Results from the PTI's studies of the spectral angular diameters of Mira variables

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ABSTRACT

A dedicated program of measuring angular sizes of Mira variables using the Palomar Testbed Interferometer has resulted in more than 13,000 measurements of over 60 stars. With visibility data in five channels across the \( K \) band, atmospheric extension and sources of opacity are evident in the dataset. Using multiple-epoch narrowband data, phase lags between wavelength-dependent angular size cycles indicate spatial extent of molecular atmospheres. A spectral angular diameter classification system is developed, based on the recurring shapes of the spectral traces seen in the dataset, which correlates to spectral type. A period-radius relationship is explored, and a refinement on the estimation of angular sizes is made.

Keywords: Interferometry, Miras, angular diameters, near infrared, stellar atmospheres, spectroscopy, oscillations, stellar chemistry.

1. INTRODUCTION

Mira variable stars are classified as long-period variables (LPV) with periods from 150 to 500 d. These giant stars (luminosity class III) are thought to be in their last stage of their evolution before ejecting their outer atmospheres as planetary nebulae, leaving exposed cores as white dwarfs. As a result of stellar pulsations, atmospheric shocks are set up which extend the outer atmospheres of these variable stars, as well as provide a mechanism for mass loss. Wideband interferometric measurements of Miras can provide insight to basic stellar parameters, such as effective temperatures and linear radii. As part of a long-term campaign to measure angular diameters of Mira variables using the Palomar Testbed Interferometer (PTI),\textsuperscript{1} we present some results of visibility measurements made with five channels across the \( K \)-band (2.0 - 2.4 \( \mu \text{m} \)), for a spectroscopic resolution of \( R \sim 22 \) at the center wavelength of 2.2 \( \mu \text{m} \). While this spectroscopic resolution is low as compared to spectrometers, visibility measurements at PTI's resolution provide better insight to the chemistry and spatial extent of opacity sources about Miras than do wideband (\( \Delta \lambda = 0.4 \mu \text{m} \)) interferometric measurements in the \( K \) band. With the extreme shortage of near-infrared narrowband diameters, the problem of testing any model at these wavelengths becomes academic. A recent review of optical interferometry\textsuperscript{2} states "time-series of measurements in well-defined narrowband filters covering several pulsation cycles will be required for a more detailed comparison between observations and theory...", not only with respect to limb-darkening and pulsation but also chemistry.

2. WAVELENGTH-DEPENDENT DIAMETERS AND PHASE LAGS

Following the analysis for RZ Peg and S Lac,\textsuperscript{3} wavelength-dependent uniform disk (UD) diameters of oxygen-rich Mira R Boo using multi-epoch data are plotted in Fig. 1 for the continuum (2.2 \( \mu \text{m} \)) and bandedge channels (2.0 \( \mu \text{m} \) and 2.4 \( \mu \text{m} \)). (See also a PTI science update,\textsuperscript{4} these proceedings.) Phases were determined using data provided by the Association Francaise des Observateurs d'Etoiles Variables (AFOEV). Fit parameters to simple sinusoids given in Fig. 1 appear in Table 1. While we do not imply that Miras vary in size in a simple sinusoidal manner, the fits in Fig. 1 allow one to extract basic amplitudes, phase offsets, and mean angular sizes with respect to visual pulsation phase.

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Table 1. The best-fit parameters of the cycloid \( \theta = a \cos(2\pi(\phi+b)) + c \) for the oxygen-rich Mira R Boo, where \( a = \) size amplitude, \( b = \) visual phase offset, and \( c = \) mean cycle size.

<table>
<thead>
<tr>
<th>Wavelength</th>
<th>a (mas)</th>
<th>b (mas)</th>
<th>c (mas)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0 ( \mu )m</td>
<td>0.24±0.04</td>
<td>0.45±0.02</td>
<td>3.65±0.03</td>
</tr>
<tr>
<td>2.2 ( \mu )m</td>
<td>0.38±0.03</td>
<td>0.52±0.01</td>
<td>3.31±0.02</td>
</tr>
<tr>
<td>2.4 ( \mu )m</td>
<td>0.19±0.02</td>
<td>0.42±0.02</td>
<td>4.03±0.02</td>
</tr>
</tbody>
</table>

Figure 1. Multi-epoch UD narrowband angular diameters of R Boo. Phases: open squares = 0.2 - 0.4, filled diamonds = 1.7 - 2.2, open diamonds = 3.1 - 3.8, crosses = 4.8 - 5.2. Visual magnitude data is overlaid on the 2.2 \( \mu \)m diameter data.

The continuum (2.2 \( \mu \)m) size of R Boo is in phase with the period established by visual magnitude data (i.e. smallest size corresponding to brightest visual magnitude). However, phase lags exist for the bandedge angular size data. In both the 2.0 \( \mu \)m and 2.4 \( \mu \)m angular size cycloids, a phase lag of \( \Delta \phi \approx +0.1 \) with respect to the visual phase is seen. The period of R Boo is 224.1 d when fit using the AFOEV data (Epoch JD 2451241), thus the maximum opacity effects in the bandedge data occur roughly 22 d after minimum visual brightness. Adopting a distance of 608±112 pc, which represents a mean from four sources,\(^5\) the corresponding mean linear radius of R Boo is 216±40 R\(_*\) . If one assumes, to first order, that these phase lags are attributed to mechanical lifting by a shock wave moving at 20 km/s, then the major sources of opacity (H\(_2\)O, for oxygen-rich Miras) would occur at 1.25 R\(_*\) . While this naive calculation does not incorporate effects due to matter infall from previous shocks nor non-gray atmospheres, the distance above the star of the opacity source is comparable to that found for oxygen-rich Mira S Lac.\(^3\)

3. SPECTRAL DIAMETER CLASSIFICATION

It is important to stress the concept that, while Mira variables occupy a distinct class of objects in the realm of stellar evolution, not all Mira variables should be treated the same. Recent evidence shows differing chemical compositions of stellar atmospheres will give rise to markedly different visibility data.\(^3\) And, when one folds in the effects of limb darkening, extended atmospheres and detached molecular atmospheres, the observational effects of these physical characteristics can make classification of the dataset problematic at best. Despite the complexity to characterize the PTI Mira narrowband diameter dataset in the \( K \) band, Thompson\(^6\) represents a
first attempt in doing such a characterization. One of the most obvious ways in which to characterize the data
is to examine the individual spectral angular diameter traces as a function of wavelength across the $K$ band. 
As evidenced in Thompson et al.,$^3$ notable differences in spectral angular diameter character exist between
the oxygen-rich S Lac ('V' shape) and the carbon-rich RZ Peg (positive slope). Further, if one compares
the spectral diameters of S Lac with that R Tri (skewed V' shape$^8$), even spectral differences within the oxygen-
rich class exist. By examining how the K-band continuum ($2.2 \mu m$) relates to the data in the edges of the
$K$ band (2.0 $\mu m$ and 2.4 $\mu m$), a quantification scheme of the spectral angular diameters can be attempted.
The shapes of the spectral angular diameters traces follow a progression if one bases this progression on their
appearance alone.

Thompson$^4$ relates the difference in bandedge sizes to that of the difference between the 2.0 $\mu m$ and con-
tinuum sizes. Following this analysis, both the carbon Mira RZ Peg and the oxygen Mira S Lac have larger
2.4 $\mu m$ angular sizes relative to the K-band continuum, while their 2.0 $\mu m$ sizes are markedly different.$^3$ Data
for each star was reduced and visibilities were normalized,$^7$ and then an ensemble nightly average of the an-
gular diameters across the $K$ band was produced. (Error bars on the ensemble average < 5\% may indicate
wavelength-dependent asymmetries across the stellar disk, as seen in the case for R Tri$^8$). Each star's nightly en-
semble average of the spectral angular diameters was then evaluated. As a result of these numerical assignments
(a "Q-factor", or numerical quality assigned to each shape), an attempt is made to qualify the observational
narrowband data. The six shapes are plotted as a function of wavelength in Fig. 2. Until a more formal analysis
of these shapes are correlated with individual stellar atmospheric chemistry and physical conditions, it must be
stressed that the Q-factor classes are based on appearance alone, and may or may not strictly lead to interpre-
tation or implication of the physics of these stellar atmospheres. A recent work compares theoretical spectral
angular size traces to actual data for oxygen-rich Miras, in which a fuller treatment of the physical modeling
is offered by Jacobs and Scholz.$^9$ However, with that disclaimer, some important results of the application of
the Q-factor can be stated. All oxygen-rich Miras are classified as O(n). Carbon-rich Miras occupy the C(n)
classes (hence the letter designations). S-type Miras occupy the C(n) classes as well, and are indistinguishable
from their carbon counterparts with respect to the wavelength-dependent angular sizes in the $K$ band. The
oxygen-rich Miras can be divided up into four distinct sub-classes, the carbon-rich Miras only two. For those
stars with multi-epoch data, an individual star may even change to another Q-factor sub-class over the course
of its pulsation cycle. Each class is now given comment.

![Figure 2. Spectral angular diameter shapes seen in the PTI Mira dataset. The absolute angular diameters are arbitrary, and the depths of the features are relative within each class.](image)
3.1. The O(n) classes

The O0 class of spectra represents only a small fraction of the PTI Mira dataset. This right-skewed “V” shape appears exclusively in the oxygen-rich Miras, but to much less degree than any other O(n) class. As seen in other narrowband visibility data of oxygen-rich Miras, the character of the bandedge angular size data in oxygen-rich Miras was attributed to H$_2$O at or just above the stellar photosphere. The O0 shape has a larger 2.0 µm size than the 2.4 µm size (UD). If both angular sizes are indeed attributed to H$_2$O, this would imply that H$_2$O opacity effects at higher temperatures (viz. 2.0 µm) are dominating those at lower temperatures. Whether this represents a Mira with “hot” (T$_{\text{eff}}$ ~ 2500K) H$_2$O at the stellar photosphere and relatively much less “cool” (T$_{\text{eff}}$ ~ 1500K) H$_2$O above the stellar photosphere can only be speculated at this time. In no case does a Mira maintain an O0 class with any frequency, nor does this class correlate with pulsational phase.

A symmetric “V” shape in the K band spectral traces characterizes the O1 class. This class represents the first of the classes with the largest number of members (N = 17, or 27% of the resolved Miras in the dataset, 40% of all O(n) Miras), and is correlated to the early- to mid-spectral types for the M-type Miras. The sources of opacity in both the 2.0 µm and the 2.4 µm bands - predominately H$_2$O - tend to be equal in strength (i.e. equivalent UD angular sizes in these bands). While some stars maintain the O1 classification almost exclusively during their pulsation cycle (10% of the PTI oxygen-rich Miras), most of the Miras designated O1 will jump to the next class (O2) as they vary in Teff as a mean effect; thus, the sources of opacity change in optical depth correspondingly in the 2.0 µm channel. We have not modeled non-LTE (local thermodynamic equilibrium) effects such as shocks, turbulence and convection and are not taken into account in the variance of T$_{\text{eff}}$ with Q-class.

A left-skewed “V” shape characterizes the spectral angular diameter shape for the O2 class. While 15 Miras are assigned O2 (24% of the Mira dataset), no Mira is ever consistently within this class; every O2 Mira spends some small fraction of time in O1 and O3. Thus, the Miras assigned to this class are on average belonging to this designation. The spectral types correlated with the O2 class tend to be of mid-M class (M4 - M6). The oxygen-rich Mira S Lac$^3$ belongs to this class. Both the O1 and the O2 Mira classes lend themselves well to the current oxygen-rich monochromatic diameter models. It is believed the data herein will help refine these models, and will be compared with the analysis of Jacobs & Scholz.

A “check mark” characterizes the spectral angular diameter shape for the O3 class. This class contains only 7 stars (11% of the Mira dataset), and is roughly correlated to late M-type Miras (M7 - M10). While members of the O1 and O2 class may show characteristics of the O3 spectral shape briefly during their pulsation cycles, the stars assigned to O3 are consistently within this latter class. Each star in the O3 class is believed to be heavily enshrouded in H$_2$O and other sources of opacity, which can account for the later spectral type correlation. The star R Tri belongs to this class. These stars, in general, are anticipated to have low temperatures (T$_{\text{eff}}$ ~ 2000K). As a result, opacity due to H$_2$O would become significant at the longer wavelengths in the K band while at the same time masking those effects at the shorter wavelengths. This could account for the minor differences between the 2.0 µm and continuum sizes and yet show dramatic differences between the continuum and 2.4 µm sizes.

3.2. The C(n) classes

The C0 class tends to be of a tilde (“~”) shape, tilted with a positive slope with respect to angular size across the K band. Only 2 of 10 carbon stars are assigned exclusively to this class, and due to low-number statistics it is marginal that this class should exist. This class differs from the C1 class (see following) in that the 2.0 µm and 2.4 µm angular sizes follow shallower slopes than the band center diameters. It is not clear whether atmospheric chemistry is truly responsible for this effect in the spectral angular diameter dataset. It may be that the C1 class may be redefined in the future to absorb this class.

A smooth positive slope with angular size across the K band characterizes the C1 class. There are 16 stars which occupy this class (26% of the Mira dataset), all of which are carbon-rich or intermediate (S-type) Miras. As demonstrated by Thompson et al. in the case of RZ Peg, the sources of opacity that give rise to this positive slope are carbon molecules such as CN, HCN, C$_2$H$_2$ and CO; RZ Peg is classified in this group.
Based on the PTI spectral angular diameter dataset alone, it appears that S-type Miras are indistinguishable from their carbon-rich counterparts. This may be reflected in the fact that many S-type stars are given subtypes such as “SC” or “CS”, which denotes a further stratification of the Mira chemical classes. 13 Where the S-type Miras do depart from their carbon-rich counterparts will be evident in a later section of this article. Due to the intermediate nature of the S-type Mira (C/O \sim 1), the evolutionary path these stars follow remains widely debated (See, for example, the discussion by van Belle et al. 14)

It should be noted that those Miras that display O3 spectral characteristics might be given a C1 classification when the numerical assignments by Thompson 6 are applied. This is due to the nature of the sensitivity of the denominator (\theta_{2,0} - \theta_{2,2}). The O3 class is defined as having the denominator positive in value, with the C1 class having a negative value. Due to the \theta_{2,0} / \theta_{2,2} ratio for both these classes are relatively close to unity (1.04 and 0.96, respectively), atmospheric conditions and unknown sources of systematic errors for a given night may allow a star to seemingly switch between these two classes suddenly. A more rigorous definition of the Q-factor by Thompson may lead to the reduction of this sensitivity between these two classes.

As seen in Thompson et al., 3 both RZ Peg and S Lac were determined to have the greatest atmospheric extension near phase 0.9. This result supports the postulate that Miras with an appreciable photosphere would appear more atmospherically extended at maximum than at minimum light. 15 In an effort to compare the Q-classes side-by-side, normalizing the narrowband angular diameters to their continuum (2.2 \mu m) sizes would enable such a comparison. By performing this normalization, the relative angular sizes within each Q-class would show just how atmospheric extension relates to pulsation phase.

### 3.3. Diameter Ratios and Phase Dependence

The narrowband UD angular size data for each of the Miras are first divided by their continuum size for each nightly ensemble average of UD angular size. The pulsation phase for each Mira's nightly ensemble average is then computed. Using the Q-class assignments, the relative UD angular sizes are then binned by pulsation phase and averaged to produce a “meta-star” that incorporates all the properties of the Miras within that Q-class.

The binning technique insures that no single Mira star dominates a particular phase bin. This is important in the establishment of global properties of a specific Q-class. Given that the S-type stars have the same spectral angular diameter characteristics as the carbon-rich stars, these intermediate Miras were folded into the carbon Mira sample. Thus, four distinct Q-classes can be compared: O1 - O3 and “CS”. (Recall that the O0 class does not hold consistently for any star in the sample, nor does C0.)

The results of this technique are depicted in Figs. 3 and 4, for the 2.0 \mu m and the 2.4 \mu m size ratios, respectively. There are four lines to each plot, each one representing the ratio to the continuum size (the continuum ratio is unity, and not displayed) of four distinct classes. Effects due to continuum pulsation are hence removed, and what remains are changes in addition to the pulsation for the other four bands. Error bars are shown only for two classes for clarity in each plot, and are used as representative error bars for the other size ratios. The ratio data is then duplicated by one full phase in order to display periodic nature.

When the 2.0 \mu m diameters are normalized to the continuum diameters (Fig. 3), the O1 class has maximum extension at phase 0.9, with minimum extension at phase 0.5. For the CS class, maximum extension is at phase 0.5, with minimum extension at 0.8, quite opposite of that in the O1 class. Performing the same normalization with respect to the continuum diameters for the 2.4 \mu m diameters (Fig. 4), both the O1 and CS classes have maximum extension at phase 0.9, and minimum extension at phase 0.5. The maximum and minimum extension recedes with respect to phase for the three oxygen-rich (O1, O2, O3) classes.

The technique of creating a “meta-star” for each Q-class is useful in determining the atmospheric extension with respect to the continuum pulsation as a function of phase. It has been shown that three of the four Q-classes of Miras do in fact have their greatest extension just before maximum visual light. Additionally, the justification of separating out the oxygen-rich stars into three distinct Q-classes is apparent in Fig. 4, with each O(n) class having slightly different cycle characteristics owing to differing abundances of opacity sources in their atmospheric chemistry. It should be stressed, especially in the case of the O2 class, that final Q-class designations were assigned if the star spent more than a majority of its pulsation cycle in that class.
Figure 3. Angular size ratios with respect to the 2.2 μm continuum angular diameter. The variation in the 2.0 μm size is comparable with that of the continuum, with the exception of the O1 class which shows variability in excess of the continuum. Phases beyond 0.9 are duplicated. Error bars are given only for O1 and CS for clarity.

Figure 4. Same as Fig. 3, but for the 2.4 μm channel. A clear dependence with phase is seen for all classes. Note that the 2.4 μm ratio minima recedes for the sequence O1–O3.

4. ESTIMATION OF ANGULAR SIZE WITH SPECTRAL TYPE

The $V$-$K$ color-size relation for Miras by van Belle$^{16}$ (hereafter vB99) was utilized in the Mira program planning for the purposes of estimating when targets would be in the resolution range of PTI. The relation for Miras by vB99 folded in empirical angular size data of semi-regular variables, Mira variables and carbon stars, with the latter broadly classified to be “variable” in this sense. Thus, the relation for “Mira variables” defined by vB99 is not strictly applied to Miras in that analysis, but rather “evolved variable stars”. This section attempts to utilize the PTI dataset to refine the $V$-$K$ vs. angular size relationship strictly for Miras.

As in the method described by vB99, K-band photometry is taken from Gezari et al.$^{17}$ While Mira variables
can change their brightness in the $K$ band by 1.5 magnitudes, this analysis will assume a static $K$-band brightness. (vB used contemporaneous photometry from various sources, as well as those published by Gezari.) The $V$-band photometry for each Mira is estimated using the best-fit sinusoid to the AFOEV data. Angular sizes in the $K$ band are taken from the 2.2 $\mu$m narrowband dataset, and represent the ensemble average for a given night per star. A best-fit function was determined ($\chi^2$ minimized) for the data within each of four Mira groups (KM, O, S and C) and is of the form

$$\log(\theta_{V=0}) = a(V - K) + b$$

which is of the same form as vB99. The best-fit parameters of four Mira groups based on the PTI data set are given in Table 1, with that of vB99 restated for comparison. The best-fit relationships for each of the four Mira groups are plotted in Fig. 5, along with the evolved-star relationship found by vB99. The class “KM” refers to four Miras in the sample for which no definite spectral type could be assigned, and have generally shorter periods than those in the oxygen (O) class. (The “KM” class was folded into the oxygen (O) class for all other analyses herein.) The S-type Mira S Lyr was excluded in the fit for that class, as the radius of this star is far larger than any other star in the entire PTI sample ($R=900$ $R_\odot$). Additionally, S Lyr has the largest $K-[12]$ color in the sample.

Table 2. The best-fit parameters of the PTI Mira data set to Eq. 1 for each of the four Mira groups, with that of van Belle\textsuperscript{16} restated for comparison. The fit for S-type Miras excludes S Lyr.

<table>
<thead>
<tr>
<th>Group</th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>KM</td>
<td>0.214±0.007</td>
<td>0.879±0.043</td>
</tr>
<tr>
<td>O</td>
<td>0.207±0.002</td>
<td>0.881±0.015</td>
</tr>
<tr>
<td>S</td>
<td>0.195±0.003</td>
<td>0.911±0.028</td>
</tr>
<tr>
<td>C</td>
<td>0.219±0.006</td>
<td>0.838±0.044</td>
</tr>
<tr>
<td>vB99</td>
<td>0.218±0.014</td>
<td>0.789±0.119</td>
</tr>
</tbody>
</table>

Figure 5. Predicted angular sizes based upon $V$-$K$ colors for the PTI Mira data set, as compared to that found for Miras by van Belle.\textsuperscript{16}
physical interpretation for any global properties of the Mira dataset. One of the more interesting tests that can be applied is to examine the relationship of linear radius with period. The results are plotted in Fig. 6. A power-law of the form

$$\log\left(\frac{R}{R_\odot}\right) = a \times \log(P) + b$$

was fit to the data and resulted in a best-fit solution with $a = 0.645 \pm 0.076$ and $b = 0.929 \pm 0.188$ ($\chi^2_{\nu} = 0.25$, with S Lyr excluded in the calculations.) This power-law relationship can be inverted such that $P \sim R^{1.55 \pm 0.18}$. This result is compared to that of Fernie,\textsuperscript{24} whereby a power-law relationship was found to be $P \sim R^{1.91 \pm 0.10}$ for Miras. Fernie determined Mira stellar radii using the method described by Sanford\textsuperscript{25}; it was assumed that for any two phases of a Mira, the color index is constant (as in Cepheids). Thus, color indices combined with radial velocity measurements were used to compute linear radii. Fernie used a relationship for Miras applicable to Cepheid variables, which assumed the atmospheric layers were in LTE. These physical assumptions do not apply to Miras, which have extended atmospheres. Additionally, color indices for Miras vary dramatically throughout pulsation phases, and effects due to the high degree convection in Miras were are not taken into account.

Conversely, Cepheids tend to be more compact than Miras. Pulsational period is coupled to rotational period via angular momentum arguments as stated in a refinement one year later by the same author.\textsuperscript{26} The product of mass and radius remains constant for Cepheids when determining the pulsation constant, owing to the more compact nature of these stars (i.e. better approximated as a rigid rotator). Such is not the case in Miras, as atmospheric extension will result in much lower densities than Cepheid atmospheres, and may result in differential rotation of the outer layers with those more associated with the stellar atmosphere (decoupling of inner and outer layers). Thus, the power law empirically determined by the PTI Mira dataset is expected to be lower than that determined for Cepheids by Fernie.\textsuperscript{26}

Almost two decades later, Wilson\textsuperscript{27} comments briefly as to the nature of a period-radius relationship in Miras. It is interesting to note that the work of Feast\textsuperscript{28} depicted by Wilson bears a similar slope to that of lines for differing pulsation constants in Fig. 6 ($a \approx 1$ in Eq. 4). Wilson then compares this to the luminosities of Eggen\textsuperscript{29} and the effective temperature scale of Ridgway et al.,\textsuperscript{30} the latter work was based on empirical
angular size measurements using the lunar occultation method. The resulting slope using this method was \( a \approx 0.64 \), and compares well to that found for the PTI Mira analysis. It would be interesting to see how the P-L relations of Feast et al.,22 when translated to linear radii, would compare to the PTI dataset.

To explore possible pulsation modes of the Miras, the analysis of Creech-Eakman & Thompson31 is applied to the 2.2 \( \mu m \) narrowband linear radii, whereby the same dataset depicted in Fig. 6 is overlaid with the relations found in Ostlie & Cox32 for Miras pulsating in the fundamental and first-overtone modes such that

\[
\log(P_0) = 1.92 - 0.73 \log(M) + 1.86 \log(R) \tag{5}
\]

\[
\log(P_1) = 1.60 - 0.51 \log(M) + 1.59 \log(R) \tag{6}
\]

where \( M \) and \( R \) are in solar units and \( P \) in days. These relations and the data are depicted in Fig. 7. While there are roughly even numbers of Miras falling between the fundamental and first-overtone lines, the error bars are too large for any conclusive answer to the pulsation mode question. This is a result of the assumed 25% error bars placed upon the distance estimates. Until better distance values for Miras are found, the question of pulsation mode - which has been debated for decades - cannot be definitively answered in this way.

![Figure 7](image_url)

**Figure 7.** The PTI mira dataset, as compared with the fundamental and first-overtone relations of Ostlie & Cox.32 Symbols are the same as in Fig. 6.


Mid-infrared data on the PTI Miras were obtained via the IRAS database and converted to magnitudes using the relation by Hickman, Sloan & Cantera33 such that \([12] = -2.5 \log(f(12\mu m) / 28.3 \text{ Jy})\). The magnitudes obtained have not been dereddened for extinction or spectral type, which for both cases can be considered negligible at this wavelength. The K-band magnitudes were obtained from Gezari et al.17 Uncertainties for the K-band magnitudes are assumed to be 0.4 mag (due to pulsation). (A mean of the [12] errors was calculated, with a value of 6%.) As seen in Fig. 8, the K-[12] colors are plotted versus linear radius. Error bars for the color index are not shown for clarity in the plot. Different relationships exist for the O and CS types. The slopes of the two lines differ, and there is a linear offset between the two groups. The \( \chi^2 \) for the CS and O best-fit lines are 0.9 and 1.2, respectively.
Figure 8. K-[12] color-radius relation for PTI Miras. When the O- and CS-types are treated separately, two different relationships are seen. The CS group maintains a higher slope as well as a larger linear offset.

The $K$-[12] color is a tracer of hot dust or gas from recent mass loss. As the $K$-[12] color index increases, the more dust-enshrouded the star. It would stand to reason that, since Miras are usually in a continual process of mass loss, both the CS and the O types might have similar relationships. However, the mean radius for the sample of CS types ($N = 18$) is $363 \pm 103 \ R_\odot$, and excludes S Lyr; the O types ($N = 43$) maintain a smaller mean radius of $318 \pm 96 \ R_\odot$. The P-L relations of Feast et al. $^{22}$ and Groenewegen & Whitelock $^{23}$ depart by $< 1\%$ from each other, which corresponds to an average distance - radius difference of $< 4\%$. Thus, this linear shift may be a result of the CS stars being larger on average (hence, larger mass loss) than the O stars. The $K$-[25] vs. radius (not shown) also follows a similar trend, as the [12] and [25] magnitudes for Miras have a definite linear relationship to each other. The [25]-[60] vs. radius relation (not shown) does depict a clustering of oxygen-rich Miras for [25]-[60] $< 0.1$, with carbon-rich and S-types occurring thereafter, which may be indicative of older mass loss events or longer-lived oxygen-rich dust. The $K$-[60] vs. radius (not shown) show no clustering effect, and both [60] relations have either marginal or no statistical significance with radius.

An alternate explanation as to the differing $K$-[12] vs. radius relations is presented. Oxygen-rich Miras are more dusty at 12 $\mu$m than the carbon-rich Miras, given the silicate features near this wavelength. $^{34,37}$ As oxygen atoms react effectively with carbon to create CO, there is no excess of oxygen to form silicates in a C-rich environment. Additionally, oxygen-rich Miras are emperically smaller in the $K$ band than the carbon-rich counterparts. The atmosphere at 12 $\mu$m is closer to the 2.2 $\mu$m atmosphere for the case of oxygen-rich Miras than for those of carbon type. In this case, oxygen-rich Miras do not loft dust as high into their outer atmospheres than for carbon-rich Miras, and as such, the dust-forming regions in O-rich stars are more centrally located and closer to the detached molecular atmospheres. This outer region would be more dense than for those of carbon type, and hence shock waves would tend to be stronger for O-rich than for C-rich types. $^{36}$

7. CONCLUSIONS

Interim results of K-band visibility data collected using the Palomar Testbed Interferometer (PTI) for over 60 Mira variables are presented. Important conclusions are summarized as follows.

- Multi-epoch data of Mira variables are used to determine linear pulsation amplitude and its relation to visible phase, as well as mean linear radii determination.
• By dispersing the K-band into five individual spectral channels, visibility measurements in narrower bandwidths provide insight to the chemistry and spatial extent of opacity sources above continuum photospheres.

• A spectral diameter classification system of the vast dataset is presented. Oxygen-rich Miras are separated into three distinct sub-classes (O1, O2, O3), which correlates to spectral type and degree of opacity of the outer atmospheres. Carbon-rich Miras are indistinguishable from the intermediate S-type Miras in the visibility data, for which both are mostly categorized by one sub-class (C1).

• A refined relationship between $V-K$ color and angular size for Mira variables is presented.
  - Oxygen-rich Miras: $\log(\theta_{V=0}) = (0.207\pm0.002) * (V-K) + (0.881\pm0.015)$
  - S-type Miras  : $\log(\theta_{V=0}) = (0.195\pm0.003) * (V-K) + (0.911\pm0.028)$
  - Carbon-rich Miras: $\log(\theta_{V=0}) = (0.219\pm0.006) * (V-K) + (0.838\pm0.044)$

• An empirical period-radius relationship, using the 2.2 $\mu$m narrowband diameters, is found such that
  - $P \sim R^{1.55\pm0.18}$

• Due to the large uncertainty in distance determination, pulsation modes of the PTI Miras could not be adequately established.

• A $K-[12]$ relationship with linear radius is found for oxygen-rich and carbon-rich/S-type stars, such that
  - C+S stars: $R/R_\odot = (96\pm30) * (K-[12]) + (178\pm54)$
  - O stars  : $R/R_\odot = (60\pm20) * (K-[12]) + (156\pm38)$

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