adjoint solution I^*. Thus the expressions below are simply borrowed, in somewhat modified form, from the work on remote sensing in thermal IR [4] (frequency ν is implied):

$$K_{IR}^{(B)}(z, \zeta) = \alpha(\zeta)(1 - \omega_0(\zeta)) \int_{-1}^{1} I^*(\zeta, u; z) du \tag{27}$$

$$K_{IR}^{(\kappa)}(z, \zeta) = \int_{-1}^{1} I^*(\zeta, u)(B(\zeta) - I(\zeta, u; z))\, du \tag{28}$$

Here $\alpha(\zeta)$ is the volume extinction coefficient of the atmosphere.

The above expressions are substituted in Eqs.(8), (10), (12) to obtain resulting sensitivities to the atmospheric parameters.

Discussion and conclusion

Sensitivity of atmospheric radiative heating/cooling rates result in first-order perturbation terms in the thermodynamics equation due to temperature and other atmospheric parameters (see Eqs.(4), (3)).

Altogether, there are eight groups of sensitivities (see Eq.(8)) of atmospheric and surface heating/cooling rates to atmospheric and surface parameters, which, in general, need to be taken into account.

To the best of the author's knowledge, there is only one effort in this area documented in literature [5] where the heating by solar radiation in a scattering atmosphere was considered using an adjoint model employing an equivalent of an adding/doubling method.

Radiative cooling due to thermal IR radiation can be a major factor in the radiation-driven atmospheres, like that of Mars, and accurate evaluation of its perturbation terms, especially due to temperature, is important, to say the least.

Linearization of the radiative heating/cooling rates is necessary to in order to completely and accurately linearize the global circulation models (GCMs).

Linearized GCMs serve as a basis of the variational assimilation approach which was pioneered in [6] and currently is widely used. It is expected that a complete linearization of GCMs will result in improved performance of corresponding variational assimilation schemes.

Acknowlegement

The research described in this paper was carried out at the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration.

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