AAS 02-139

LOW-THRUST ORBIT TRANSFER AROUND MINOR PLANETS

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Several methods for determining the characteristics of low-thrust transfers around minor planets are examined and compared. The methods range from simple analytic approximations to sophisticated optimization. The ΔV required for a given transfer generally decreases with increasing flight time. Allowing a relatively short coasting period can significantly decrease the required ΔV over a minimum time (continuous thrust) transfer. A simple analytic approximation provides a good estimate of the ΔV, and a Hohmann-type transfer, which can be achieved in a reasonable time, establishes a lower bound.

INTRODUCTION

The use of solar electric propulsion (SEP) has been shown to be highly effective for transferring from Earth to rendezvous with main-belt asteroids.[1,2] Once the spacecraft is captured at the asteroid, the SEP system can be used to transfer between orbits. In this paper we present analyses of these types of transfers between coplanar, circular orbits using low-thrust propulsion systems such as SEP. We assume an inverse square gravity field.

Exact, analytic solutions to this orbit transfer problem not being available, we examine various analytic approximations as well as numerical methods. Approximate expressions for multi-revolution, constant-thrust-acceleration trajectories, starting from circular orbits, have been known since the 1950s.[3,4] In the formidable body of research conducted in the following two decades, constant-thrust analogues were obtained,[5] and averaging and multiple-time-scale methods were employed to model analytically the oscillations of the osculating orbit elements.[6,7] Exact analytic results are known for various special cases which require variable thrust (often of realizable maximum magnitude).[8-11] However, the above results are only imperfectly applicable to the problem of circle-to-circle transfers using constant thrust. Some of these, along with other approximations and simple numerical control laws, are discussed below.

A small collection of analytic results based on optimization methods is also available. Again using averaging, partly analytic results are known for the problem of propellant-optimal, constant-power, variable-thrust orbit transfer.[12,13] More recently,

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exact planar solutions were obtained for propellant-optimal transfer between ellipses using engines whose thrust and I_{sp} are variable and bounded but not on the bounds.[14]

A large number of works have focused on numerical solutions to the problem of optimal orbit transfer; a small sampling is presented in Refs. 15-20. Kechichian,[15] for example, uses a shooting method, with coordinates chosen so as to permit faster numerical integration, to solve the two-point boundary value problem which arises from a calculus of variations formulation of the optimization problem. Geffroy and Epenoy[16] use averaging techniques and the calculus of variations, solving the resulting problem by continuation and other methods. Betts[19] applies collocation techniques to the full dynamics of the problem, subsequently using sequential quadratic programming. Kluever and Oleson[20] also use a direct method, but use averaged dynamics to avoid the large size of the full-dynamics problem. Whiffen and Sims[21] describe software based on static/dynamic control which can use a full n-body gravity model to optimize orbit transfers either alone or as part of larger problems involving a succession of several primary gravitational centers. This software, which is used in the present paper, works very well when the number of revolutions required for the orbit transfer is small. Computation times become increasingly longer as the number of revolutions increases. However, for the problem of orbit transfer around a small body, thrust levels are often high enough so that only a few revolutions are required.

APPROACH

We examine several methods for determining the characteristics (e.g., ΔV and flight time) of low-thrust transfers around relatively large asteroids, or minor planets.

Analytic Techniques

We examine several analytic techniques. These include shape-based methods, where the spacecraft is assumed to follow a predetermined shape, such as the logarithmic spiral or the exponential sinusoid,[11] and optimization-based methods affording analytic solutions. For example, Lawden's spiral,[22] although now known to be sub-optimal, can provide some qualitative insights thanks to its simple analytic form. Also examined are analytic solutions to the minimum-fuel problem based on averaging techniques.[12,13] All of these analytic techniques reduce to the following equation as an estimate for the ΔV required to transfer between coplanar, circular orbits in the limit of constant infinitesimal thrust over an infinite flight time:

$$\Delta V = V_{c1} - V_{c2} \tag{1}$$

where V_{c1} is the velocity in the lower circular orbit and V_{c2} is the velocity in the higher circular orbit. For finite flight times, Lawden's spiral, the logarithmic spiral and the exponential sinusoid all require two impulsive ΔVs, once to leave the circular orbit and enjoin the transfer arc, and once to depart from the transfer arc and enjoin the target circular orbit. As the number of revolutions decreases, the impulsive ΔVs for the former two shapes are increasingly non-tangential and increasingly larger than the impulsive ΔVs for the exponential sinusoid. For the exponential sinusoid, the sum of the impulsive ΔVs and the ΔV needed on the transfer arc is increasingly less than the ΔV of Eq. (1), the fewer the revolutions, while for the other two shapes this sum is increasingly greater than the ΔV of Eq. (1). For all three shapes, however, the required thrust acceleration varies significantly over the course of the transfer, and reaches increasingly large maximum values as the number of revolutions decreases. Thus, while the exponential sinusoid provides the best performance of the three shapes, some interpretation is required to apply the results to the case of constant thrust. For example, the average thrust over the transfer may be constrained to lie below the available thrust from the engines.

Hohmann Transfer. Another point of comparison that we use is the well-known Hohmann transfer that assumes impulsive ΔVs. If infinite transfer time were allowed, the Hohmann ΔV itself could be attained for the transfer. For example, to enter a higher circular orbit, a series of infinitesimal impulses at periapse could be used to raise apoapse to the desired circular radius, whereupon another series of impulses would be applied at apoapse to circularize the orbit. A much less efficient way of using an infinite transfer time is to transfer repeatedly between a succession of circular orbits using infinitesimal Hohmann transfers. The total ΔV required in this case is given by Eq. (1) also.

Simple Control – Thrust Aligned with Velocity

Another method uses a simple control scheme in which the thrust is aligned with the velocity relative to the minor planet. The two-body equations of motion are integrated with the thrusting included as an additional force.

Infinitesimal continuous thrust along the velocity vector results in a trajectory that remains differentially close to circular throughout the circle-to-circle transfer. However, for any finite continuous thrust, the trajectory does not remain circular, so continuous thrusting strictly along the velocity vector cannot be used to transfer from one circular orbit to another. Finding a simple thrusting strategy for a nearly continuous low-thrust transfer between circular orbits was challenging. The strategy described here is very simple, employing constant thrusting along the velocity vector, except for one short (less than one orbit) coasting period at some point during the transfer. The short coast enables this simple strategy to result in a circular orbit at the end of the transfer. The orbits must be coplanar and the phasing in the final orbit (relative to the initial orbit) is not free. There may, however, be multiple solutions from which to choose, each having different final orbit phasing. Each of the multiple solutions has a different total ΔV requirement (although the differences have been very small in the test cases we have examined). In general, we choose the one with the lowest ΔV, although we could also choose the one with the shortest total flight time.

Given an initial and final circular orbit radius, the problem is to find the duration of the first thrusting period (Δt_1), the duration of the required coasting period, and the duration of the second thrusting period (Δt_2) to transfer from a state on the initial orbit to a state on the final orbit. The solution technique involves integrating forward in time from the initial orbit and backward in time from the final orbit. Integrating forward in time with thrusting from a state on the initial orbit for a time interval Δt_1 results in an intermediate orbit with a particular semimajor axis and eccentricity. Integrating backward in time from a state on the final orbit for a time interval $-\Delta t_2$, we arrive at an intermediate orbit with the same semimajor axis and eccentricity. Except for a rotation, these represent the same orbit. The rotation problem is solved by simply rotating the spacecraft position in orbit for the start of the backward integration (this is the reason why the phasing in the final orbit is not free). With this rotation, the shape and orientation of the intermediate orbits for the forward and backward integrations are identical, so they are the same orbit. This orbit is called the *transition orbit.* But, the transition orbit still has a phasing problem. For the forward integration, the spacecraft enters the transition orbit with a particular mean anomaly (position in the orbit). For the backward integration, the spacecraft enters the transition orbit at a different mean anomaly, which (since integration is backwards) is actually the mean anomaly of the spacecraft at the point where it must leave the transition orbit. The total thrusting time is $\Delta t_1 + \Delta t_2$. The total transfer time is the total thrusting time plus the time spent coasting on the transition orbit (less than one revolution). Using an iterative method, we solve for Δt_1, Δt_2, the parameters of the transition orbit, and the required coast time in that orbit. There may be several solutions as illustrated in Figure 1.

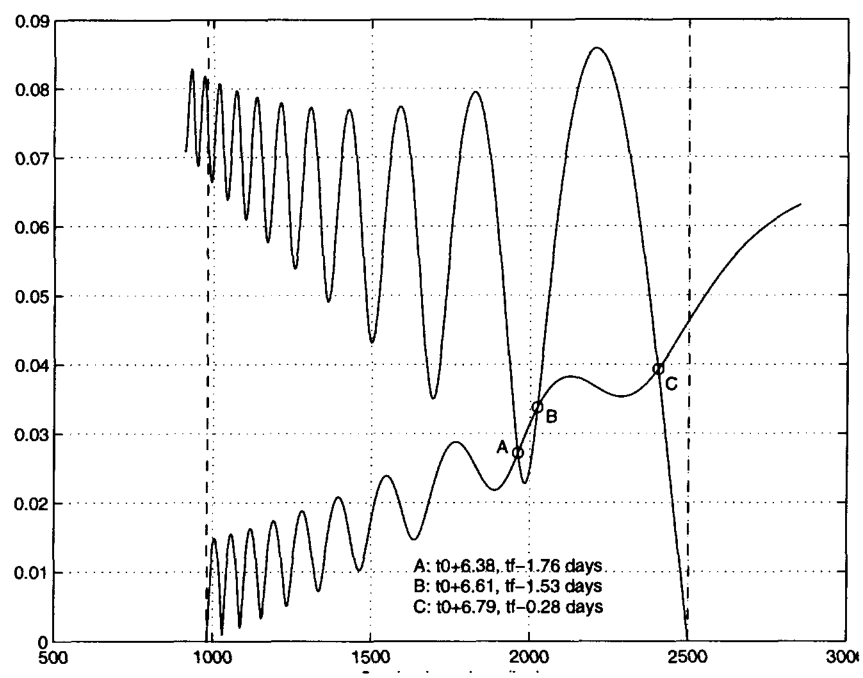

Figure 1 Eccentricity versus Semimajor Axis for Thrusting Aligned with Velocity

There is one final complication. For the backward integration, the final mass (which is required for the integration) is not known. The value must be guessed and then compared at the transition orbit with the spacecraft mass from the forward integration. These two values must be equal. By iteration the correct final mass is found.

Numerical Optimization

The third primary method is an application of the optimization algorithm called Static/Dynamic Control (SDC).[21] SDC is a general, gradient-based optimization method that is distinct from both parameter optimization and the calculus of variations. Trajectories are integrated with a multi-body force model and engine operation is modeled as finite burns.

SDC is a new, general optimization algorithm that was derived to address a class of problems with the same structure as low-thrust optimization. SDC best fits into the direct method category. However, unlike other direct methods, the explicit time dependence of the optimization problem is not removed by parameterization. The SDC optimization algorithm is a form of optimal control. The SDC optimization algorithm is based in part on the Hamilton, Bellman, Jacobi dynamic programming equation.[23] Unlike traditional differential dynamic programming methods, SDC is constructed to solve highly nonlinear and non-convex problems with a dual dynamic and parametric structure. Optimal solutions generated by SDC satisfy both the necessary and sufficient conditions of optimality.

RESULTS

We present detailed results from two primary cases. The first case has a thrust acceleration to gravitational acceleration (T/W) ratio ranging from 0.3 to 0.07. In the second case, T/W ranges from 0.02 to 0.004.

Case 1

The first case is a transfer between circular orbits from 3000 km radius to 1500 km radius around a body with a gravitational constant of 1.016 km^3/s^2. The initial spacecraft mass is 1550 kg, and the engine parameters correspond to an NSTAR 30 cm ion thruster[24] as demonstrated on Deep Space 1: thrust = 0.05225 N and mass flow rate = 1.751×10^{-6} kg/s.

A plot of propellant mass versus flight time using SDC is presented in Figure 2. The minimum flight time achievable is 4.31 days and requires 0.65 kg of propellant. The corresponding trajectory is shown in Figure 3. The thrust direction (represented by the arrows) is significantly different than the velocity direction for most of the transfer. In

Figure 2 we see that the propellant mass can be significantly reduced with only a small increase in flight time. A trajectory with a 5-day flight time is shown in Figure 4. The thrust direction is aligned very closely with the velocity direction in this case. As we increase the flight time, the performance approaches that corresponding to a Hohmann transfer. The trajectory with a 30-day flight time is shown in Figure 5. The periapse is reduced to 1500 km with two short duration thrust arcs, and the apoapse is then reduced with three thrust arcs, approximating a Hohmann transfer in stages. The line labeled "Analytic" in Figure 2 corresponds to Eq. (1). Using the method in which we align the thrust with the velocity has only one solution in this case and results in a propellant mass slightly less than the analytic value.

Case 2

The second case that we examined in detail has a T/W an order of magnitude less than in Case 1 and requires several revolutions (>10) in order to complete the transfer. We transfer from a 2500 km circular orbit to a 982 km circular orbit around a body with a gravitational constant of 20.016 km^3/s^2. The initial spacecraft mass is 690.55 kg, and the thrust and mass flow rate are the same as in Case 1.

A plot of propellant mass versus flight time is shown in Figure 6. The minimum flight time is 8.27 days, requiring 1.251 kg of propellant. The required propellant decreases quickly as the flight time increases to about 8.6 days and then slowly approaches the value corresponding to the Hohmann transfer. Figure 7 compares the minimum flight time trajectory to a trajectory with a flight time of 8.56 days (near where the performance starts to level off). Note the thrust angle with respect to the velocity for the two cases. The minimum flight time objective results in significant thrusting inefficiency compared to a slightly longer flight time solution. Avoiding this performance penalty requires only a short (relative to total flight time) coast. The performance penalty is indicated by a significant component of the thrust being perpendicular to the velocity. The penalty is worse for fewer revolutions as we saw in Case 1.

The line labeled "Analytic" in Figure 6 corresponds to Eq. (1). Using the method in which we align the thrust with the velocity has three solutions in this case (see Figure 1). The one with the minimum total ΔV is represented in Figure 6.

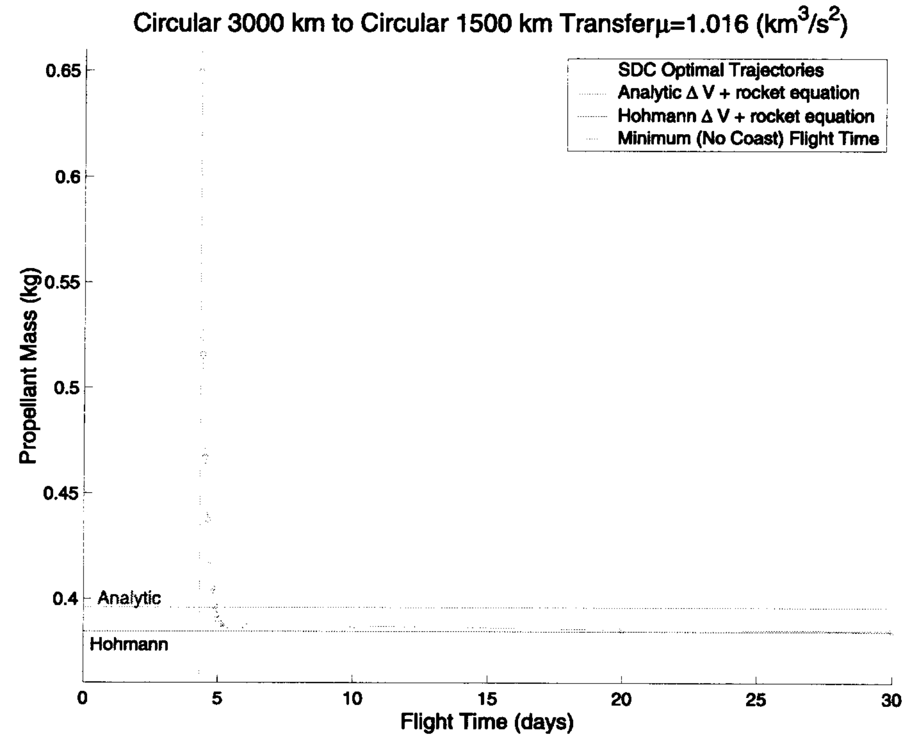

Figure 2 Propellant Mass versus Flight Time for Transfer between Circular Orbits

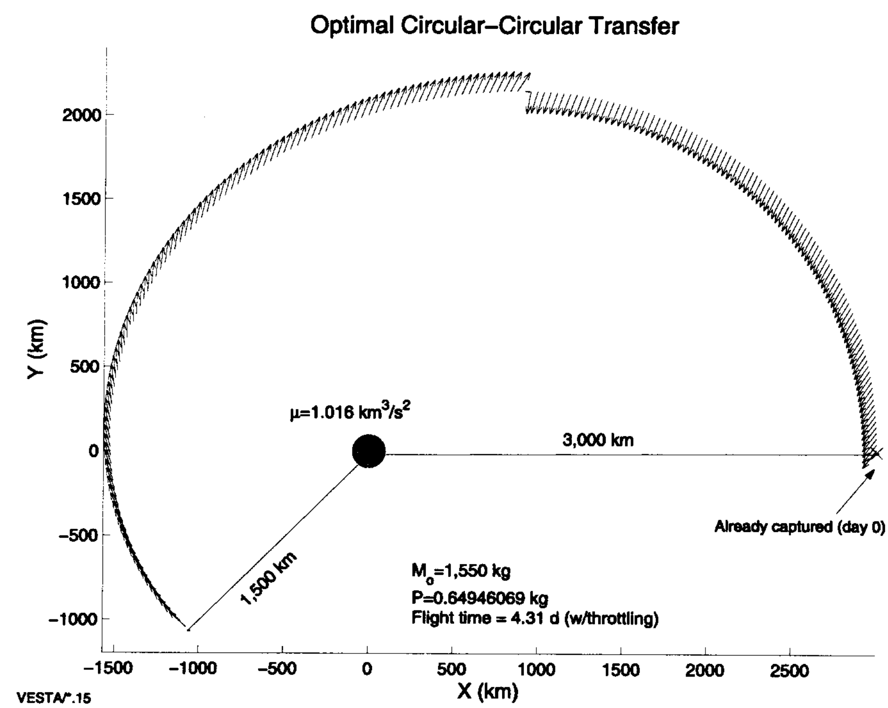

Figure 3 Minimum Time Transfer between Circular Orbits

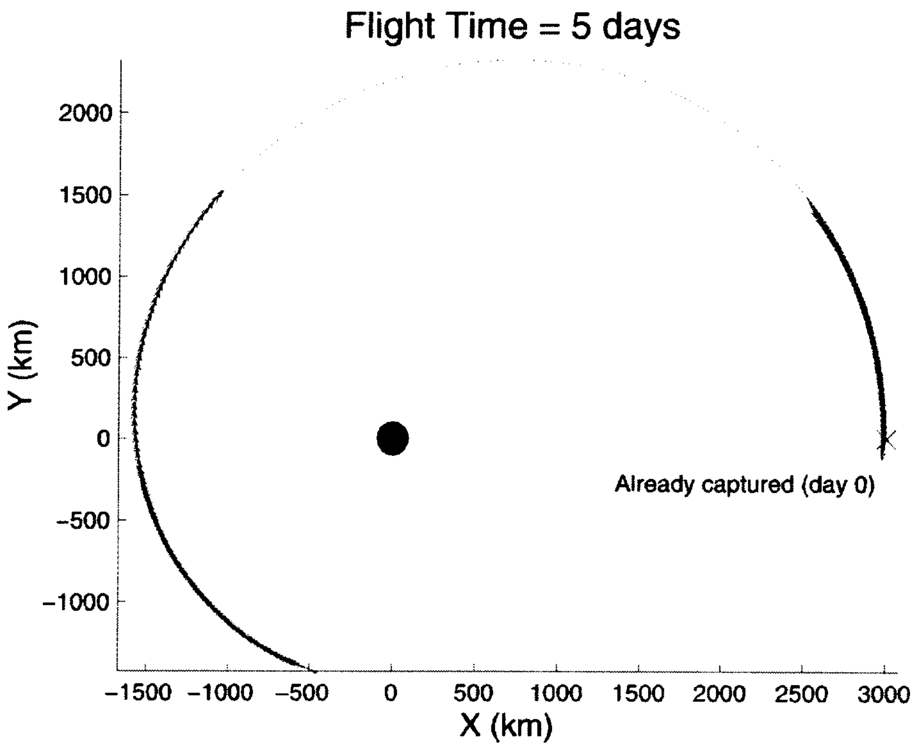

Figure 4 5-Day Transfer between Circular Orbits

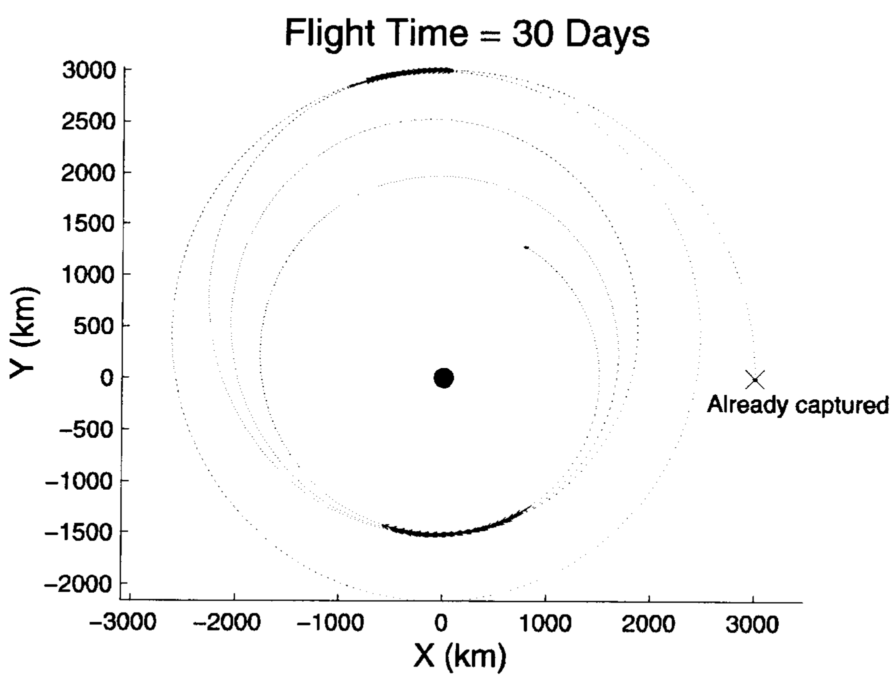

Figure 5 30-Day Transfer between Circular Orbits

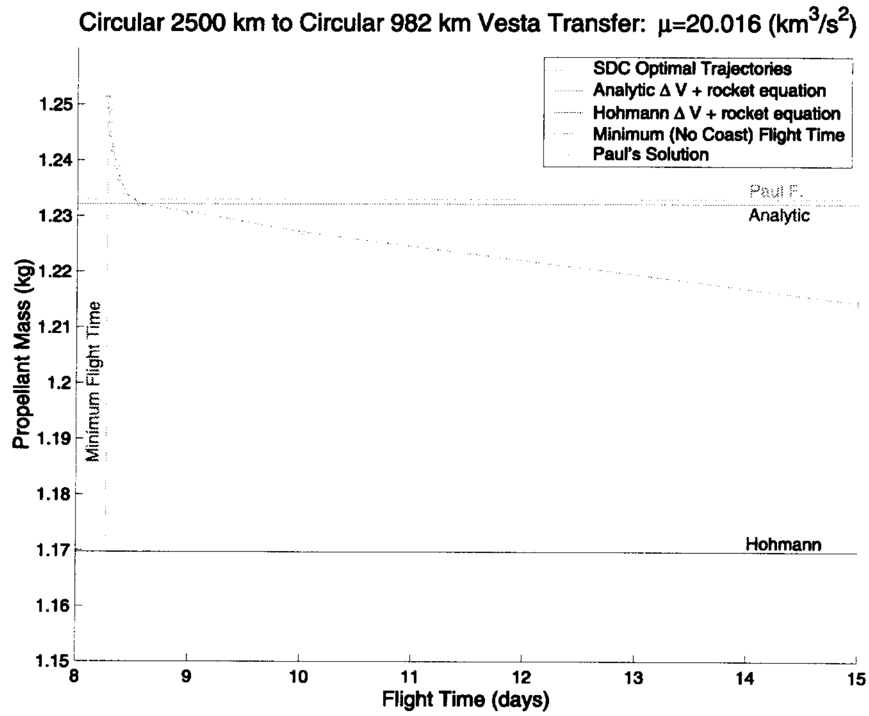

Figure 6 Propellant Mass versus Flight Time for Transfer between Circular Orbits

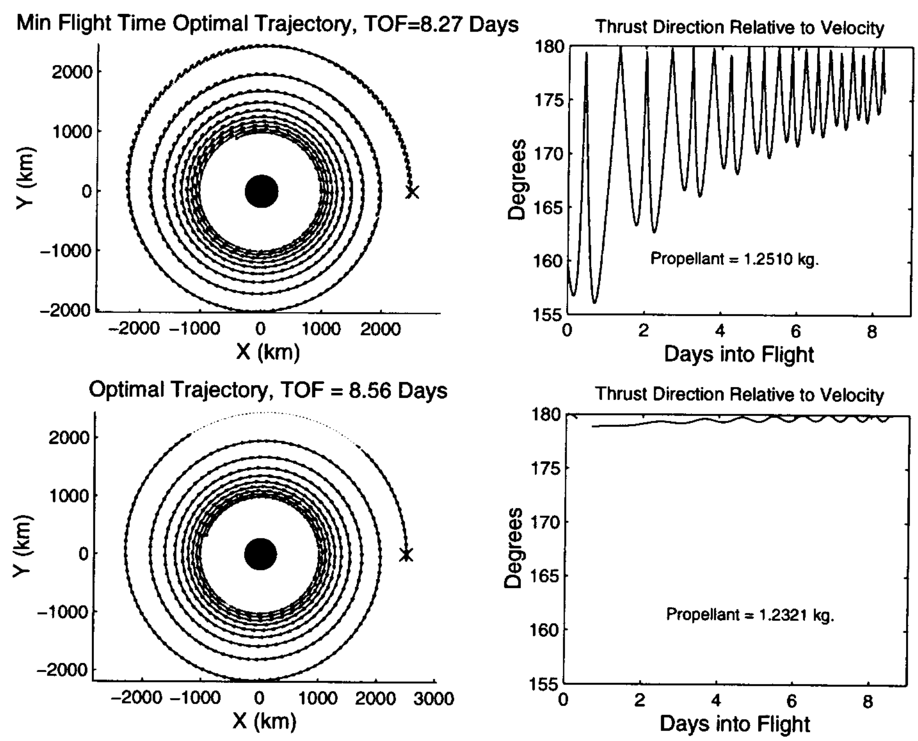

Figure 7 Minimum Time and 8.56-Day Transfer between Circular Orbits

CONCLUSION

We have examined several different methods for determining the characteristics of low-thrust transfers around minor planets. The methods range from simple analytic approximations to sophisticated optimization.

The ΔV required for a given transfer generally decreases with increasing flight time. Allowing a relatively short coasting period can significantly decrease the required ΔV over a minimum time (continuous thrust) transfer. A simple analytic approximation provides a good estimate of the ΔV, and a Hohmann-type transfer, which can be achieved in a reasonable time, establishes a lower bound.

ACKNOWLEDGMENT

The research described in this paper was carried out at the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration.

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