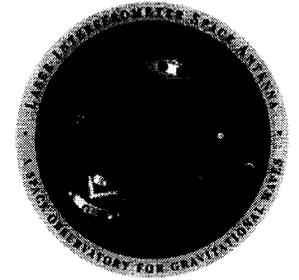


Laser Interferometer Space Antenna (LISA)



Time-Delay Interferometry for LISA

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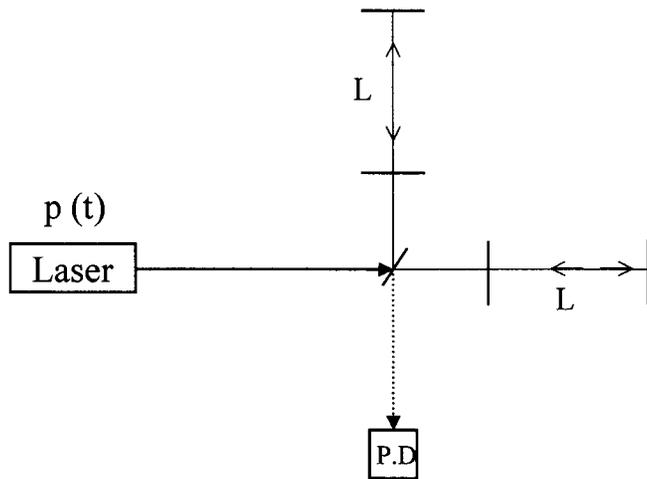


Earth vs. Space-Based Interferometers

- Earth-based interferometers have arm lengths essentially equal. This is in order to directly remove laser frequency fluctuations at the photodetector, where the two beams interfere.
- They operate in the long-wavelength limit ($L \ll \lambda$).
- By contrast, LISA will have arm lengths significantly different ($\Delta L/L \sim 10^{-2}$), with $L = 5 \times 10^6$ km.
- Over much of its sensitivity frequency-band, it will **not** operate in the long-wavelength regime.
- Time-of-flight delays in the response to the wave, and travel times along the beams in the detector must be allowed for, in order to derive a correct theory of the detector response.

Statement of The Problem

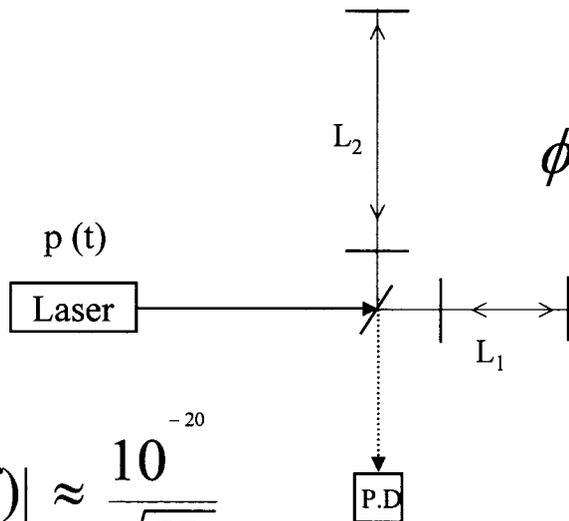
$p(t)$ = Laser phase fluctuations



$$\frac{1}{2\pi\nu_0} \frac{dp(t)}{dt} = \left[\frac{\Delta\nu(t)}{\nu_0} \right]_{\text{Laser}} \equiv C(t)$$

$$\phi_1(t) = h_1(t) + p(t - 2L_1) + n_1(t)$$

$$\phi_2(t) = h_2(t) + p(t - 2L_2) + n_2(t)$$

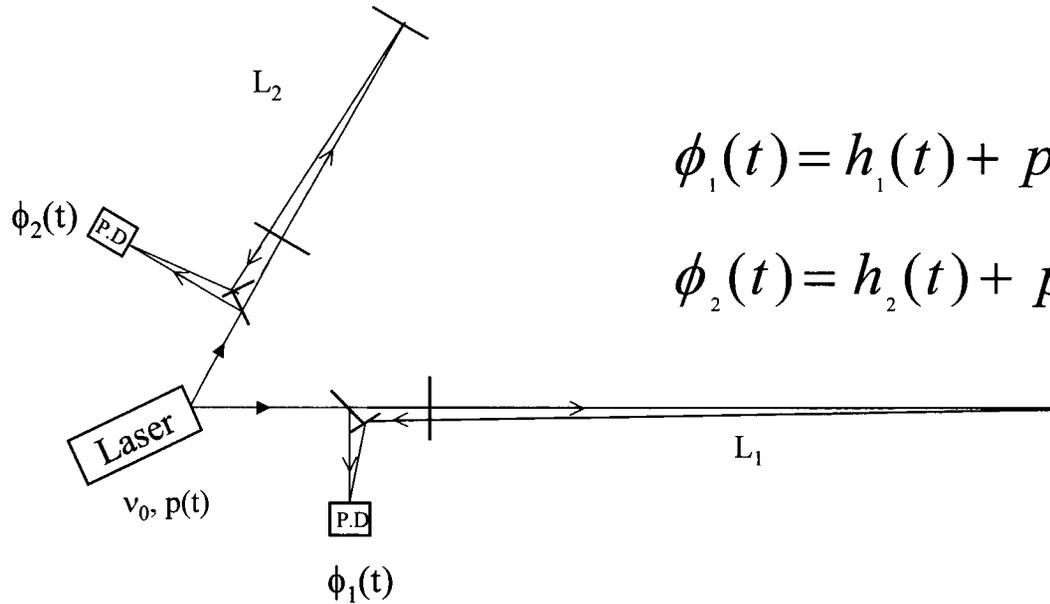


$$\phi_1(t) - \phi_2(t) \Rightarrow p(t - 2L_1) - p(t - 2L_2) \cong 2 \frac{dp}{dt} \varepsilon L_1$$

$$|\tilde{h}(f)| \approx \frac{10^{-20}}{\sqrt{\text{Hz}}}$$

$$|\tilde{C}(f)| \approx \frac{10^{-13}}{\sqrt{\text{Hz}}}, \quad \varepsilon \cong 3 \times 10^{-2} \Rightarrow \frac{5 \times 10^{-16}}{\sqrt{\text{Hz}}}$$

Unequal-arm Interferometers



$$\phi_1(t) = h_1(t) + p(t - 2L_1) - p(t) + n_1(t)$$

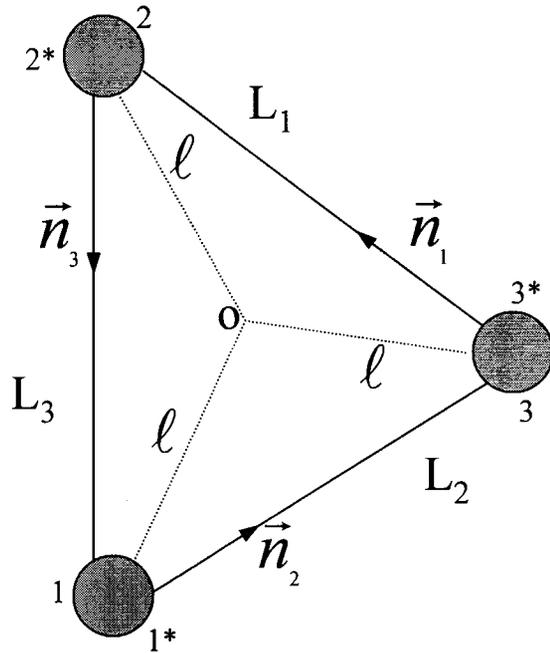
$$\phi_2(t) = h_2(t) + p(t - 2L_2) - p(t) + n_2(t)$$

$$\phi_1(t) - \phi_2(t) = h_1(t) - h_2(t) + p(t - 2L_1) - p(t - 2L_2) + n_1(t) - n_2(t)$$

$$\begin{aligned} \phi_1(t - 2L_2) - \phi_2(t - 2L_1) &= h_1(t - 2L_2) - h_2(t - 2L_1) + \\ & p(t - 2L_1) - p(t - 2L_2) + n_1(t - 2L_2) - n_2(t - 2L_1) \end{aligned}$$

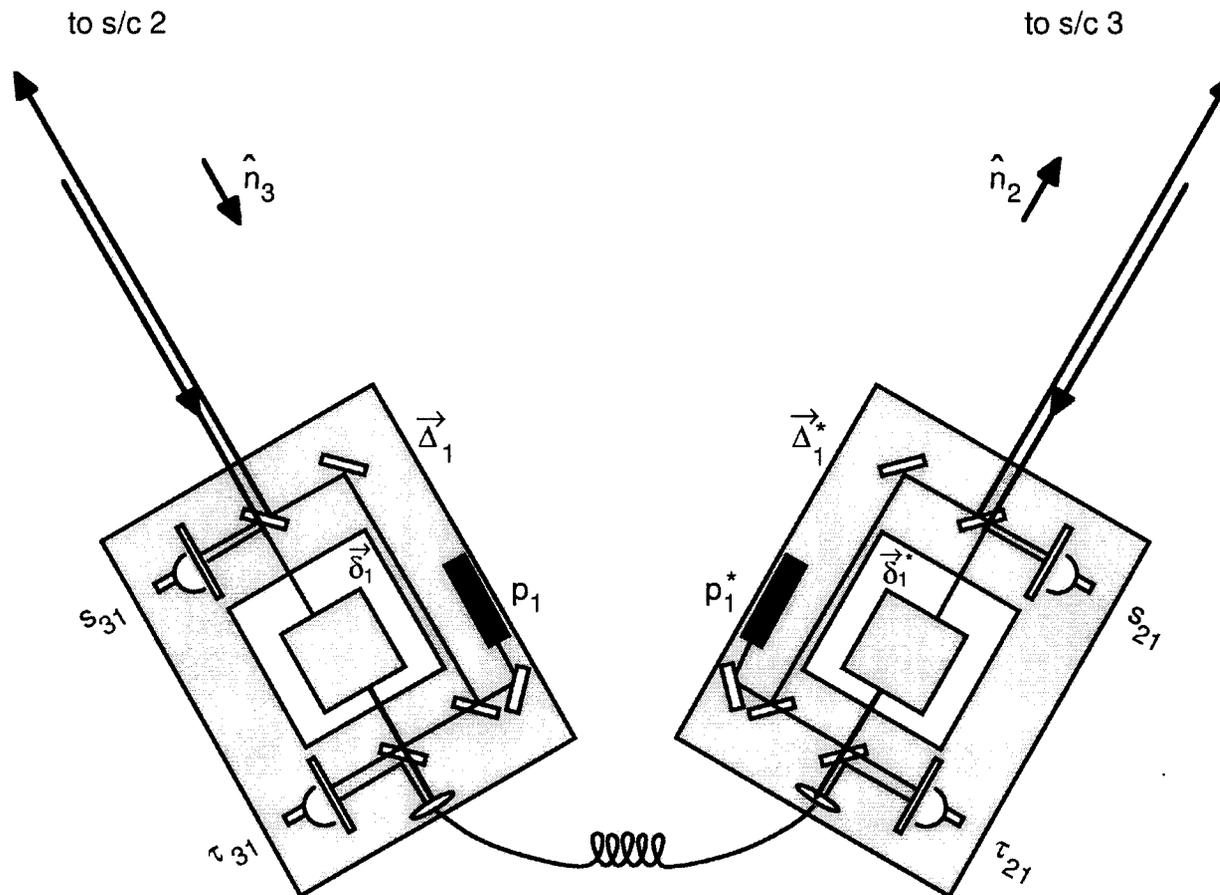
$$X(t) \equiv [\phi_1(t) - \phi_2(t)] - [\phi_1(t - 2L_2) - \phi_2(t - 2L_1)]$$

Time-Delay Interferometry (T.D.I.)



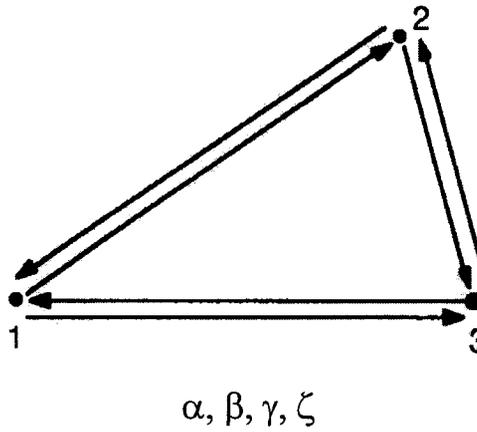
- It is best to think of LISA as a closed array of six one-way delay lines between the test masses.
- This approach allows us to reconstruct the unequal-arm Michelson interferometer, as well as new interferometric combinations, which offer advantages in hardware design, in robustness to failures of single links, and in redundancy of data.

Time-delay Interferometry & The Drag-free Configuration

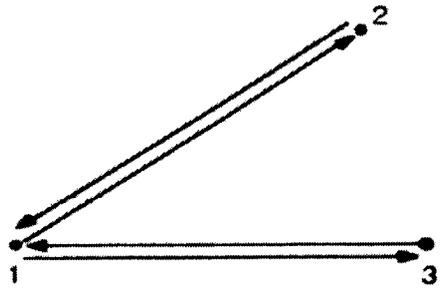


F.B. Estabrook, M. Tinto, and J.W. Armstrong: *Phys. Rev. D*, **62**, 042002 (2000)
M. Tinto, F.B. Estabrook, and J.W. Armstrong: *Phys. Rev. D*, **65**, 082003, (2002)

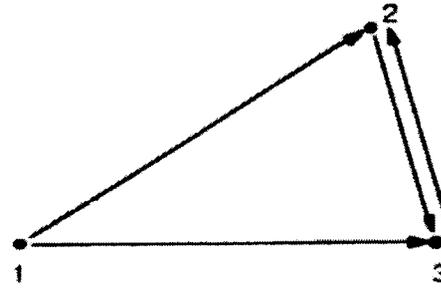
Six-Pulse Data Combinations



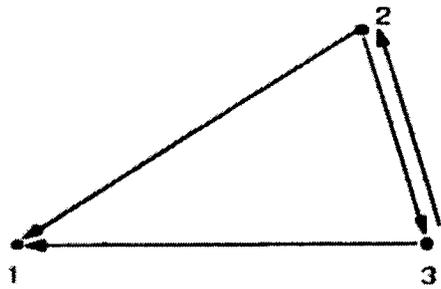
Eight-Pulse Data Combinations



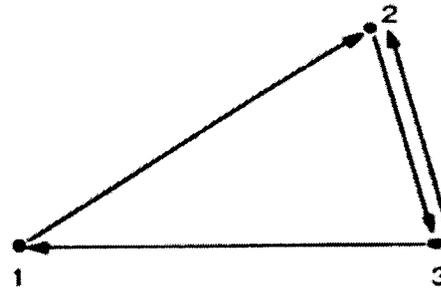
INTERFEROMETER (X, Y, Z)



BEACON (P, Q, R)



MONITOR (E, F, G)



RELAY (U, V, W)

Data Combinations (Cont.)

- There are 6 optical benches, 6 lasers, 3 Ultra Stable Oscillators (USO), and a total of 18 Doppler time series observed.
- The 6 beams exchanged between distant spacecraft contain the information about the GW signal; the other 12 signals are for comparison of the lasers, relative optical bench motions within the spacecraft, and calibration of the USO phase noises affecting the interferometric observables.
- Four combinations $(\zeta, \alpha, \beta, \gamma)$ can be regarded as the generators of the functional space of interferometric combinations.

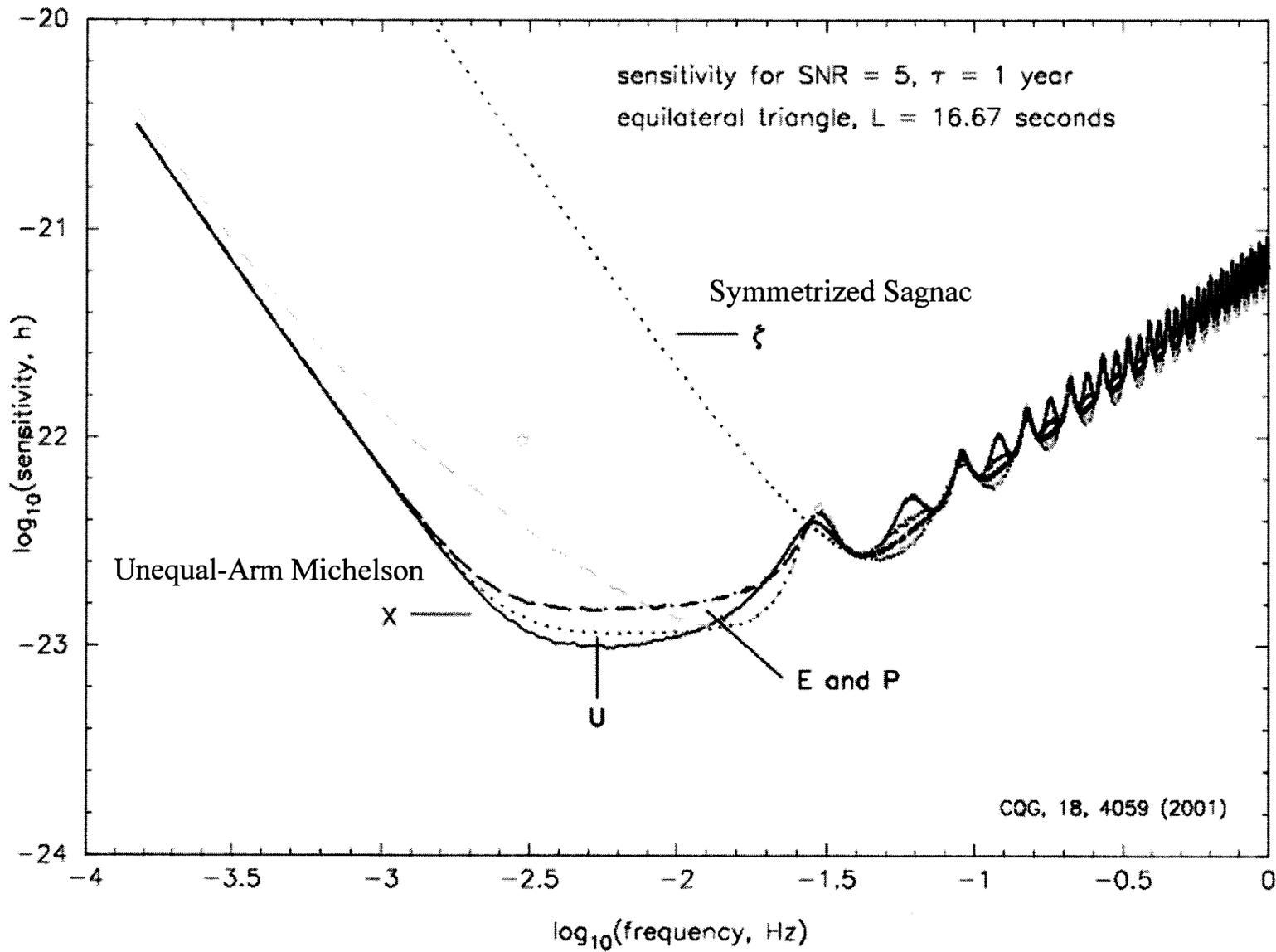
$$\zeta - \zeta_{,123} = \alpha_{,1} - \alpha_{,23} + \beta_{,2} - \beta_{,31} + \gamma_{,3} - \gamma_{,12}$$

$$X_{,1} = \alpha_{,32} - \beta_{,2} - \gamma_{,3} + \zeta$$

$$P = \zeta - \alpha_{,1}$$

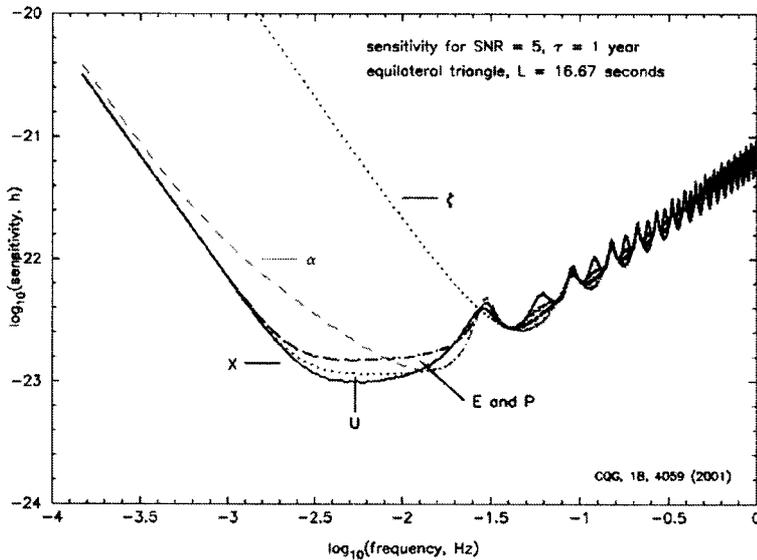
$$E = \alpha_{,1} - \zeta_{,1}$$

$$U = \gamma_{,1} - \beta$$



Optimal Interferometric Combinations

Work in collaboration with J.W. Armstrong, S. Larson, T. Prince



$$\eta = a_1(f)\tilde{\alpha} + a_2(f)\tilde{\beta} + a_3(f)\tilde{\gamma} + a_4(f)\tilde{\zeta}$$

$$\begin{aligned} [1 - e^{2\pi i f(L_1 + L_2 + L_3)}] \tilde{\zeta} = & [e^{2\pi i f L_1} - e^{2\pi i f(L_2 + L_3)}] \tilde{\alpha} \\ & + [e^{2\pi i f L_2} - e^{2\pi i f(L_1 + L_3)}] \tilde{\beta} \\ & + [e^{2\pi i f L_3} - e^{2\pi i f(L_1 + L_2)}] \tilde{\gamma} \end{aligned}$$

$$SNR_{\eta}^2 = \int \frac{|a_1(f)\tilde{\alpha}_s + a_2(f)\tilde{\beta}_s + a_3(f)\tilde{\gamma}_s|^2}{\langle |a_1(f)\tilde{\alpha}_n + a_2(f)\tilde{\beta}_n + a_3(f)\tilde{\gamma}_n|^2 \rangle} df$$

- One should regard the SNR as a functional of the functions $a_i(f)$, and extremize it with respect of them.

Optimal Interferometric...(Cont.)

$$SNR_{\eta}^2 = \int \frac{a_i A_{ij} a_j^*}{a_r C_{rt} a_t^*} df$$

$$A_{ij} = X_i^{(s)} X_j^{(s)*} ; C_{ij} = \langle X_i^{(n)} X_j^{(n)*} \rangle$$

$$(X_1, X_2, X_3) \equiv (\alpha, \beta, \gamma)$$

- The stationary values of the SNR_{η} corresponds to the stationary points of the integrand.
- Since the quadratic form \mathbf{C} is non-singular, we can identify the stationary points of the integrand by using the **Rayleigh's Principle for Quadratic Forms**: *The stationary values of the integrand are attained at the eigenvalues of the matrix $(\mathbf{C}^{-1}\mathbf{A})$*

Optimal Interferometric...(Cont.)

- Since \mathbf{A} has rank 1, it follows that also $\mathbf{C}^{-1}\mathbf{A}$ has rank 1.
- The only non-zero eigenvalue of the matrix $\mathbf{C}^{-1}\mathbf{A}$ is therefore equal to $\text{Tr}(\mathbf{C}^{-1}\mathbf{A})$. This implies the following expression for the optimal signal-to-noise ratio:

$$SNR_{\eta_{opt.}}^2 = \int X_i^{(s)*} C^{-1}_{ij} X_j^{(s)} df$$

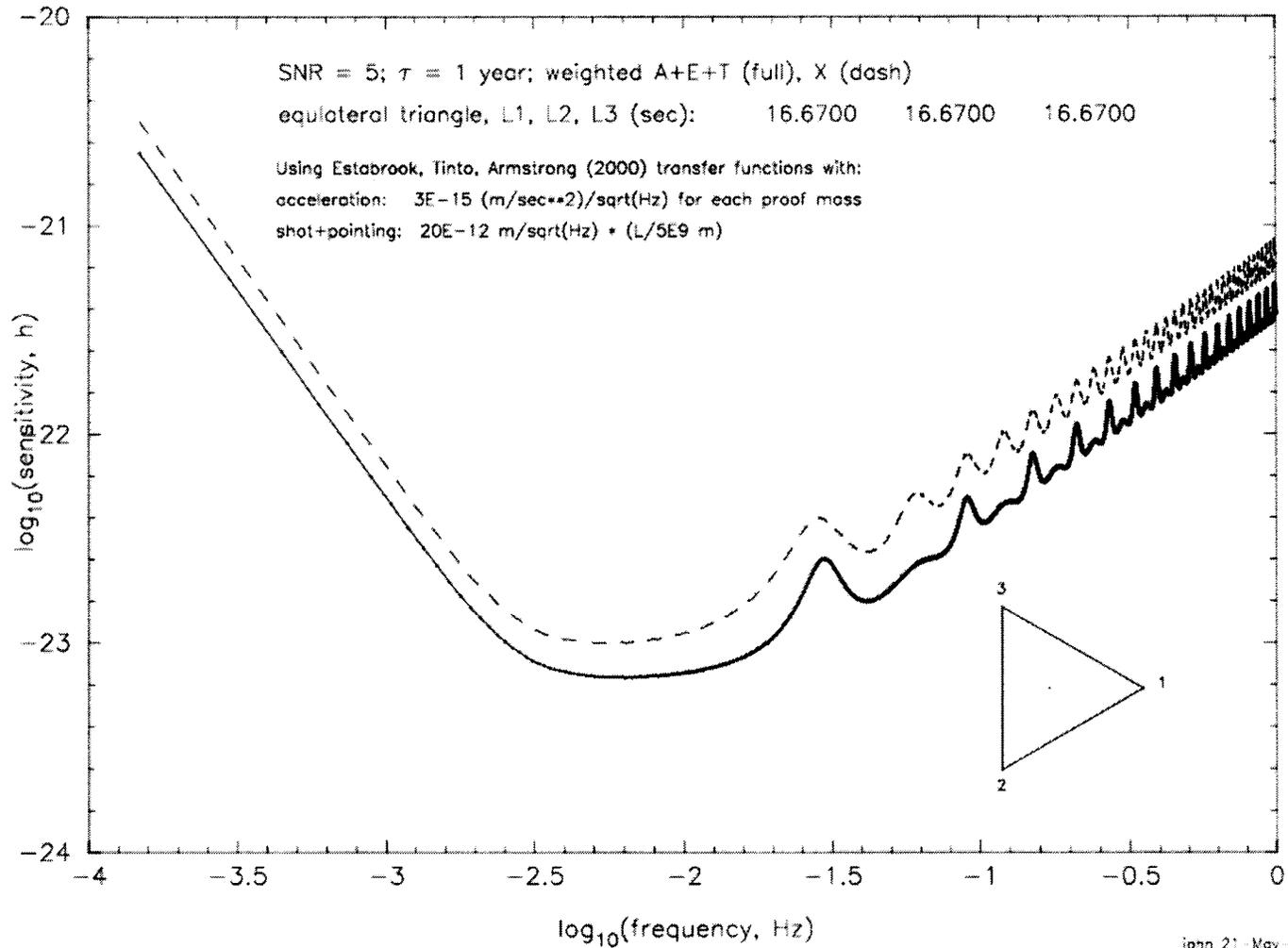
- We can change basis, and work with a triad of interferometric observables that have diagonal covariance matrix.

Optimal Interferometric...(Cont.)

$$\begin{bmatrix} A \\ E \\ T \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}}(\tilde{\gamma} - \tilde{\alpha}) \\ \frac{1}{\sqrt{6}}(2\tilde{\beta} - \tilde{\alpha} - \tilde{\gamma}) \\ \frac{1}{\sqrt{3}}(\tilde{\alpha} + \tilde{\beta} + \tilde{\gamma}) \end{bmatrix}$$

$$SNR_{\eta}^2 = \int \left[\frac{A_s A_s^*}{\langle A_n A_n^* \rangle} + \frac{E_s E_s^*}{\langle E_n E_n^* \rangle} + \frac{T_s T_s^*}{\langle T_n T_n^* \rangle} \right] df$$

Optimal Interferometric...(Cont.)



john 21 May 2002 08:09

Conclusions

- T.D.I. provides an exact method for canceling the leading noise source – laser phase fluctuations – in an interferometer with unequal, time-variable arms.
- It allows analysis of signals, noises, achievable sensitivities, and architectural design (including system-level tradeoffs between, e.g. Laser stability, arm length accuracy, stability of optical bench, Doppler shifts due to chosen orbits, USO stability).
- It provides robustness of the mission with respect to failures of subsystems.
- It shows existence of alternate LISA configurations offering potential design, implementation, or cost advantages.
- It gives a data combination (ζ) for assessing the LISA on-orbit instrumental noise performance.
- The LISA optimal sensitivity can be obtained by regarding LISA as a network of three Interferometers.