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Title: All-sky search for gravitational waves

Abstract: We present a set of general tools to search for multiparameter gravitational-wave signals buried in the noise of a detector. Our analysis is based on the maximum likelihood detection method that is related to the matched-filtering technique which is the optimal method in the sense that it maximizes the probability of detection of the signal. We apply our techniques to perform an all-sky search for gravitational-wave signals from spinning neutron stars in the data of the EXPLORER bar detector. We present the results of our search.
All-sky Search for Gravitational Waves

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1. Response of the detector to a continuous GW signal

\[ h(t) = A_2 a(t) \cos \phi(t) + A_2 b(t) \cos \Phi(t) + A_3 a(t) \sin \phi(t) + A_3 b(t) \sin \Phi(t) \]

\[ A_0 \sim h_0 = 4.2 \times 10^{-23} \left( \frac{c}{10^5 \text{omin}^2} \right) \left( \frac{1 \text{kyr}}{r} \right)^2 \]

\[ \Phi(t) = \sum_{k=0}^{s_2} \frac{t^{k+1}}{k!(k+1)!} + \frac{n_0}{c} \sum_{k=0}^{s_2} a_k \frac{t^k}{k!} \]

\[ \Phi_s(t) = p_0 t + \sum_{k=1}^{s} p_k t^{k+1} + A \cos (N_0 t) + B \sin (N_0 t) \]

Linear model of the phase

2. Maximum likelihood detection

\[ x(t) = n(t) + h(t) \]

data noise signal

\[ \mathcal{F} = \frac{2}{S_h(t_0)} \left( \frac{|F_a|^2}{<a^2>} + \frac{|F_b|^2}{<b^2>} \right) \]

\[ F_a = \int_{t_0}^{T_0} x(t) a(t) \exp [-i \Phi(t)] dt \]

In the case of a linear phase model \( \Phi \) is a homogeneous random field

\[ C(\tau) = \frac{1}{2} C(0) \]

characteristic correlation time

\[ N_{eff} = \frac{V_{parametric space}}{V_{all}} \]

\[ P_F (N_0) = 1 - (1 - P_F (N_0))^N_0 \]
3. Grid of templates in parameter space

\((p_0, p_2, A, B)\)

4-dimensional prism, the two bases of which are 3-dimensional hexagonal prisms.

The size of the prism is constrained by the fitting factor (FF) of the FFT.

\[
\text{FF for FFT is } \sim 0.63 \\
\text{FF for twicet as fine} \\
\text{a Fourier grid is } \sim 0.90
\]

4. Two-step search procedure

1. Coarse search

2. Fine search \(\sim\) Nelder-Mead simplex algorithm

(Can be hundreds, has been varied)
Analysis of the EXPLORER detector

Explorer bar detector is ran by Italian collaboration called ROC and based in Rome.

Explorer has collected year of data with good sensitivity and high duty cycle (eg. 75% in 1991).

1. Parameters of the search

\[ f_0 = 922 \text{ Hz} \quad A_\nu = 0.74 \text{ Hz} \quad T_0 = 2 \text{ days} \]

Parameters of the filter: frequency, 1 spindown, position in one day (2 parameter).

\[ \text{No. of filters:} \quad N = 1.85 \times 10^9 \quad \text{all-day, } T_{\text{min}} = \frac{f_{\text{max}}}{2 f_{\text{max}}} = 100 \text{ days} \]

Sensitivity of the search!

\[ \text{for } \text{SNR} = 1 \quad h_0 = 3.4 \times 10^{-25} \]
\[ \text{for } \text{SNR} = 3 \quad h_0 = 2.9 \times 10^{-23} \]

Search threshold: 6.6 corresponds to 99% confidence of detection.

2. Random search

Advantages

a) straightforward calculation of false alarm probability: \[ N_2 = \frac{N_1 N_p}{2} \]

b) No need to set up an elaborate grid

c) More effective coverage of parameter space than for systematic search.

d) Effective use of computing resources
3. Results of the search

9 x 10^5 trials \( (P < 0) \)

No significant signals found

We have 231 threshold crossings
We expect 228 threshold crossings
assuming due to Gaussian white noise

Verification procedure

c) Transformation to astrophysical parameters
\( (f, f_{tot}, x, \tau) \)

d) Search on a grid around astrophysical parameter

b) Search in 4-day stretch of data including the original one

c) Search in a different 2-day stretch of data
Power spectrum of EXPLORER data
\( f_{\text{dot}} = -1.3235 \times 10^{-08} \text{ Hz s}^{-1} \), \( \delta = 30.199 \text{ deg} \), \( \alpha = 302.6834 \text{ deg} \)

Signal-to-noise ratio = 7.7

Frequency [Hz]