

AN OPTIMAL MODIFICATION OF A TIME SCALE KALMAN FILTER

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The Kalman filter in question was developed in the early 1980s by Jones and Tryon, and was implemented in the time scale algorithm TA(NIST). This algorithm produced time scales with poor short-term stability. By incorporating a simple modification of the Kalman covariance matrix into the algorithm, we can produce time scales with good stability for all averaging times, as verified by simulation.

For simplicity we treat ensembles of clocks whose noise is a sum of phase and frequency random walks with known noise levels; the results also hold for clocks with random walk of drift. The measurements consist of the phase differences of all clock pairs at a sequence of dates. Because the measurements are assumed to be noiseless, the *corrected clocks*, i.e., the times of the physical clock pulses minus the Kalman phase estimates, are all the same; this common value is called the *raw Kalman scale*. Simulations confirm its poor short-term stability and show that the error variances of the Kalman phase estimates grow fast. To obtain a better time scale, we can neglect the Kalman phase estimates and use only the Kalman frequency estimates in a conventional weighted-average time scale, with clock weights inversely proportional to the phase process-noise variances. The resulting scale, called Kalman-plus-weights (KPW), has good long-term and short-term stability.

The covariance modification, called *x-reduction*, consists of zeroing out the rows and columns that correspond to phase states. By carrying out x-reduction after each measurement update, while otherwise following the standard Kalman algorithm and using the corrected clocks as before, we obtain another time scale, called the *reduced Kalman scale*. We have proved the following assertions: 1) the raw and reduced scales have the same Kalman frequency estimates; 2) the two scales are both weighted-average scales, but with different weights; 3) among all weighted-average scales that use the Kalman frequency estimates (such as KPW), the weights of the reduced scale are those that minimize the variance of the ensemble time increment from one measurement date to the next. The optimal weights need not be computed explicitly; they are automatically implied by the x-reduced Kalman filter. For the ensembles we have simulated, the reduced scale has slightly better overall stability than the KPW scale.

This work was carried out by the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration.