Reliability Modeling for the Space Interferometry Mission[1][2]

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Abstract—Top-level reliability models have been developed for several recent configurations of the Space Interferometry Mission (SIM) and the results used in the decision process for selecting viable configurations for further study. For one configuration, closed-form solutions were obtained. For three configurations, Excel-based Monte Carlo models were developed. The agreement between the closed form and the Monte Carlo models was excellent, verifying that the Excel-based approach had been successfully implemented. The Excel model has the flexibility to extend the model to more complicated arrangements for which it would be impractical to develop closed-form solutions.

The Space Interferometry Mission in NASA's Origins Program is a 10m-baseline space-based Michelson interferometer scheduled for launch in 2009. This large instrument will measure the angles between stars to an accuracy of about one billionth of a degree of arc. This is an improvement of about two orders of magnitude over current astrometric instruments.

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1. INTRODUCTION

The Space Interferometry Mission (SIM) is a joint effort of NASA Jet Propulsion Laboratory, California Institute of Technology, Lockheed Martin Missiles and Space, and TRW. SIM will use Interferometry to measure the angles between pairs of stars to the unprecedented accuracy of about 1 μ arc second (μas). Analysis of these measurements will enable several scientific objectives to be realized. A key objective is to infer the orbital parameters of planets around nearby stars based on the reflex motion of the star. SIM should be able to detect planets as small as the earth in favorable orbits, and will easily detect Saturn mass planets. These measurements will complement radial velocity measurements already made using earth-based telescopes, but will extend to smaller masses in longer orbits, and will resolve the inclination of the orbits, something that cannot be done using the radial velocity technique. Besides planet-defection, SIM will investigate many other celestial phenomena [1]. Additional general information about SIM can be found at the SIM Website: http://sim.jpl.nasa.gov [2].

SIM is currently in Phase A (Conceptual Design). In this phase, many different configurations are assessed at a fairly high (coarse) level. At this stage, it is not appropriate to develop very detailed reliability models. The overall design changes too rapidly and the changes are so great that it would be both impractical and too costly to develop very detailed models. However, it has been very useful to develop high-level reliability models, which can be used to compare the relative reliability of different configurations. The models are composed of a handful of large blocks. At this level of modeling, reliability databases do not exist, so actual probabilities cannot be used. However, by varying the reliability parametrically over a reasonable range, one can identify arrangements that are particular sensitive to failures or that are particularly insensitive to failures. Even though the actual probability numbers that come out of the models cannot be considered realistic estimates of the actual mission success probability, the relative reliabilities among various arrangements of the elements are useful in a comparative sense. This information has been used as part of a larger decision making process to select among various configurations. Also, within a configuration, it is possible to consider different arrangements of the elements to enhance reliability or to identify deficiencies that should be addressed.

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2. OVERVIEW OF SIM

SIM performs astrometry (measurement of star locations) by using a white light Michelson interferometer with a 10 m baseline. Groups of optical elements (similar to telescopes) are located 10 m apart on opposite ends of a Precision Support Structure (PSS) to collect the starlight. Light from these telescope-like assemblies is combined in an Astrometric Beam Combiner (ABC) in the middle of this large instrument. Optical Delay Lines (ODLs) are used to adjust the path length followed by the starlight so that the wavefronts from both arms of the interferometer arrive at the detector at precisely the same time. The path lengths within the instrument are then measured to a precision (not accuracy) of a few tens of picometers (1 pm = 10^{-12}m) using infrared lasers metrology gauges. Based on these measurements and other laser gauge measurements of the baseline length, the angle between the target star and the baseline is determined. In order to determine the orientation of the astrometric baseline, two other similar astrometric interferometers are used. The baselines for all the interferometers are kept as parallel as possible. The laser metrology system measures the small amount of deviation from parallelism to make corrections to the results.

The laser gauges are not absolute gauges. They do not measure the actual distances involved, but rather the changes in the distances with a precision of tens of picometers. The absolute lengths are basically calibrated by using measurements of stars around the sky. We measure a gridwork of stars spanning a large part of the celestial sphere, and then adjust the scale factor for the instrument to "close the grid." This is somewhat analogous to a surveyor measuring angles around a full circle and verifying that the total is equal to 360° and adjusting the scale factor to make it so.

More description about SIM and how it performs astrometry can be found in previous IEEE Aerospace Conference papers, "SIM Configuration Evolution" [3] and "Space Interferometry Mission Instrument Mechanical Layout" [4].

3. RELIABILITY MODELING

High-level (i.e. coarse) reliability models were developed for three competing configurations for SIM. The reliability models were used to help chose among these various options. The design of SIM has now evolved away from any of these designs, but the examples are still illustrative of the value of the relatively simple reliability models. They also illustrate that the methods presented here can be applied to reasonably complicated situations successfully without a tremendous expenditure of resources.

The three configurations modeled were named SIM Classic, Shared Baseline, and ParaSIM. At the time, SIM Classic was the reference design and the other two were proposed alternative designs. There were also several variants on each of these, but the three basic arrangements were modeled to help understand the aspects that might be sensitive in a reliability sense. This paper will focus on the SIM Classic Models.

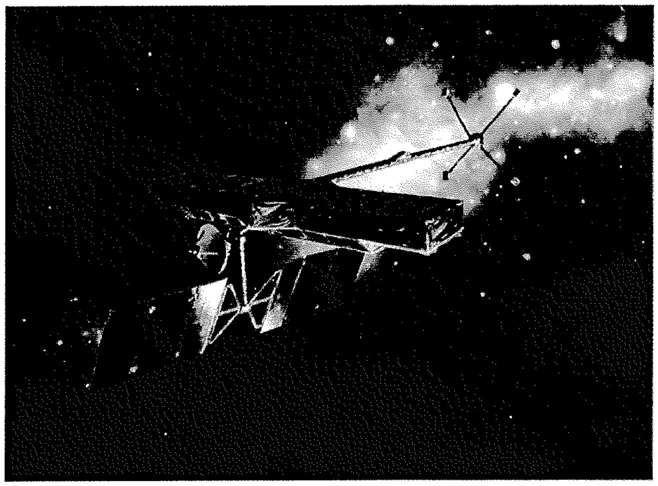

Figure 1 SIM Classic

The current SIM Configuration is a variant of Shared Baseline and it is being described in a companion paper in a different session at this same conference [5].

The first reliability model developed was for SIM Classic. At first, we did not plan to develop a reliability model, *per se*. Instead, we were trying to break down the system into relatively large logical blocks and looking at the interactions to get a sense for how they interacted from a reliability point of view. We expected only to develop a qualitative feel for these interactions. One of the authors (Kim Aaron) developed a block diagram showing some connectivity (series and parallel) among the various elements discussing how the system could continue to operate in the face of various types of failures. Another of the authors (Don Ebbeler) realized that it might be possible, at least in principle, to develop a closed-form solution for the reliability of the overall system based on assumed values for the individual blocks. After a significant effort with a fair amount of review and revision, a closed-form solution was actually developed for SIM Classic. The resultant equation, while not being trivial and obvious, was not as formidable as we had expected it to be. A second closed-form solution was also developed for SIM Classic, but with a different set of assumptions about which of the many redundant arrangements of the metrology kite could actually be considered operational. It turned out to be much more difficult to develop the closed-form solution for that case, but in the end, this problem also succumbed. However, it was clear that if we tried to increase the complexity of the model much beyond the point we were at, analytic solutions would rapidly become intractable.

Because we expected to need more complicated reliability models eventually (and because the commercially available reliability models were not set up to handle the unusual connectivity among the SIM elements) we decided to develop an Excel-based Monte-Carlo simulation instead. George Fox had been developing such capability for other projects and had created several macros/subroutines/add-ins for Excel. Starting with the same SIM block diagram, he was able to develop a model in Excel very rapidly (in just a few hours). This was particularly impressive since it had taken weeks to get the closed-form solution to the point that we believed it was correct. A little debugging was required, but very soon, the results from the Monte Carlo model were matching the predictions of the closed-form equations. Furthermore, it was reasonably easy for George to modify the connections of the elements in the model to represent the more complex alternative.

Having gained some confidence in the results of the Excel modeling approach, we developed similar Excel models for the other two SIM design configurations (Shared Baseline and ParaSIM).

Because we do not have any true reliability data for the kinds of blocks used in these models, we simply varied the reliability values for each block parametrically over a reasonably representative range. The model was exercised and the predicted overall reliability calculated for various combinations of parameters. This provided us with useful sensitivity data. In fact, some design changes were made as a direct consequence of seeing these results.

4. SIM Building Blocks

The reliability models comprise several large blocks. Each will be described briefly here. A schematic, Figure 2 below, shows how these elements are connected together in the reliability model for SIM Classic.

Siderostat Bay

The Siderostat Bay, or Sid Bay for short, is a combination of major optical elements mounted on a very precise and stable optical bench. In the version of SIM Classic modeled here, there are seven Sid Bays. The Sid Bay is not one of the reliability model blocks, but it is an important structure housing some of the reliability model blocks, so it is mentioned here.

Beam Compressor—A major subassembly mounted within the Sid Bay is the Beam Compressor. A beam compressor is similar to a telescope in that it has several powered optics. However, a telescope typically focuses the incoming light to form an image on a detector. In contrast, a beam compressor is afocal; instead of focusing the beam, it merely reduces the size of the beam. The output of this assembly is a smaller bundle of parallel light rays that are much easier to manipulate and guide throughout the rest of the instrument using small flat relay mirrors.

Siderostat Mirror—A Siderostat is a large flat mirror mounted on gimbals. It is also a major element mounted in the Sid Bay. It takes light from the target star of interest and reflects it along the optical axis of the Beam Compressor. A Siderostat is so named because it keeps the pointing direction <u>stat</u>ionary in <u>sid</u>ereal space.

Residual Siderostat Bay Elements

After starlight has bounced off the Siderostat Mirror, it follows a path dictated by many successive optical elements (starting with the Beam Compressor). Since these elements are all in series, in both an optical sense and a reliability sense, they are lumped together as a single reliability block and called Residual Siderostat Bay Elements. In a more detailed reliability model, one might choose to model the actuators for the fast steering mirror, for example. In the current model, that level of detail would have been inappropriate and unwieldy.

Optical Switchyard

SIM operates by combining starlight from two Sid Bays in an Astrometric Beam Combiner (described below). In SIM Classic, there are seven Sid Bays and three interferometers must be formed. The Optical Switchyard is a set of rotating flat mirrors that can used to channel the light from three pairs of Sid Bays into any three of the four Astrometric Beam Combiners in any combination desired. Provided the Switchyard itself is highly reliable, this arrangement provides system level robustness. The extra Sid Bay and extra Beam Combiner are included to provide redundancy.

Optical Delay Line

An Optical Delay Line (ODL) is a group of optical elements mounted on a moving trolley. Its purpose is simply to change the path length followed by the starlight. It is sometimes referred to as a trombone. Delay Lines are used to adjust the distance traveled by starlight in the two arms of the interferometer. In order to form white light fringes in the Astrometric Beam Combiner, the light must travel exactly the same distance from the source (the star) through each arm of the interferometer to the fringe detector. The ODL performs this function for SIM. In principle, only one delay line is required. However, in SIM, the delay line operates at a sufficiently high frequency that it can actively damp vibrations due to the spacecraft and instrument. We have chosen to split the function of the delay line into two halves: one handles the low bandwidth long stroke portion, which is a few meters; the other handles the high bandwidth short stroke motions. This way, the voice coil and piezoelectric actuators on the high bandwidth delay line do not travel over several meters, trailing cables as they go. Instead, they are mounted in a fixed location. The long stroke low bandwidth device carries only optics, and so the cable handling is simplified. Another advantage of having two delay lines, one in each arm of the interferometer, is that the light will then experience exactly the same number of reflections in exactly the same sequence in each arm. This is desirable for matching polarization and intensity of the starlight in the two arms.

In SIM, there are eight total delay lines, four low bandwidth and four high bandwidth. There are also four Astrometric Beam Combiners (ABC). In SIM Classic, a particular low bandwidth delay line and a particular high bandwidth delay line are always connected to a specific ABC. In reliability terms, these elements are in series.

Astrometric Beam Combiner

After the starlight has progressed past all the Residual Siderostat Bay elements and Optical Delay Lines, it eventually enters the Astrometric Beam Combiner (ABC). This device is a further collection of optical elements, detectors, etc. and forms the heart of the astrometric interferometer. It takes the star light from two different Siderostat Bays and combines them in an interferometric manner and focuses the light onto detectors. By measuring the phase difference between the starlight from the two arms, one can deduce the angle to the star. The ABC also houses internal metrology beam launchers, which measure the internal path length from the beam combiner to the Siderostats.

Triple Corner Cube

A corner cube is an arrangement of three mirrors, each perpendicular to the other two. The point of intersection of the three planes is called the vertex of the corner cube. A corner cube has the property that any beam of light entering the corner will undergo three bounces, one from each surface, and the outgoing beam will then be parallel to the incoming beam. This characteristic holds for any direction of the incoming beam entering the corner. A single corner cube is mounted on the face of each Sid Mirror. This Corner Cube must be mounted precisely so its vertex is within a few μm of the reflective surface of the Sid Mirror.

A triple corner cube (TCC) is an arrangement of optical prisms with mirrored surfaces and arranged to form three different corner cubes all sharing a common vertex. There are several ways of achieving this. SIM Classic uses an arrangement in which three 30° wedges with their transverse surfaces coated with reflective material (bare gold for SIM) are bonded onto a reflective optical flat. This triple corner

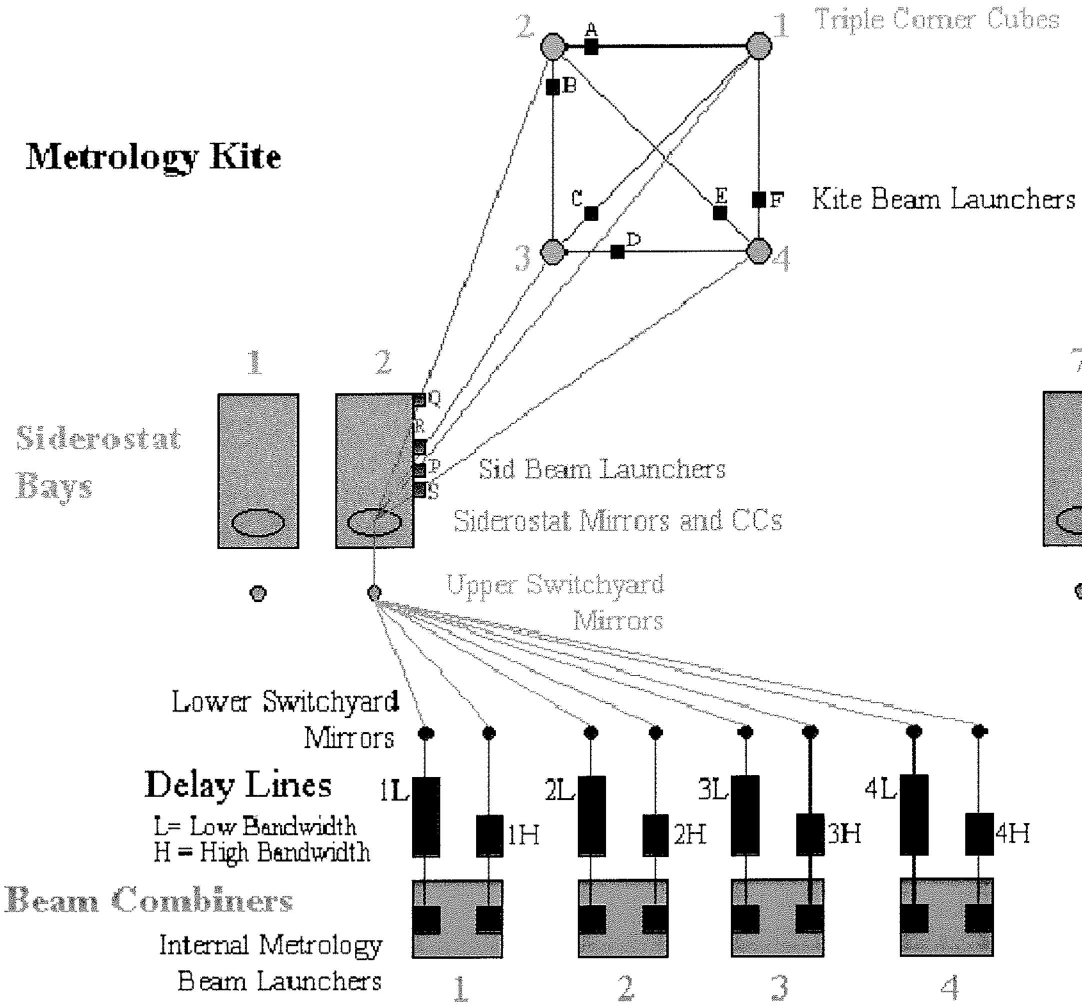

Figure 2 SIM Classic Reliability Model Schematic

cubes resembles three wedges of cheese, sitting on a flat platter, with their points meeting in the center. In SIM Classic, a triple corner cube is mounted at each of the four corners of the External Metrology Kite.

Beam Launchers

A beam launcher is an optical device used to measure the distance between two corner cubes. Actually, in SIM, the beam launchers only measure changes in the distance between the corner cubes precisely. There are many beam launchers in SIM. Laser light (infrared light, actually, with a wavelength of 1.3 μm) is fed into the beam launcher using optical fibers. The light from the fiber tip is collimated and "launched" out of the beam launcher. The light is aimed towards a corner cube. After hitting the corner cube, the beam returns to the beam launcher along a direction parallel to the outgoing beam. A similar beam is launched from the beam launcher in the opposite direction towards a second corner cube, which also returns the beam parallel to the incident beam. The two returning beams are combined inside the beam launcher with a reference beam at a slightly shifted wavelength to produce a heterodyne signal. The beam launcher operates as a laser interferometer. The phase of the heterodyne signal (effectively the fringe position) is resolved to about 1 part in 10,000 of a wavelength. Thus, changes in the distance between the two corner cubes is measured with a precision of 1.3 um/10,000/2 = 65 pm. By sampling the detectors as frequently as 100,000 times per second and averaging, the effective precision is reduced to a few picometers, assuming the noise in the system is uncorrelated, incoherent and stationary (in a statistical sense).

Kite Beam Launchers—Many Beam Launchers are used in SIM. Kite Beam Launchers are used to measure the distances between various pairs of triple corner cubes, which are situated at the vertices of the External Metrology Kite on SIM Classic.

Siderostat Bay Beam Launchers—Four beam launchers, as described above, are mounted in each Siderostat Bay. These beam launchers resolve changes in the distances between the corner cubes attached to the Siderostat Mirrors and the triple corner cubes located at the vertices of the External Metrology Kite. Sid Bay Beam Launchers (SBBLs) and Kite Beam Launchers (KBLs) are physically identical, but they are used differently in the reliability model, and so they are named differently here to distinguish them.

Kite Triangles

The External Metrology Kite on SIM Classic is a flat square perpendicular to the baseline of seven Siderostats. At each corner (or kite vertex) is a triple corner cube (TCC). Six Kite Beam Launchers (KBLs) measure the intravertex distances between vertices.

The distances from the kite vertices to the corner cube on each Siderostat Mirror are measured using the Sid Bay Beam Launchers (SBBLs). These distances are used to triangulate the position of the Siderostat with respect to the plane of the kite. Only three of the four kite vertices are required for SIM to operate. These three vertices would naturally form a triangle. There are only four ways to form a triangle connecting the corners of a square. Each of these triangles contains exactly three TCCs and three KBLs. Although it now seems obvious, it was the recognition that we could break the square down into these triangles that enabled us to proceed with the closed form solution. Prior to that, the connectivity among the various elements forming SIM seemed too complicated to deal with in a simple manner.

These kite triangles are not independent, and they are not treated as being independent in the reliability model. Still, it was helpful to decompose the elements into these triangles and consider the logical connections of these kite triangles to the Siderostats.

Triangle Usage—Any triangle can be used as a base for triangulating the positions of all the Siderostat Mirrors. If one measures the distances from each of the three corners down to a Sid Mirror, then one can determine the x, y, and z coordinates of that Sid Mirror with respect to a local frame of reference attached to the triangle. Of course, these three measurements can only be made if the three Sid Bay Beam Launchers (SBBLs) aimed at the three corners of the triangle are still operating.

Initially, there are four operational SBBLs in each Sid Bay. In this initial condition, the coordinates of the Siderostat Mirror can be found using any of the four triangles. Throughout the mission, it is possible that some of the SBBLs might fail. This is undesirable, but one can never guarantee there will be no failures. In fact, it is to protect against such failures that redundant SBBLs are used.

If two SBBLs fail in any particular Sid Bay, then that Sid Bay is effectively inoperative because it is not possible to determine all three coordinates of the Sid Mirror using just two linear measurements. Even in this unlikely situation, the whole system can continue to operate because there are still six operational Sid Bays. The design of the instrument includes redundancy at several levels.

There is a question as to whether the positions of all Sid Mirrors must be related to the same triangle, or if it is possible to use different triangles for different Sid Bays. The closed-form reliability expressions are derived to cover each of these situations.

5. CLOSED-FORM RELIABILITY EQUATIONS

Reliability Equations are derived for SIM Classic for the two operational constraints: 1) All Sid Bays must use the same External Metrology Triangle; 2) Sid Bays can use any triangle available to them.

The four vertices of the External Metrology Kite are nominally in the same plane, but small deformations of the structure are unavoidable due to temperature differences in the structure, for instance. By just measuring the in-plane distances, it is not possible, to first order, to determine the out of plane displacement of the vertices. However, if at least one Sid Bay has all four SBBLs working (a very likely situation), then it is possible to determine the out of plane displacements of the kite and thus relate any triangle to any other triangle. When this work was first performed, it was not clear that this would give acceptable accuracy in the knowledge of the positions of the Sid Mirrors. A later analysis confirmed that the knowledge of Sid Mirror positions only degrades about 10% when more than one triangle is used versus the situation in which all Sid Bays use the same triangle. Because we were unsure about the ability to use multiple triangles, we developed the closed-form equations for both cases. Allowing the use of multiple triangles improves the probability that the system will remain operational.

Reliability

Since we will be using the term, “reliability” frequently in this paper, we will describe what we mean by the term. We mean the probability that something is (still) working. Usually, we will mean the probability that the device or system is still functioning at the end of the nominal space mission 5½ years after launch. The expressions, however, do not include time dependence. The expressions we derive are simply combinations of the reliabilities of the various blocks. The expressions could just as easily be evaluated at any intermediate point in the mission (using appropriately higher component reliabilities consistent with that point in the mission lifetime).

We’ll use “ R ” to denote reliability. Although we will sometimes use P to mean probability in a general sense, often we will use it as the complement of R : $(R = 1 - P)$. Thus, P will represent the probability that the device or system has failed.

In our analyses here, we assume that if a device fails at some point in the mission, it remains inoperative for the remainder of the mission. In reality, there is some chance that a defective device might recover.

Triangles Available

In deriving the reliability relations, we started with the kite triangles. We found it helpful to consider the possible states of the External Metrology Kite in terms of the number of triangles available (*i.e.*, operating).

A kite requires three triple corner cubes (TCC) and the three specific corresponding Kite Beam Launchers (KBLs) all to be operational for the triangle to be available. This can be seen by examining any of the four larger triangles in Figure 3 below. For example, triangle 1-2-3 will only be available if TCCs 1, 2, 3 and KBLs A, B, C are all operational.

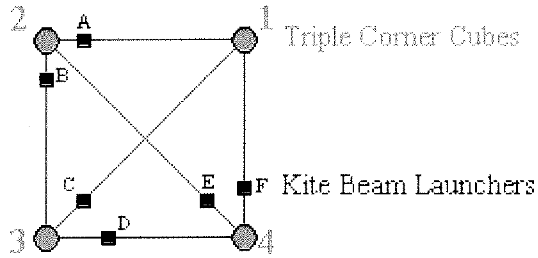

Figure 3 External Metrology Kite

Triple Corner Cube Reliability—Let P_T be the probability that a TCC fails. There is a TCC at each of the four corners of the Met Kite, and three are required to form any triangle, so the useful states of the Met Kite are zero TCC failures and one TCC failure. If two or more TCCs fail, then the entire system fails. The probability of zero TCC failures is $R_T{}^4 = (1 - P_T)^4$ and of exactly one TCC failure is $4P_T(1 - P_T)^3$. Thus, from the view of just availability of triangles based on TCCs, the probability of being in a useful state is

$$P\{\text{useful state}\} = (1 - P_T)^4 + 4P_T(1 - P_T)^3$$

This is illustrated schematically in Figure 4 and Figure 5 below.

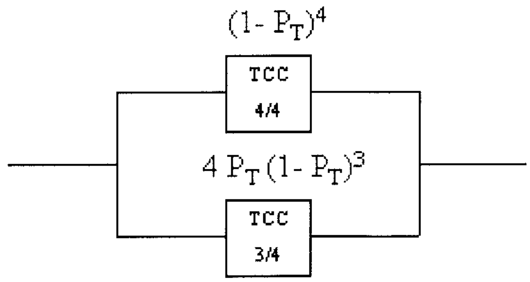

Figure 4 Reliability Diagram for Triple Corner Cube

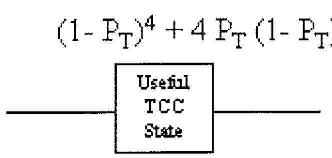

Figure 5 Equivalent Combined Block for Triple Corner Cube Reliability

Kite Beam Launcher Reliability—Let P_K be the probability that a KBL fails. Each KBL is shared by exactly two different triangles. If a KBL fails, then two of the triangles will become unavailable. There is no state in which exactly three triangles are available. The only states are 0, 1, 2, 4 triangles available. We found it easiest to consider the conditional probabilities of the KBLs being in a useful state given either of the useful states based on the TCCs.

If one of the TCCs has failed, then the three associated KBLs must be operational. The probability that three specific KBLs is operational is $R_K{}^3 = (1 - P_K)^3$. We use this as a conditional probability in the overall reliability expression.

When zero TCCs have failed, there are three mutually exclusive states of the KBLs that will still enable the interferometer to work (one, two or four triangles operating). The probability that all four triangles are available (all six KBLs operational) is $(1 - P_K)^6$. The probability that two triangles are available (exactly five KBLs operational) is $6P_K(1 - P_K)^5$. The probability that one triangle is available (three or four KBLs operational) is $4(1 - P_K)^3[P_K{}^3 + 3\,P_K{}^2(1 - P_K)]$. Kite reliability for the three triangle operational states is shown in Figure 6.

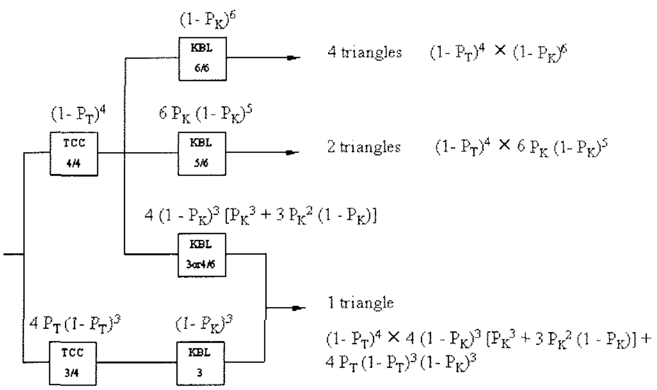

Figure 6 Reliability Diagram for Metrology Kite Operational States

The reliability for each branch is found by multiplying the TCC probabilities by the probabilities of the KBL states conditional on the TCC states. The overall reliability of kite availability is then found by summing all the branches on the right hand side of Figure 6.

Siderostat Bay Reliability

In this section, we discuss the probability that a Sid Bay is considered functioning conditional on the various states of the Metrology Kite. Here, we assume that if a Sid Bay can "see" any triangle, then it is functional. In a later section, we will derive a different expression based on the assumption that all Sid Bays must use the *same* triangle.

In each Sid Bay, there are four Sid Bay Beam Launchers (SBBLs). Each SBBL is aimed at a different corner of the Met Kite. If just one particular triangle is available, then the three SBBLs aimed at the corners of that triangle must be operating for the Sid Bay to be considered operational.

Let P_B be the probability that a SBBL fails. Let $R_S = (1 - P_S)$ be the probability that the rest of the Sid Bay elements are working.

Four Triangles Available—If four triangles are available, then the Sid Bay will be operational if any three of the four SBBLs are operational. This can be decomposed into two cases: zero failures and one failure of a SBBL. The probability of zero failures is $(1 - P_B)^4$. The probability of exactly one failure out of the four SBBLs is $4\,P_B(1 - P_B)^3$. Thus, the probability that the Sid Bay is working conditional on four triangles being available is

$$P\{\text{Sid Bay Working} \mid \text{4 Triangles Working}\} = \left[(1 - P_B)^4 + 4\,P_B(1 - P_B)^3\right](1 - P_S)$$

For notational convenience, we will define this expression to be equal to $(1 - Q_4)$. Since this is the probability that a Sid Bay can be considered operational given that four triangles are available, then Q_4 is the probability that a Sid Bay has failed, given that four triangles are available. There are seven Sid Bays, and only six are required for the SIM Instrument to be considered functional. The probability that six out of seven Sid Bays are working (given four triangles) is $(1 - Q_4)^7 + 7\,Q_4(1 - Q_4)^6$. We will keep this more compact notation, but it would be easy to expand this to display the full expression.

Two Triangles Available—If two triangles are available, the corresponding probability that the Siderostat Bay is working is $(1 - P_S)[(1 - P_B)^4 + 2\,P_B(1 - P_B)^3]$. The factor of 2 occurs because there are just two ways to select the SBBLs that can interact successfully with the two available triangles. We define this expression to be $(1 - Q_2)$. The subscript on the Q refers to the number of triangles available. The probability that six Sid Bays are operational, conditional on exactly two triangles being available is

$$(1 - Q_2)^7 + 7\,Q_2(1 - Q_2)^6.$$

One Triangle Available—If only one triangle is available, the corresponding probability that the siderostat bay is working is $(1-P_S)[(1-P_B)^4+P_B(1-P_B)^3]$. This, in turn, is defined to be $(1-Q_1)$. The probability that at least six of the siderostat bays are working, given that only one triangle is available, is:

$$(1-Q_1)^7+7\,Q_1(1-Q_1)^6.$$

Optical Switchyard

The Optical Switchyard is composed of two banks of steering mirrors. The upper bank has seven steerable mirrors. The lower bank has eight steerable mirrors. Each of the upper switchyard mirrors takes the output of one of the seven Sid Bays and diverts it down to any of the eight lower switchyard mirrors. Each lower switchyard mirror, receives the beam of light from an upper switchyard mirror and diverts it into one of the eight optical delay lines. Each upper switchyard mirror is always associated with a specific Sid Bay. Similarly, each lower switchyard mirror is always associated with a particular delay line. Pairs of delay lines are always associated with a particular Astrometric Beam Combiner (ABC). Rather than defining separate reliability parameters for the switchyard and delay line elements, we have lumped them with the major elements with which they are in series. The upper switchyard reliability is incorporated into the Sid Bay residual elements reliability, defined above. Similarly, we lump the reliability of the lower switchyard mirrors and delay lines into the Beam Combiner Reliability,

Let the probability of a Beam Combiner Assembly (beam combiner, delay lines and lower switchyard mirrors) failure be P_C. At least three of the four Beam Combiner Assemblies must be working for the system to be operational. The probability of that event is $(1-P_C)^4+4\,P_C(1-P_C)^3$. The resulting Classic SIM reliability structure is given in Figure 4.

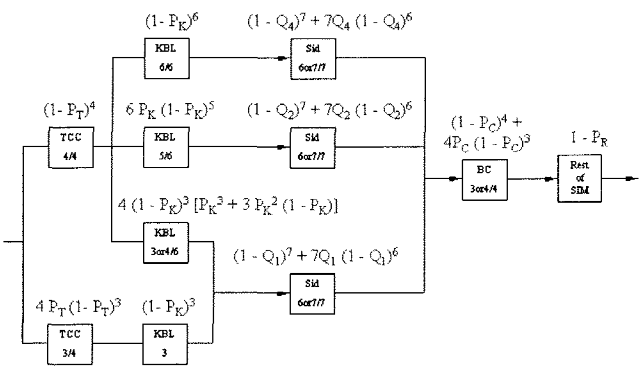

Figure 7 Reliability Diagram for SIM Classic assuming Sid Bays can use any Met Triangle

The Optical Switchyard does not explicitly appear in the reliability diagram. However, it manifests itself as the confluence of the three branches on the right hand side of the diagram. We have added a block representing the probability that the Rest of SIM is operating: $(1-P_R)$. The full reliability expression can be generated from the reliability diagram. Elements in series are multiplied, whereas branches in parallel are summed. The full expression for SIM Classic assuming Sid Bays can use any available triangle is given by:

$$\begin{aligned}
&\{(1-P_T)^4(1-P_K)^6[(1-Q_4)^7+7Q_4(1-Q_4)^6]+\\
&6(1-P_T)^4P_K(1-P_K)^5[(1-Q_2)^7+7Q_2(1-Q_2)^6]+\\
&4(1-P_T)^4(1-P_K)^3\,[P_K{}^3+3P_K{}^2(1-P_K)][(1-Q_1)^7+7Q_1(1-Q_1)^6]+\\
&4P_T(1-P_T)^3(1-P_K)^3[(1-Q_1)^7+7Q_1(1-Q_1)^6]\}\\
&\times\;[(1-P_C)^4+4P_C(1-P_C)^3](1-P_R)
\end{aligned}$$

All Sid Bays Must Use the Same Triangle

Now we consider the constraint that all Sid Bays must use the same triangle. The derivation of the reliability expression was more challenging for this case. We used a different approach based on the union of four events, as described in the following section.

Union of Events—We need a general expression for the probability that at least six Siderostat Bays can all see at least one of the available triangles. As a step along the way, let E_i be the event that at least six Siderostat Bays can all see the i^{th} triangle. The notation is a little confusing. This subscript refers to a specific triangle (1 to 4) rather than to the number of triangles available. The union of the four events E_1, E_2, E_3, and E_4 is exactly the condition for which we wish to assess the probability. Of course, there are intersections among these events, which must be accounted for. We will use symmetries to simplify the solution. For instance, $P(E_1) = P(E_2) = P(E_3) = P(E_4)$. The following is a general expression for the probability of the union of $\{E_i\}$ as a function of their intersections when there is symmetry of the four events:

$$4P(E_1)-6P(E_1\cap E_2)+4P(E_1\cap E_2\cap E_3)-P(E_1\cap E_2\cap E_3\cap E_4).$$

If we develop expressions for the appropriate intersections, then we will be able to evaluate the probability of the union.

It is helpful to consider different states of the Metrology Kite in terms of the number of triangles available.

One Triangle Available—If Sid Bays are constrained to use a common triangle and only one is available, then obviously they must use that one. The result is thus identical to the previous case with one triangle available as we will see towards the end of this section. However, for the cases of two and four triangles available a different approach is now necessary in order to derive the closed form expression for reliability. We will introduce and apply a consistent notation for all three cases under the constraint that all Siderostat Bays must use the same triangle.

Let $(1-S_1)$ be the probability that a given Sid Bay can see a specific single triangle. The subscript, 1, refers to the number of triangles specified and not to a specific triangle. This is the same as the probability $(1-Q_1)$ developed earlier:

$$(1-S_1)=(1-P_S)[(1-P_B)^4+P_B(1-P_B)^3].$$

That is, in order to see a specific triangle, the residual Sid Bay elements must be working and either 4 SBBL must be functioning, or else the right 3 must be functioning and the fourth one failed. Actually, an equivalent approach is simply that the correct three right SBBLs must be working, so the term in square brackets could be replaced by $[(1-P_B)^3]$. One can see that with very simple manipulation, the original expression in square brackets can be reduced to this form, but we have maintained the original form to maintain consistency with the general form of the similar expressions elsewhere that do not reduce in this fashion.

The probability that a least six Sid Bays can see this specific triangle, in this notation, is $(1-S_1)^7+7\ S_1(1-S_1)^6$. This is the probability that six or seven Sid Bays can see one specific triangle without regard to whether or not that particular triangle is actually available. This expression depends only on the state of the Sid Bay. There are four triangles. When it is known that exactly one triangle is available, then the probability that a specific triangle, say triangle 3, is operational is 1/4. Thus, given that one triangle is available, the probability of E_3 is

$$P(E_3)=0.25[(1-S_1)^7+7\ S_1(1-S_1)^6]$$

When this is substituted back into the expression for the union of four events, it is multiplied by 4, and so the factor of 0.25 disappears yielding exactly the same expression found earlier with S replaced by Q.

When only one triangle is available, the intersection terms in the expression for the union are all zero. When only one triangle is available, it is obviously not possible to see triangles 1 and 3 for example. Therefore

$$P(E_1\cap E_3)=P(E_1\cap E_2)=0.$$

Similarly, the probabilities of the higher order intersections are all zero. With one triangle available, the probability of the union is:

$$P(E_1\cup E_2\cup E_3\cup E_4)=[(1-S_1)^7+7\ S_1(1-S_1)^6]$$

Two Triangles Available—As before, we define $(1-S_1)$ to be the probability that a given Sid Bay can see one specific triangle. The probability that six or seven of the Sid Bays can see this one specific Triangle, say triangle number 3, is

$$P(E_3)=[(1-S_1)^7+7S_1(1-S_1)^6]$$

However, this assumes that triangle 3 is functioning. This is a conditional probability. We need to multiply by the probability that this particular bay is indeed functioning given that two triangles of the four are working. This probability is 0.5. Thus, the correct expression to use in the equation for the probability of the union is

$$P(E_3)=0.5[(1-S_1)^7+7S_1(1-S_1)^6]$$

Recall that by symmetry, $\mathrm{P(E_1)=P(E_2)=P(E_3)=P(E_4)}$.

Next, we need expressions for the intersection terms.

If only two triangles are available, then there is no way that three or four triangles can be seen by even one Sid Bay, never mind six or seven. Therefore, the triple and quadruple intersection events have zero probability (given that only two triangles are available). That is,

$$P(E_1\cap E_2\cap E_3)=P(E_1\cap E_2\cap E_3\cap E_4)=0.$$

This leaves just the double intersections to be evaluated.

A Siderostat Bay can see two triangles only if all four SBBLs are working. Let $(1-S_2)$ be the probability that a given Sid Bay can see two specific triangles:

$$(1-S_2)=(1-P_S)(1-P_B)^4.$$

Note that this is just an expression about the state of he Sid Bay; it does not include the probability that triangle 1 and 2 are actually operational. That will be factored in later. The probability that six or seven Sid Bays can see these two specific triangles is

$$(1-S_2)^7+7S_2(1-S_2)^6$$

However, this is only part of the term $\mathrm{P(E_1\cap E_2)}$.

Event E_1 is six or seven Sid Bays being able to see triangle 1 and event E_2 is six or seven Sid Bays being able to see triangle 2. The intersection of these two events, $(\mathrm{E_1\cap E_2})$, also occurs when five Sid Bays can see both triangles, the sixth Sid Bay can see triangle 1 and the seventh Sid Bay can see triangle 2. At first, this particular term eluded us and the closed form solution gave different results from the Monte Carlo model. By carefully enumerating states for a test case with fewer Sid Bays, we were able to identify the missing term.

The probability that five Sid Bays can each see two specific triangles is

$$(1-S_2)^5.$$

The two extra Sid Bays that can see only one triangle are in a state such that their Sid Bay Residual Elements are operational and one SBBL has failed. For one bay, the probability of being in this particular state is

$$(1-P_S)P_B(1=P_B)^3$$

There are two Sid Bays in this state, so this term will be squared.

Next we need to understand the combinations. There are five Sid Bays able to see two triangles. The number of such combinations is 7 taken 5 at a time, or $7!5!/2!=21$. Then there are two Sid Bays each able to see one triangle. There are 2 combinations. The extra term is thus

$$21(1-S_2)^5 \times 2[(1-P_S)P_B(1-P_B)^3]^2$$

The overall expression for $P(E_1 \cap E_2)$ is

$$(1-S_2)^7+7S_2(1-S_2)^6+21(1-S_2)^5 \times 2[(1-P_S)P_B(1-P_B)^3]^2$$

given that triangles 1 and 2 are actually operational. .

When two triangles are available, it will be two specific triangles depending on which Kite Beam Launcher has failed. There are six different ways of selecting the two particular triangles, and each is equally likely to occur. Thus, given that there are two random triangles available,

$$P(E_1 \cap E_2)=$$

$$\frac{1}{6}\{(1-S_2)^7+7S_2(1-S_2)^6+42(1-S_2)^5 \times [(1-P_S)P_B(1-P_B)^3]^2\}.$$

Recalling that $P(E_1 \cap E_2 \cap E_3)$ and $P(E_1 \cap E_2 \cap E_3 \cap E_4)$ are zero, and substituting back into the expression for the probability of the union of the four events E_i, we have the probability that at least six Siderostat Bays can all see at least one of the available triangles, when two triangles are available, is

$$P(E_1 \cup E_2 \cup E_3 \cup E_4)=$$

$$4 \times 0.5[(1-S_1)^7+7S_1(1-S_1)^6]-$$

$$6 \times \frac{1}{6}\{(1-S_2)^7+7S_2(1-S_2)^6+42(1-S_2)^5 \times [(1-P_S)P_B(1-P_B)^3]^2\}$$

$$=2[(1-S_1)^7+7S_1(1-S_1)^6]-$$

$$\{(1-S_2)^7+7S_2(1-S_2)^6+42(1-S_2)^5 \times [(1-P_S)P_B(1-P_B)^3]^2\}.$$

Four Triangles Available— A siderostat bay can see all four triangles only if all four SBBLs are working. This state of the Sid Bay is identical to the state discussed in the previous section. That is, if a Sid Bay can see two different triangles, it can also see four triangles. The difference now is the number of triangles available. We can reuse several of the results from that section.

As before, the probability that a Sid Bay is in a state such that it can see a specific triangle, say triangle 3, is $(1-S_1)$. The probability that a least six Sid Bays can see this specific triangle is $(1-S_1)^7+7\,S_1(1-S_1)^6$. Now, however, all four triangles are available. Therefore, the probability that six or seven of the Sid Bays can see a particular triangle just equal to this expression:

$$P(E_1)=P(E_3)=(1-S_1)^7+7\,S_1(1-S_1)^6.$$

Continuing to recycle expressions from above, the probability that a Sid Bay can see two specific triangles is

$$(1-S_2)=(1-P_S)(1-P_B)^4.$$

Continuing as above, we arrive at the same expression as for when two triangles were available but this time, given that all four are operational, the probability that any particular two are operating is 1 so there is no extra adjustment.:

$$P(E_1 \cap E_2)=$$

$$(1-S_2)^7+7S_2(1-S_2)^6+42(1-S_2)^5[(1-P_S)P_B(1-P_B)^3]^2$$

As mentioned much earlier in the paper, there is no way for a Sid bay to be able to see exactly three triangles. If it can see three triangles, then it can also see four triangles. A consequence of this is that when four triangles are available, $P(E_1 \cap E_2 \cap E_3)=P(E_1 \cap E_2 \cap E_3 \cap E_4)$.

Let $(1-S_4)$ be the probability that a given Siderostat Bay can see all four triangles:

$$(1-S_4)=(1-P_S)(1-P_B)^4=(1-S_2).$$

This is identical to the expression for seeing two triangles because everything in the Sid Bay must be operational to see two different triangles. The probability that six or seven of the Sid Bays are in this state is:

$$P(E_1 \cap E_2 \cap E_3)=P(E_1 \cap E_2 \cap E_3 \cap E_4)=$$

$$(1-S_4)^7+7S_4(1-S_4)^6.$$

The various expressions are substituted into the expression for the union to yield the probability that at least six Siderostat Bays can all see at least one of the triangles, when all four triangles are available:

$$4\{(1-S_1)^7+7\,S_1(1-S_1)^6\}-$$

$$6\{(1-S_2)^7+7S_2(1-S_2)^6+42(1-S_2)^5[(1-P_S)P_B(1-P_B)^3]^2\}+$$

$$4\{(1-S_4)^7+7S_4(1-S_4)^6\}-$$

$$\{(1-S_4)^7+7S_4(1-S_4)^6\}$$

This expression is simplified and S_4 is replaced by S_2:

$$4\{(1-S_1)^7+7\,S_1(1-S_1)^6\}-$$

$$3\{(1-S_2)^7+7S_2(1-S_2)^6+84(1-S_2)^5[(1-P_S)P_B(1-P_B)^3]^2\}$$

The analytical expression for SIM Classic optical interferometer reliability when all siderostat bays must use the same triangle is then given by:

$$
\begin{aligned}
&< (1-P_T)^4(1-P_K)^6[4 \text{ triangles}]+ \\
&6(1-P_T)^4 P_K(1-P_K)^5[2 \text{ triangles}]+ \\
&\{4(1-P_T)^4(1-P_K)^3\,[P_K{}^3+3P_K{}^2(1-P_K)]+4P_T(1-P_T)^3(1-P_K)^3\} \\
&\qquad\qquad\qquad [1 \text{ triangle}] > \\
&\times\ [(1-P_C)^4+4P_C(1-P_C)^3](1-P_R)
\end{aligned}
$$

where the square bracket terms are replaced by the appropriate term for the number of triangles specified, yielding the following expression:

$$
\begin{aligned}
&< (1-P_T)^4(1-P_K)^6[4\{(1-S_1)^7+7\,S_1(1-S_1)^6\}- \\
&3\{(1-S_2)^7+7S_2(1-S_2)^6+84(1-S_2)^5[(1-P_S)P_B(1-P_B)^3]^2\}]+ \\
&6(1-P_T)^4 P_K(1-P_K)^5[2[(1-S_1)^7+7S_1(1-S_1)^6]- \\
&\{(1-S_2)^7+7S_2(1-S_2)^6+42(1-S_2)^5\times[(1-P_S)P_B(1-P_B)^3]^2\}]+ \\
&\{4(1-P_T)^4(1-P_K)^3\,[P_K{}^3+3P_K{}^2(1-P_K)]+4P_T(1-P_T)^3(1-P_K)^3\} \\
&\qquad\qquad \times[(1-S_1)^7+7\,S_1(1-S_1)^6] > \\
&\times\ [(1-P_C)^4+4P_C(1-P_C)^3](1-P_R)
\end{aligned}
$$

Figure 8 illustrate a simple case in which the component failure rates were all set to the same value. This value was then varied over the range 0 to 2%. The overall system reliability was calculated using the closed form solution for the case when all Sid Bays must use the same triangle and when the Sid Bays can use any triangle. As expected, the reliability is a little lower with the more restricted constraint that all Sid Bays must use the same triangle. However, if one imposes this constraint, it does not drastically reduce the overall system reliability.

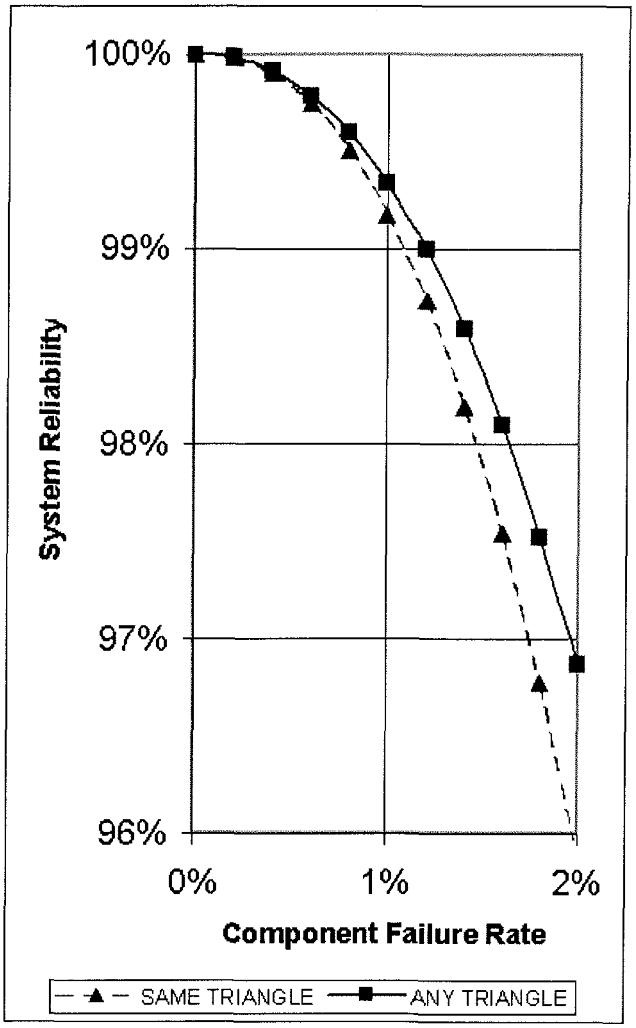

Figure 8 System Reliability Sensitivity to Component Failure Rates

In order to use more than one triangle at the same time, the project would have to perform some additional analysis and develop algorithms, along with testing and verifying this approach. Depending on how risk averse the project wishes to be, and based on available resources, one might decide not to perform this additional work, and thereby save some resources for use elsewhere. There is a good chance that the entire mission could be performed never needing the capability to use more than one triangle. If, during the mission, the unfortunate situation were to arise that six Sid Bays could not simultaneously interrogate the same triangle, then one would have to perform the analyses and develop the necessary algorithms at that point in order to continue the mission. Throughout the mission, if some components failed but the system were still operational, then the model could be exercised setting the failure rate to 100% for the failed components. If the results at that point indicate a greater difference between using one triangle and any triangle, then one might initiate the extra work to develop the capability to use any triangle.

6. MONTE CARLO SIMULATION USING EXCEL

One of the authors (George Fox) has developed some Excel add-ins and macros for use in performing Monte Carlo simulations using Excel. He developed a method of representing interconnections among elements of a system graphically at the same time as representing the probabilistic relations among the elements. This pseudo intuitive approach makes it easier to understand the functioning of the system. The graphical representation is not essential. One could use the relationships without adding on the layer of graphical representation.

For the SIM model, we represent each block as a logical entry in the spreadsheet with a True value (1) representing an operational block, and a False value (0) representing a failed unit. The built-in Excel RAND( ) function is used to generate random values in the range (0,1). In the cell, the random value is compared with a failure probability stored in a different cell. If the random value is lower than the probability of failure, then the cell for that block is marked False. Otherwise it is marked as True (operational). Some cells are used to represent aggregates of elements. These are considered operational only when the appropriate relations among the sub blocks are satisfied. This allows very complex relations to be represented. Most of the commercially available reliability assessment tools allow parallel combinations of series elements, but do not easily allow the relations among Sid Bays and the Metrology Kite to be represented. Using the Excel-based approach, these relationships were modeled easily.

Each time the spreadsheet is recalculated, the random function returns a different value between 0 and 1. In a sense, each time the spreadsheet is recalculated, one has an instance that represents a complete occurrence of the SIM mission. By running the model many times (we chose 10,000 samples) one can estimate the reliability of the entire system. The greater the number of samples, the greater will be the precision in the final result. Of course, the accuracy depends upon the accuracy of the assumed component reliabilities. In our case, we did not have realistic values. Instead, we used the model parametrically and varied the input component reliabilities to investigate sensitivities and to compare different arrangements of the major elements. We also wanted to compare the results with the analytic method to help verify both the analytic approach and the Monte Carlo approach. Agreement between the two does not guarantee that there are no errors, but if there are significant differences, then it is clear that at least one of the models is incorrect.

The add-ins automate the ability to run the model a large number of times. They also allow desired cells to be monitored and tabulated in a separate sheet in the Excel file. They also include the ability to display the results in a consistent way.

For SIM Classic, once the basic model was assembled in Excel, it turned out to be relatively easy to modify which condition of the Metrology Triangles was considered operational. In fact, both arrangements were assessed simultaneously in separate cells of the same model file. This is in distinct contrast to the additional effort required during development of the closed-form solution. It took weeks to develop the modification going from allowing any Sid Bay to use any available triangle to restricting all Sid Bays to use a common triangle. In the end, the results agreed to three significant digits for both the analytic and the Monte Carlo models for both constraints.

MCTool

The custom add-in developed by George Fox is named MCTool (Monte Carlo Tool). It includes many capabilities, of which only a few were used in the current effort. For instance, one can select among a number of different probability distribution functions for random variables. In the SIM modeling, we just used uniform distributions between 0 and 1. During the simulation, results for several variables can be collected and plotted or tabulated in different ways. MCTool includes several standard formats, but the data can easily be manipulated using general Excel capabilities to generate customized views if desired. Random variables are named starting with "RV" to denote variables that the tool will monitor and collect

Simple Cells—If a fundamental element - such as a Sid Bay Beam Launcher (SBBL) for SIM - has a simple probability of failure rate, say 1% for the mission, then that value (0.01) is entered into a cell in Excel. It is convenient to create a table showing the various simple elements along with their values. Figure 9 shows a sample from the SIM Classic model.

The lines labeled P, S, Q, R are the SBBLs for each Sid Bay. We assumed that all SBBLs are identical and have equal failure rates. If there were some differences among the various SBBLs, then different entries could have been used in the table. The values in this table are all static since they represent the end-of-mission reliability for these elements, but in a more complicated model, these might change for different phases of the mission.

Elsewhere in the spreadsheet there is a separate cell for each of the SBBLs. Each of these contains an equation similar to this: =IF(RAND()<P6,0,1). P6 is the cell in the table above for one specific SBBL. Every time the spreadsheet is updated, a different random variable is returned by the RAND function. This value is compared with the value in P6, in this case 0.01. Ninety-nine percent of the time, the random variable will be greater than 0.01 and the value of

Pr{SBL failure}	Sid Bay #1	Sid Bay #2	Sid Bay #3	Sid Bay #4	Sid Bay #5	Sid Bay #6	Sid Bay #7
P	0.01	0.01	0.01	0.01	0.01	0.01	0.01
S	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Q	0.01	0.01	0.01	0.01	0.01	0.01	0.01
R	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Pr{SBL failure}	Sid Bay #1	Sid Bay #2	Sid Bay #3	Sid Bay #4	Sid Bay #5	Sid Bay #6	Sid Bay #7
Rest of Sid Bay	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Pr{Beam Combiner failure}	B C #1	B C #2	B C #3	B C #4			
	0.01	0.01	0.01	0.01			

Figure 9 Sample Table of Reliability Values (Specific Values are not necessarily representative of real hardware)

the cell will be set to 1. One percent of the time, the random number will be less than 0.01 and the cell will be set to 0 meaning not operating. The add-in automates the repeated updating of the spreadsheet and monitoring of which cases correspond to failed systems.

Series Cells—When two or more simple cells must all be operating for some portion of the system to be considered operational, then the cells are said to be in series. If any device represented by the cell fails, then the combination is considered failed. In Excel, this can easily be incorporated into a cell representing the combination by using the AND function. Here's an example from the SIM Classic reliability model spreadsheet:
=IF(AND(TCC_1,TCC_2,TCC_4,KBL_12,KBL_24,KBL_14),1,0)
This is a cell that represents Kite Triangle number 2. For this particular triangle to be considered operational, the three specific triple corner cubes and the three specific kite beam launchers must all be working.. Note that the cells are named and these names are used rather than cell references using row and column designators. For instance, KBL_24 is really cell J19 in the spreadsheet. This cell represents the beam launcher measuring the diagonal member of the kite running between TCC_2 and TCC_4.

An alternative way of representing this series condition is simply to multiply the individual cells. The cells are all either 0 or 1. The product of the cells will be 0 if any of the individual cells is 0. The combination is only 1 if all the series elements are 1. The choice is a matter of style and personal preference. Using names can help one to understand the functionality when editing the spreadsheet, especially for complex combinations.

Parallel Cells—When any one of two or more simple cells can be operating for some portion of the system to be considered operational, then the cells are said to be in parallel. In Excel, the OR function can be used. Here's an example that could represent the situation in which any of the four triangles is operational:
=IF(OR(Tri_A,Tri_B,Tri_C,Tri_D),1,0)
As before, the cells are named and the names are used in the equation rather than explicit references to the cell row and column.
An alternative approach is simply to sum the cells in parallel. The sum can only be zero if all the parallel elements are zero (failed). Excel interprets zero as false and any other value as true. If the sum is greater than zero, then at least one of the parallel elements is operational. However, it is probably wise to normalize true to be equal to one. Therefore, rather than using the straight sum of several parallel elements, one could test for a positive sum and set the result to one when satisfied and to zero otherwise. For example, =IF(SUM(Tri_A,Tri_B,Tri_C,Tri_D)>0,1,0) will achieve this objective.

N of M Operational—SIM has a single point failure tolerance failure. No single failure is allowed to cause failure of the entire system. However, in some situations, there are additional elements, and often two or more must fail before a larger system is taken down. If a larger part of a system can operate whenever N elements are operating out of a total of M, then one can simply sum the cells representing the elements and the larger part of the system will be operational whenever the sum of the elements is equal to or greater than N. The larger part of the system could be represented by =IF(sum(range)>=N,1,0).

Complex Cells—Some cells in the model refer to combinations of cells in a more complicated manner than the situations discussed above. Here's an example
=IF(OR(V40>=6,V42>=6,V44>=6,V46>=6),1,0)
The four cells referenced contain the number of Sid Bays that can see a particular Kite Triangle. For instance, cell V44 is simply the sum of the number of Sid Bays that can see triangle 4-3-2. If this sum is greater than or equal to 6, then the front end of the interferometer can be considered operational under the constraint that all Sid Bays must share a common triangle. There are additional "downstream" elements that must also be operational for the interferometer to be considered functioning. By "front-end elements" we are basically referring to the first elements that star light encounters as it progresses towards the detectors inside the Beam Combiner.

Fairly complex relations among elements can be constructed by developing appropriate intermediate cells and combining them to represent the correct operation of the overall system.

Model Visualizations—the discussion above of the Excel modeling describes the construction of the formulas entered into the cells of the spreadsheet to represent the reliability of the system. Like any program, debugging is required. That is one of the reasons it is recommended to name the cells descriptively and use the names rather than direct cell references. This especially holds for large complex models. A further refinement that increases the ability to understand the model is to color the cells representing different elements different colors and to draw lines connecting elements to represent the connectivity in the system being modeled. This is even more helpful if the layout corresponds to the physical layout although this is not essential. One could start with a block diagram and lay out the elements according to the block diagram instead of the physical arrangement of the elements. Text entries in adjacent cells can be used as labels. This further decreases the chance of making errors during the construction and debugging of the model. A segment of the model

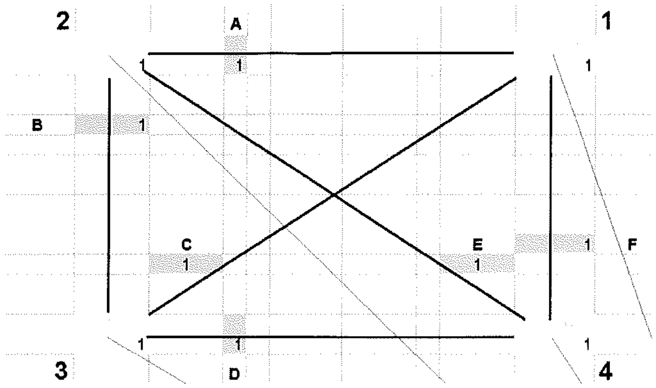

Figure 10 Portion Representing Metrology Kite of SIM Reliability Model in Excel

Figure 10 illustrates the techniques suggested. The figure is a screen capture of the portion of the Excel spreadsheet representing the External Metrology Kite. The four yellow cells represent the TCCs. Each of the TCCs is labeled by a simple number 1, 2, 3, 4. One could as easily have used TCC_1, TCC_2, etc. The green cells represent the six KBLs. These are labeled A though F. The values in all of the cells happen to be 1 indicating that all elements are operating. For a particular instance, one of more of these might be zero indicating failure of that component. One can press the F9 function (on a PC) to force the spreadsheet to recalcualte. This cause a new set of random numbers to be generated. Sometimes this can be helpful during debugging, because the eye is sometimes notices patterns that would otherwise be difficult to deduce. For instance, if all the TCCs seem to fail at the same time, then one might suspect that the logic is not set up correctly and perhaps all cells are referring to a common cell instead of separate cells.

The diagonal lines in the figure represent the laser beams connecting the TCCs. Each passes through a cell representing the corresponding Kite Beam Launcher. These lines are merely graphical elements drawn on the spreadsheet. They serve only to make it easier to understand the connectivity among the TCCs and the KBLs.

The thin red lines slanting down and to the right illustrate the laser beams connecting the TCCs to one Sid Bay. It would be overwhelming to show all the connections among the TCCs and all seven Sid Bays. However, the link to one Sid Bay makes it easier to develop the correct logic linking the operation of the Sid Bays and the Kite Triangles. In fact, once the equations are developed for one element, they can often be copied to the cells representing the other elements. Using absolute references and mixed references can be very helpful in performing this task. If one has never explored absolute and mixed references, it is recommended that one do so if one wishes to develop Excel models of any complexity. The investment of the time necessary will be repaid many times when an equation is simply dragged across rows and columns instead of having to enter each cell manually.

Figure 11 shows an adjacent portion of the spreadsheet. The red diagonal lines connect to the Metrology Kite illustrated in Figure 10. The pale blue horizontal boxes each contain seven boxes corresponding to the seven Siderostat Bays. The four rows contain the operational state of the four Sid Bay Beam Launchers (SBBLs) in that Sid Bay. These cells contain simple random variables.

Below the pale blue boxes are four green boxes, which will be described shortly. To the left of the green boxes are four pink cells. Each of the pink boxes contains the state of one of the four Met Triangles. A triangle is considered operational if its three TCCs and the three KBLs connecting these three TCCs are all operational. That is, the six elements are in series and are thus represented using the AND() function as discussed above.

The four green rows each correspond to a different Met Triangle. A one in a cell indicates that the specific Sid Bay and the corresponding triangle are operational. To the right of the green boxes is a cell which sums the number of Sid Bays that can see the triangle corresponding to that row. If the sum is six or more, then those operating Sid Bays can all see a common triangle. Below the four sums, a cell checks to see if any triangle is in this state. This cell is labeled "Case 2: All Sid Bays must use the same Triangle." When this cell contains a one, then at least one triangle can be seen by at least six Siderostats. This is the situation for which it was quite difficult to develop a closed-form solution.

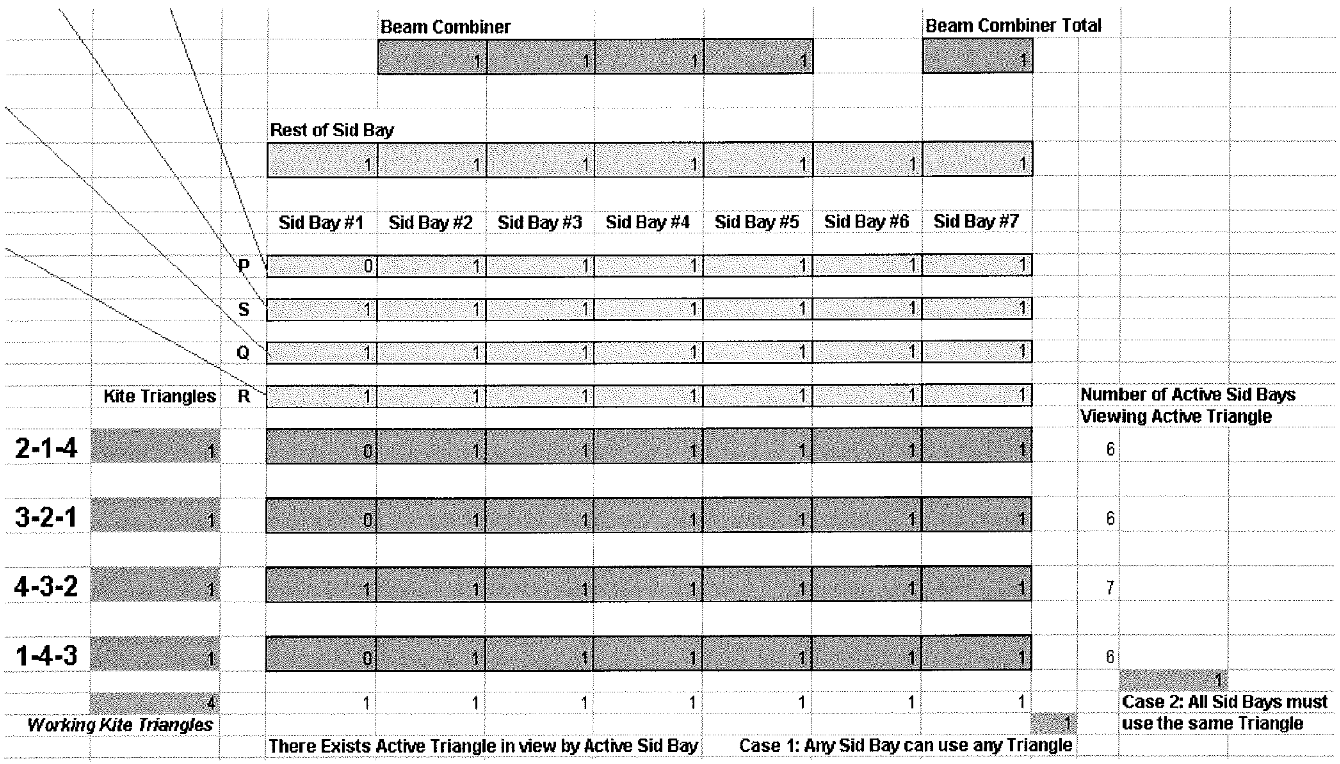

Figure 11 Portion of Spreadsheet representing Combinations of Sid Bays and Kite Triangles

Below the green boxes is a row of seven cells. Each of these cells tests if the Sid Bay above it can see any triangle. To the right of these seven boxes is a cell which checks to see if at least six Sid Bays are in this state. This cell is labeled, : Case 1: Any Sid Bay can use any Triangle." It can be seen that both Case 1 and Case 2 can be tested at the same time. The subroutines contained in the MCTool add-in count the number of instances out of the 10,000 trials for which the system is in this operational state.

Above the pale blue boxes, there is a row of cells entitled "Rest of Sid Bay." This is a simple random variable denoting when the elements in series with the Sid Bay are operational. The green boxes below indicating the status of the Sid Bays includes a link to the corresponding pink box with the Rest of Sid Bay Elements corresponding to that Sid Bay.

Above the pink boxes are four blue cells labeled "Beam Combiner." These cells contain simple entries for the four beam combiners. To their right, a cell tests if three or more beam combiners are operational. Since the beam combiners are in series with the rest of the system (by way of the Switchyard) this cell is simply multiplied into the final operational state of the system. Since there are the two cases being run simultaneously, they are multiplied into each of the summary cells corresponding to a fully operational system.

7. Comparison of Results from Monte Carlo Model and Closed-From Solution

In order to confirm that the different modeling approaches were consistent, we evaluated the overall system reliability using the Monte Carlo approach and using the closed-form equations. The input reliability values for the various elements were varied over a wide range.

Table 1 Comparison of Closed Form Solution and Monte Carlo Simulation with 10,000 instances

	System Reliability		
P(ABC)	Closed Form	Monte Carlo (10,000)	Difference
0.00	0.9990	0.9989	0.0001
0.01	0.9984	0.9983	0.0001
0.02	0.9967	0.9970	-0.0002
0.03	0.9939	0.9928	0.0011
0.04	0.9900	0.9897	0.0003
0.05	0.9850	0.9853	-0.0002
0.06	0.9791		
0.07	0.9723		
0.08	0.9646		
0.09	0.9561		
0.10	0.9468	0.9484	-0.0016

Table 1 shows a particular comparison. The failure rate for the Astrometic Beam Combiner (ABC) was varied from 0 to 0.1 while the failure rates for the remaining elements in the system were held constant. The agreement between the two models is about what would be expected based on the 10,000 random instances used. Table 2 lists the nominal values used in the Monte Carlo modeling. This table lists the constant values used for each of the parameters while the others were varied parametrically one by one. These values should not be construed as being representative of actual failure rates for the actual flight hardware.

Reliability Model Element	Pf
Triple Corner Cube	0.0005
Residual Siderostat Bay Elements	0.001
Kite Beam Launcher	0.01
Siderostat Bay Beam Launcher	0.01
Astrometric Beam Combiner	0.02

Table 2 Nominal Failure Rates Used in Monte Carlo Modeling

8. CONCLUSION

High-level (coarse) reliability models of the Space Interferometry Mission (SIM) have been developed. For two operational constraints of the SIM Classic configuration, closed form relations were developed. Monte Carlo models were also developed using Excel. The two models agree to the level expected for the number of instances run in the Monte Carlo model (10,000). The derivation of the closed-form solution has been derived in some detail. The use of Excel as a tool for modeling probabilities has also been described in significant detail. The Excel technique is more easily extended than the closed-form approach.

9. ACKNOWLEDGEMENT

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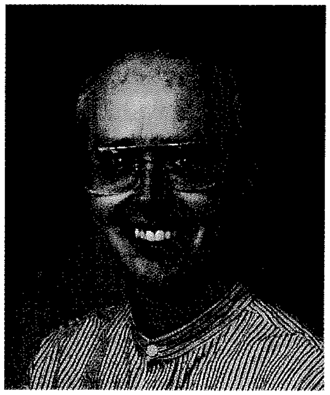
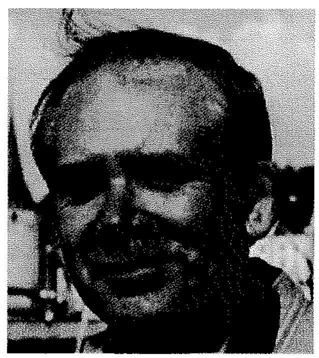
11. BIOGRAPHIES

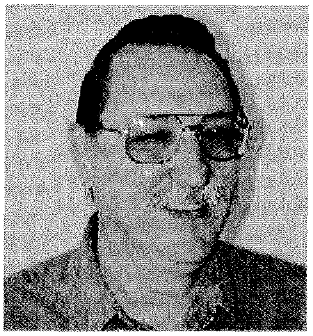


Kim Aaron is the SIM Flight System Architect. He has worked on the SIM project for almost five years. During his 16-year tenure at JPL, he has been involved in the conceptual design phase of about thirty different space missions. On a recent project, he designed and flight-qualified a vibration isolation system to operate at 2 kelvin for a superfluid helium experiment. Earlier, he was responsible for hardware design, fabrication and assembly of portions of the Rocky 4 microrover, predecessor to the Mars Pathfinder Sojourner rover. In 1985, he graduated from Caltech with a Master of Science and PhD in Aeronautics. He earned a Bachelor of Engineering degree in Honors Mechanical Engineering from McGill University in Montreal, Canada in 1979.

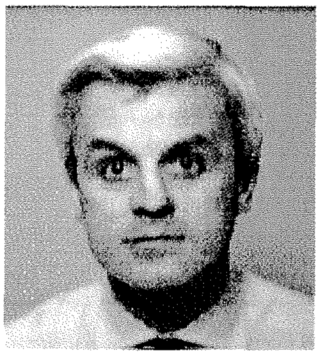


Don Ebbeler is an Operations Research analyst at JPL. He has been involved in the modeling and analysis of both the engineering reliability and economic aspects of solar energy, war-gaming, cost prediction, aerospace propulsion, electro-optical, structural, mechanical, chemical and advanced automobile air bag systems and, for SIM, the optical interferometer system designs. Before coming to JPL in 1980 he worked as an aerospace research engineer for General Motors, served as an assistant and associate professor of economics at Georgia Tech, Claremont Graduate School and Iowa, and as an economist at Southern California Edison. He has been on the editorial board of the Journal of Macroeconomics since its founding in 1977. He graduated from Purdue University in 1970 with an MS and PhD in Economics. In 1964 he earned a BS in Electrical Engineering in the R&D option, with concentrations in physics and mathematics, and in 1965 earned an MS in Electrical Engineering, specializing in statistical communication theory with a concentration in statistics, both from Purdue University.

George Fox: Engineers used slide rules when George entered Caltech as a Freshman in 1965. After certification as a Quantum Mechanic & Micro Economist (Caltech PhD, Applied Physics and Economics, 1979) George joined JPL to determine the value of solar cell power for the national PV solar energy program. When Ronald Reagan killed the program George changed sides, doing military acquisition policy analysis & war-gaming modeling for the highly successful JESS, validated during the Gulf War. As an original developer of the Space Station System Design Tradeoff Model and an analyst for JPL's Project Design Center, George failed in educating NASA about cost performance tradeoffs. He currently does reliability modeling of complex systems (SIM, NEPTUNE) for hire and is entering the quagmire of cost-risk modeling to convince management that it can be done without (irrelevant) historical data and (inappropriate) regression models.



W. John Walker received the B.Sc. degree in Electrical and Electronic Engineering from Queen's University, Belfast, Northern Ireland in 1980 and the M.S. degree in Aeronautics and Astronautics from MIT in 1990.



His previous employers include GEC Marconi in Scotland and Litton Guidance & Control in California.

Since 1990, he has been a technical staff member at the Jet Propulsion Laboratory in Pasadena, California, and has contributed to the design and integration of attitude control hardware for the Cassini Mission to Saturn, Deep Space 1 and Europa Orbiter spacecraft.

He is currently involved with the development of real-time control systems for the Space Interferometry Mission.