Practical quantum repeaters with linear optics and double-photon guns

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We show how to create practical efficient quantum repeaters for optical quantum communication with double-photon guns. These latter devices create event-ready polarization-entangled photon pairs. One such source might be a semiconducter quantum dot, which has the distinct advantage over parametric down-conversion that the probability of creating a photon pair is close to one, whereas the probability of creating multiple pairs vanishes. The swapping component is implemented by probabilistic optical CNOTs.

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Quantum repeaters [1,2] are essential for optical quantum comunication over distances longer than the attenuation length of the used channels. These devices are a combination of entanglement swapping [3] and entanglement purification or distillation [4]: multiple pairs of degraded entanglement are condensed into (fewer) maximally entangled states, after which swapping is used to extend the (now maximal) entanglement over greater distances. Both entanglement distillation and swapping has been demonstrated experimentally [5,6]. In this Letter, we present a practical protocol for optical quantum repeaters based on linear optics and a double-photon gun.

Until now, the source of polarization-entangled photons has mostly been parametric down-conversion. However, the output of these devices are not 'clean' maximally entangled two-photon states, but rather a coherent superposition of multiple pairs: $|\Psi_{out}\rangle \propto \sum_{n=0}^{\infty} N_n(\epsilon L_+)^n|0\rangle$, where L_+ is the operator that creates a photon pair (analogous to the ordinary creation operator, with normalisation N_n), $|0\rangle$ is the vacuum and ϵ is the (complex) probability amplitude of creating the maximally entangled state [7]. Therefore, down-converters can only be used when $|\epsilon| \ll 1$, and this puts a severe practical limit on their usefulness. Both because this yields a very weak source of entanglement (the major contribution to the state is the vacuum $|0\rangle$) and the fact that when several down-converters are used together, the double-pair production might seriously affect the fidelity of the optical quantum computation [7,8]. We would therefore like to have a source with the following properties: (1) whenever we push the button of our entanglement source, we produce a polarization-entangled photon-pair with high probability. (2) The fidelity of the output of our entanglement source is very close to one. A source with these properties is what we call a double-photon gun.

One entanglement source that very nearly meets our requirements has been proposed by Yamamoto and coworkers [9] (see Fig. 1). A quantum dot separating p-type and n-type GaAs is sandwiched between two Bragg mirrors. The entire structure is an optical microcavity, and electron-hole recombination will result in the creation of an entangled photon pair. Critically, due to Pauli's exclusion principle, only one electron and one hole are recombined at the time, resulting in at most one photon pair. Furthermore, this process is triggered by applying a potential difference over the microcavity, which allows for greater control over the creation of a pair. In particular, the probability of creating a pair can be as high as $p_s = 0.9$. Consequently, this source satisfies the required properties outlined above.

The two entangled photons from this source have different frequencies, which allows us to spatially separate them by means of a dichroic mirror. Interference phenomena at (polarization) beam splitters, however, rely on the indistinguishability of the incoming photons, and the non-degenerate frequencies might render the photons distinguishable. Special care need to be taken to arrange the setup in such a way that only photons with equal frequency enter any particular optical element.

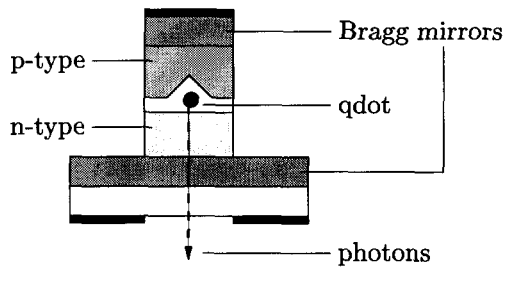

FIG. 1. The entanglement source. A quantum dot separating p-type and n-type GaAs is sandwiched between two Bragg mirrors. Electron-hole recombination will result in the creation of an entangled photon pair. Due to the Pauli exclusion principle, multiple pair production is suppressed. The efficiency of the source reaches values up to 90%.

In practice, quantum communication protocols may use any of the four two-qubit Bell states [$|\Phi^{\pm}\rangle = (|H,H\rangle \pm |V,V\rangle)/\sqrt{2}$ and $|\Psi^{\pm}\rangle = (|H,V\rangle \pm |V,H\rangle)/\sqrt{2}$, with H and V the polarization directions]. These states are locally transformed into each other by means of simple qubit operations, and we therefore assume that the

above entanglement source can make any of the four Bell states. Now let us look at the other ingredients of our quantum repeater.

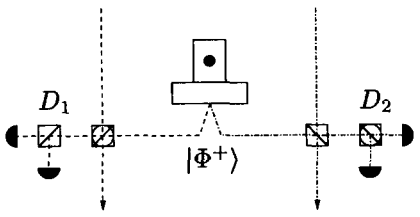

FIG. 2. The probabilistic CNOT gate. Conditioned on a specific detector outcome in D_1 and D_2, the setup performs a controlled not. The boxed beam splitter is a linear polarisation beam splitter and the circled box is a beam splitter around circular polarisation.

The entanglement-swapping part of the quantum repeater is essentially nothing more than a Bell detection. It is well known that it is impossible to make a deterministic complete Bell measurement with linear optics [10], but one can distinguish two out of the four two-qubit Bell states with a simple beam splitter configuration [11]. Recently, it has been shown that a controlled-not (CNOT) —and hence a Bell measurement— is possible ***probabilistically*** with projective measurements and entangled input states [12]. The probability of success for this CNOT is not large enough to make the Bell measurement more efficient, but it will be an essential component of the purifier.

In order to distill maximally entangled quantum states, one needs entanglement purification. Suppose Alice and Bob share two non-maximally entangled states. Bennett *et al.* showed that with some finite probability it is possible to extract a single maximally entangled state [4]. Both Alice and Bob apply a CNOT, where the halves of the first entangled state serves as the control qubit, and the halves of the second as the target. The target qubits are then measured in the computational basis (determined prior to operation by the participants, e.g., $|H\rangle$ and $|V\rangle$), and conditioned on a parallel coincidence, Alice and Bob now share a maximally entangled state in the remaining two qubits. The probability of purification depends on the fidelity of the incoming entangled state, and therefore on the channel noise γ.

Additionally, in some cases the modes of the control qubit might be empty, because the entanglement source failed to create a photon. In order to rule out these events, we can employ the interferometric quantum nondemolition (QND) measurement scheme proposed by Kok *et al.* [13]. This is a probabilistic scheme that can be set up to signal a single photon in an optical mode without destroying its polarization. The success rate of this device is $p_{\mathrm{qnd}} = \frac{1}{8}$. This device employs two entanglement sources to create the auxiliary single-photon input states and four photodetectors.

Essential for this protocol is the ability to perform the controlled-not operation. In Fig. 2 we show the schematic setup for the probabilistic CNOT designed by Pittman *et al.* [12]. The main ingredients are a $|\Phi^+\rangle$ source and four polarization beam splitters, two of which seprate ***circular*** polarization. The control qubit enters a polarizing beam splitter, and the target enters a circular polarizing beam splitter. The secondary input ports of these two beam splitters are fed by the two parts of a $|\Phi^+\rangle$ Bell state.

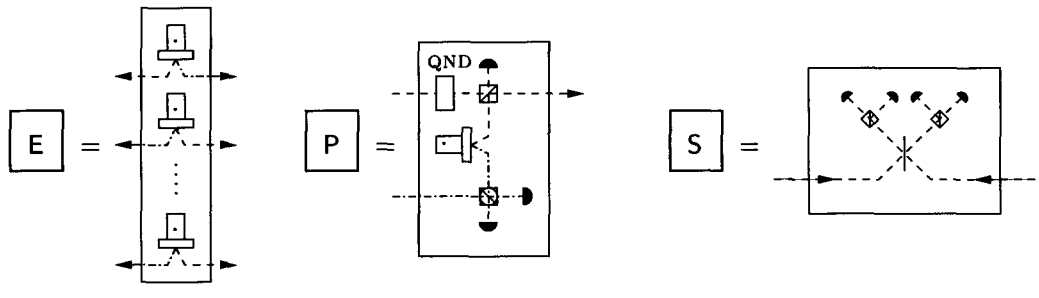

FIG. 3. The components of the quantum repeater. The three boxes E, P, and S denote the entanglement station, the purifier and the swapping element respectively. The entanglement sources are drawn as little high hats with a qdot inside. The dashed and the dash-dotted lines represent the fact that the two output modes have different frequencies. The purifier element contains a QND measurement, an optical CNOT and the detection of one output mode, and the swapping element consists of a partial Bell measurement.

A successful CNOT operation is now conditioned on detecting a linearly polarized photon after the circular polarization beam splitter (D_1 in Fig. 2), and a circularly polarized photon after the linear polarizing beam splitter (D_2 in Fig. 2). These detections can be implemented with suitable polarization beam splitters and ordinary photodetectors [12]. The probability of this CNOT operation is given by $p_{c\neg} = \frac{1}{4}$.

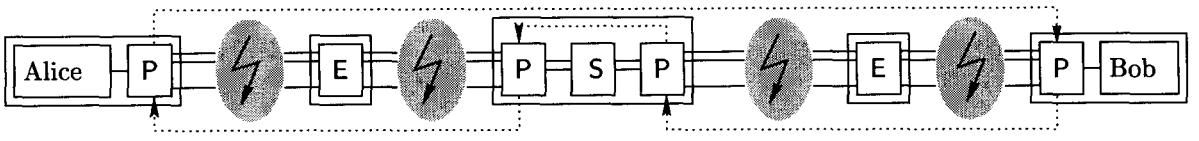

FIG. 4. The assembled quantum repeater. The solid lines represent the quantum channels and the dotted lines denote classical communication channels. The distance is included in the shaded region. This is also where the "decoherence devil" resides. Note that the entanglement sources are separate stations. Alternatively, the entanglement station can be placed near Alice and Bob.

In order to build a complete quantum repeater, we have to integrate the components described above into a circuit [1,2]. The separate components are (see Fig. 3) the *entangler* E, the *purifier* P and the *swapper* S. The assembled quantum repeater, shown in Fig. 4, is a circuit involving E, P and S, together with classical communication between the different stations. This classical channel is necessary to exchange information about the measurement outcomes of the purifiers (success of the purifiers is conditioned on parallel measurement outcomes between remote stations) and about the location of the purified (and swapped) entanglement.

Finally, we address the success rate of the quantum repeater. The success rate is given by the reciprocal value of the single-pair creation between Alice and Bob. It turns out that this value is highly dependent on the quantum efficiency of the photodetectors.

To purify two entangled photon pair we have to take into account the probability of success for the individual components and the losses in the system. Suppose we have six double-photon guns (two for the photon sources, two in the QND device and one for every CNOT) and ten detectors with quantum efficiency η (three per CNOT and four in the QND device). Furthermore, let the noise parameters due to the attenuation be given by γ for the dephasing (reducing the fidelity) and ζ for the photon loss over the channel. The probability for purifying a single pair of entangled photons is then given by

$$p_{\text{pur}} = p_s^6\, \eta^{10}\, (1-\gamma)^2\, \zeta^2\, p_{c\neg}^2\, p_{\text{qnd}}\ . \tag{1}$$

It is immediately obvious that a reduced quantum efficiency η will strongly contribute to the deterioration of the success rate.

In order to make a repeater, we need two purified pairs and perform entanglement swapping on two of their halves. This swapping protocol is not deterministic, and subject to losses as well. As can be seen in Fig. 3, the swapping element requires a two-fold detector coincidence. Furthermore, a complete Bell detection occurs only 50% of the time. The probability of success for entanglement swapping is therefore given by

$$p_{\text{swap}} = \frac{\eta^2}{2}\ . \tag{2}$$

Let us now insert some values of the several components. We will use different values for the detector efficiency, since this is the most important parameter. Choose for example $p_s = 0.9$, $\gamma = \frac{1}{2}$, $\zeta = \frac{1}{2}\sqrt{2}$, $p_{c\neg} = \frac{1}{4}$ and $p_{\text{qnd}} = \frac{1}{8}$. For three different values of η, this gives rise to Table I (with $N_{\text{pur}} = p_{\text{pur}}^{-1}$ and $N_{\text{swap}} = p_{\text{swap}}^{-1}$). Since a repeater needs two purifiers and one swapper, the total number of components N_{total} is given by $N_{\text{total}} = 2N_{\text{pur}}N_{\text{swap}}$. By comparison, the number of transistors on a pentium chip is of the order 10^7. It is immediately clear that an improvement in the detector efficiency yields a substantial gain in the efficiency of the protocol. Even though efficiencies of 0.8 are calculated, experimental values are closer to 0.3. Therefore, in order to operate the repeater more efficiently, better detectors are needed.

There are two possible drawbacks with this quantum repeater protocol. First, the semiconductor quantum dot entanglement sources operate in the temerature range of 20–50 Kelvin [9]. Secondly, intelligent switching conditioned on detector outcomes and classical communication between the components is needed both to purify and to match purified entanglement in the swapping procedure. This requires an overhead in the number of components.

In conclusion, we proposed a practical implementation for a quantum repeater with double-photon guns, probabilistic CNOT operations and quantum nondemolition

measurements. The protocol requires available and almost available technology. Possible drawbacks are the low operating temperatures, conditional switching and the quantum efficiencies of photodetectors.

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η	N_{pur}	N_{swap}	N_{total}
0.3	$3 \cdot 10^7$	20	$\sim 10^9$
0.8	$2 \cdot 10^3$	3	$\sim 10^4$
1	250	2	$\sim 10^3$

TABLE I. The number of components in the quantum repeater for different values of the detector efficiency η.

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